Name:______________________________ Class #:______

8. PRACTICE FACTORING DIFFERENCES OF SQUARES

Remember the difference of square formula is given by:

$$(a^2 - b^2) = (a - b)(a + b)$$

Factor each of the polynomials given below. Use the difference of squares formula.

8A. $y^2 - 64$
8B. $4m^2 - 9$
8C. $9x^2 - 16$

Let's take a look at the difference of squares formula

subtraction

$$\underbrace{a}_{\text{perfect square}}{}^2 - \underbrace{b}_{\text{perfect square}}{}^2 = (a - b)\cdot(a + b)$$

$\boxed{\text{Problem 8A}}$

Let's transform the given expression into general form using the perfect square format:

$$y^2 - 64 = \underbrace{y^2}_{a^2} - \underbrace{8^2}_{b^2}$$

$\boxed{\begin{matrix} a = y \\ b = 8 \end{matrix}}$

$$\Rightarrow y^2 - 64 = \boxed{\underbrace{(y - 8)\cdot(y + 8)}_{(a - b)\cdot(a + b)}} \checkmark$$

4:35p – 4:39pm

Problem 8b

Let's transform the given expression into the general form. Notice

- $4m^2 = 2 \cdot 2 \cdot m \cdot m = 2 \cdot m \cdot 2 \cdot m = \boxed{(2m)^2}$ ← perfect square
- $9 = 3 \cdot 3 = \boxed{3^2}$ ← perfect square

We see that we can write this

$$4m^2 - 9 = (2m)^2 - 3^2$$

$$a^2 - b^2$$

$\boxed{\begin{array}{l} a = 2m \\ b = 9 \end{array}}$

$$\Rightarrow 4m^2 - 9 = \boxed{(2m - 3) \cdot (2m + 3)}$$

$$(a - b) \cdot (a + b)$$

$\boxed{\text{Problem 8c}}$

Let's transform the given statement

$$9x^2 - 16$$

into general form. To do so, we note

- $9x^2 = 3 \cdot 3 \cdot x \cdot x = 3 \cdot x \cdot 3 \cdot x = \boxed{(3x)^2}$ ← perfect square

- $16 = 4 \cdot 4 = \boxed{4^2}$ ← perfect square

Now we can write this:

$$9x^2 - 16 = \underbrace{(3x)^2 - 4^2}_{a^2 - b^2} \qquad \boxed{\begin{array}{l} a = 3x \\ b = 4 \end{array}}$$

$$\Rightarrow 9x^2 - 16 = \underbrace{(3x - 4) \cdot (3x + 4)}_{(a - b) \cdot (a + b)}$$

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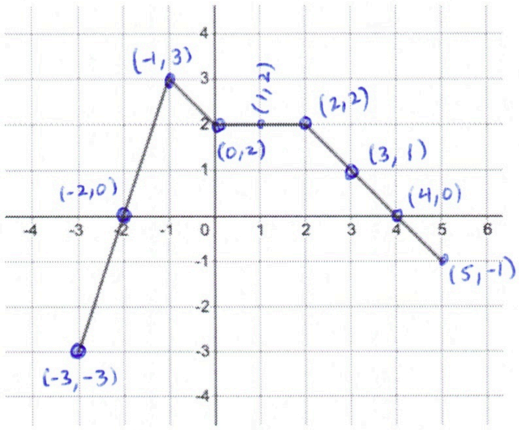
9. TRANSFORMATIONS OF GRAPHS (from Lessons 11 - 12)

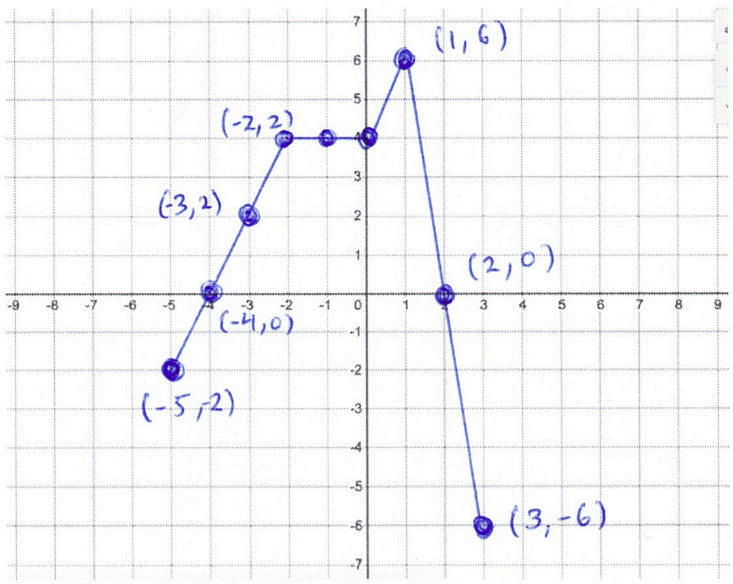
Below is a graph of a function $f(x)$.

Let's begin by analyzing the behavior of the graph of f(x). To Do so, let's identify some points on the graph.



Now let's create a table of values and a piecewise description of this function. See next pages for this work.

9A. Sketch a graph of $g(x) = 2f(-x)$.



9B. Explain, in words, the effect of each transformation in the new function $g(x) = 2f(-x)$

Notice that for the ordered pair

$$(x, g(x)) = (x, 2 \cdot f(-x))$$

$$= (\underbrace{-x}_{\substack{\text{transform} \\ \text{input into negative} \\ \text{values}}}, \underbrace{2 \cdot f(x)}_{\substack{\text{multply output} \\ \text{values by 2}}})$$

4:33am - 5:20am

Problem 9, continued ...

Let's take a look at the input-output relationship for the function $f(x)$. To this end, consider

Input-values x	output values $f(x)$
-3	-3
-2	0
-1	3
0	2
1	2
2	2
3	1
4	0
5	-1

Now let's generate this same information using Desmos.com and make a testable conjecture for the piecewise description of $f(x)$

4:37am - 4:46am

Problem 9, continued...

Below we see an image of $f(x)$ that highlights 4 distinct pieces of this function:

Piece #1: $-3 \le x < -1$

Piece #2: $-1 \le x < 0$

Piece #3: $0 \le x < 2$

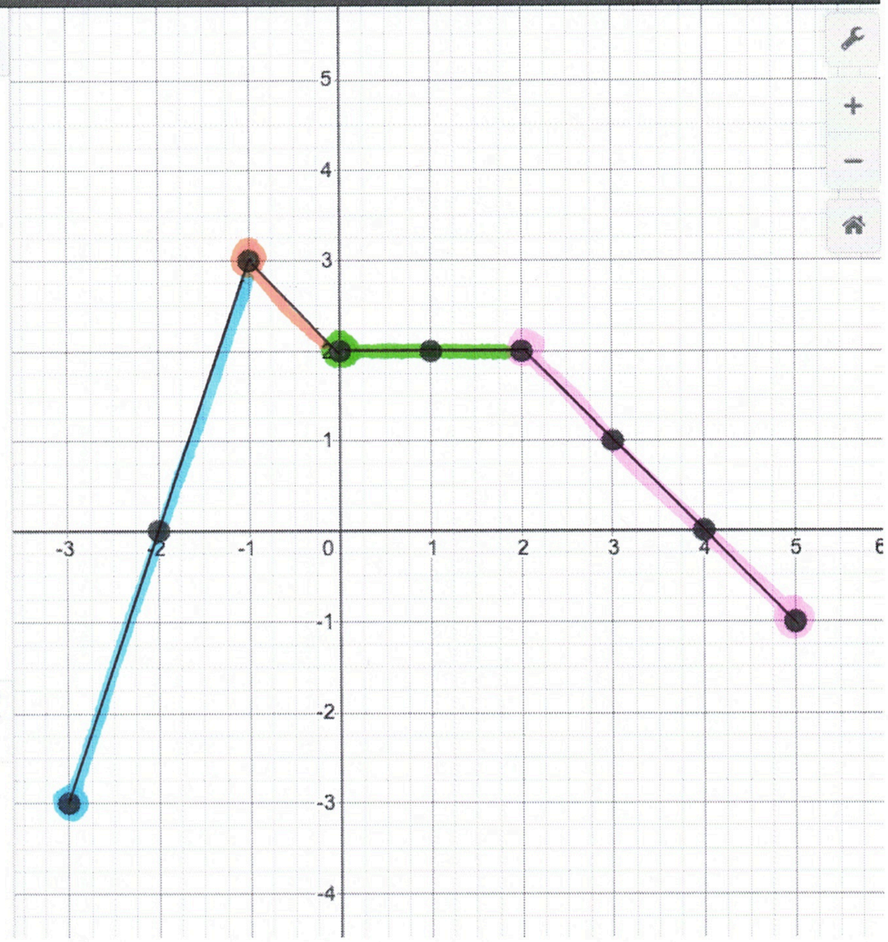
Piece #4: $2 \le x \le 5$

w21, Math 48A, Exam 3, P9

desmos

Jeff

	x	y
Piece #1 →	−3	−3
	−2	0
Piece #2 →	−1	3
Piece #3 →	0	2
	1	2
Piece #4 →	2	2
	3	1
	4	0
	5	−1



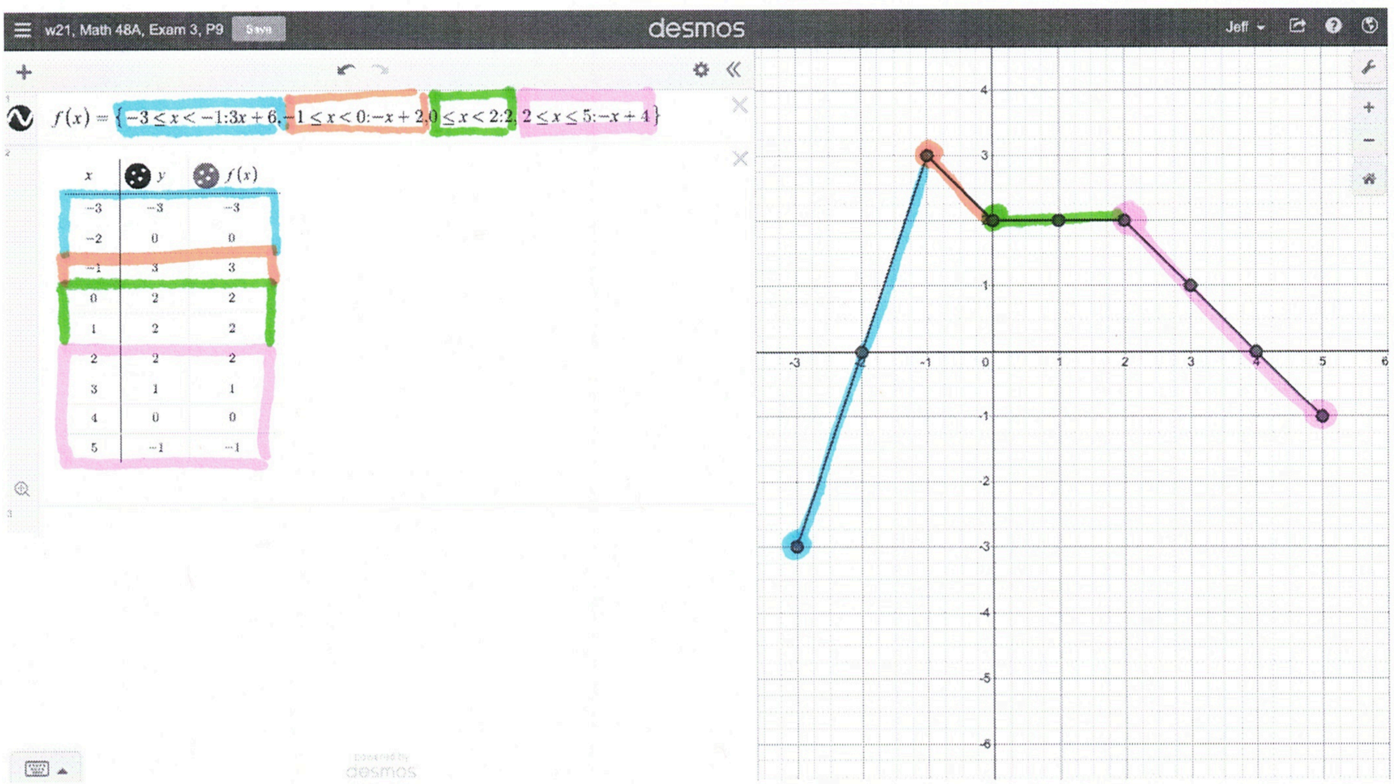
Then, we can write the piece wise definition for this function

$$f(x) = \begin{cases} 3x + 6 & \text{if } -3 \le x < -1 \quad \leftarrow \text{piece 1} \\ -x + 2 & \text{if } -1 \le x < 0 \quad \leftarrow \text{piece 2} \\ 2 & \text{if } 0 \le x < 2 \quad \leftarrow \text{piece 3} \\ -x + 4 & \text{if } 2 \le x \le 5 \quad \leftarrow \text{piece 4} \end{cases}$$

4:46am - 4:54am

Problem 9, continued...

Now we confirm using Desmos.com that our guess for f(x) was valid (see screen shot below):



Now that we've come up with an equivalent description for the outputs of f(x), we have a mechanism to make guesses (conjectures) about the shape of the graphs for problems 9a - 9d and check our work quickly.

4:55am - 5:02am

Problem 9 continued ...

Let's continue with the two shifted functions from this problem given by

$$g(x) = \underbrace{2}_{\substack{\text{vertical} \\ \text{stretch} \\ \text{by factor} \\ \text{of 2}}} \cdot f(\underbrace{-x}_{\substack{\text{Horizontal} \\ \text{reflection} \\ \text{about y-axis}}})$$

$$h(x) = \underbrace{3}_{\substack{\text{vertical shift} \\ \text{upward by} \\ \text{three units}}} \underbrace{-}_{\substack{\text{vertical reflection} \\ \text{about x-axis}}} f(\underbrace{x+2}_{\substack{\text{horizontal shift} \\ \text{towards left (negative direction)} \\ \text{by 2 units}}})$$

We can use Desmos.com to confirm these descriptions and draw our desired graphs.

5:03am – 5:10am

Problem 9a, continued...

Below we see the graph of $g(x) = 2 \cdot f(-x)$

combined with the associated table of values.

Notice we've hidden all other data that is not related

to $g(x)$.

w21, Math 48A, Exam 3, P9 Save desmos Jeff

1 $f(x) = \{-3 \le x < -1: 3x+6, -1 \le x < 0: -x+2, 0 \le x < 2: 2, 2 \le x \le 5: -x+4\}$

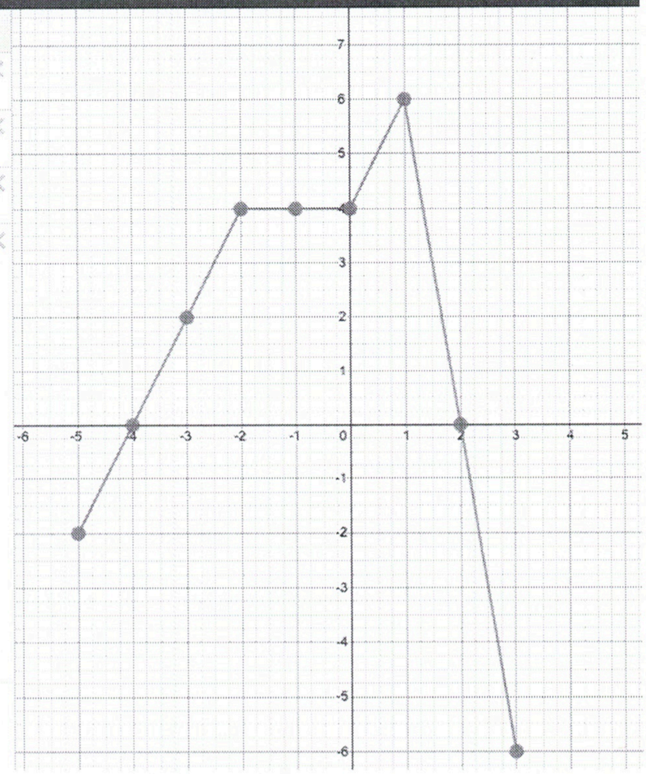
2 $g(x) = 2 \cdot f(-x)$

3 $h(x) = 3 - f(x+2)$

4

x	y	$f(x)$	$g(x)$	$h(x)$
−5	....	undefined	−2	6
−4	....	undefined	0	3
−3	−3	−3	2	0
−2	0	0	4	1
−1	3	3	4	1
0	2	2	4	1
1	2	2	6	2
2	2	2	0	3
3	1	1	−6	4
4	0	0	undefined	undefined
5	−1	−1	undefined	undefined

5



We see $g(x) = 2 \cdot f(-x)$ has two transformations

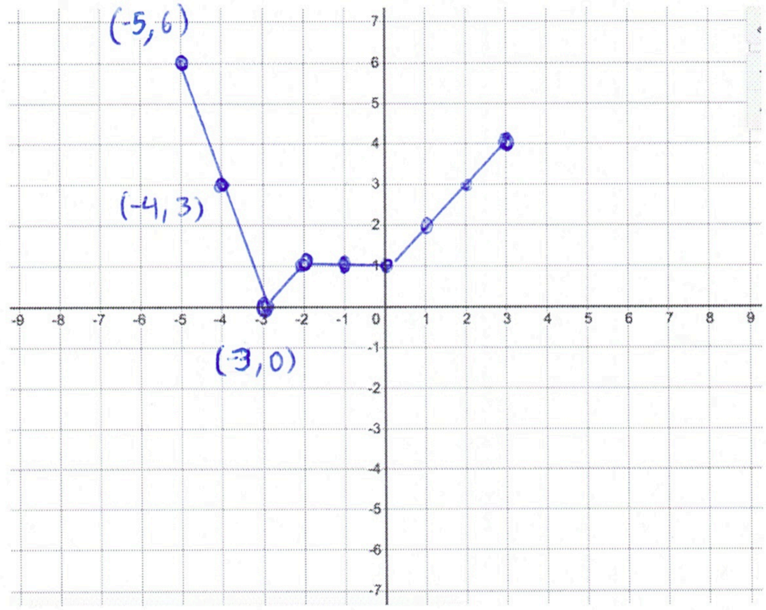
1. Horizontal reflection about y-axis $f(x) \to f(-x)$

2. Vertical scaling by factor of 2 $f(-x) \to 2 \cdot f(-x)$

5:11am - 5:16am

Name:______________________________ Class #:_______

9C. Sketch a graph of $h(x) = 3 - f(x+2)$.



9D. Explain, in words, the effect of each transformation in the new function $h(x) = 3 - f(x+2)$

Notice that for the ordered pair

$$(x,\ h(x)) = (x,\ 3 - f(x+2))$$

$$= (\underbrace{x-2}_{\text{inputs shift to the left by two units}},\ \underbrace{3 + -f(x)}_{\text{outputs flip signs and then shift upward by 3 units.}})$$

For more on this, see next page.

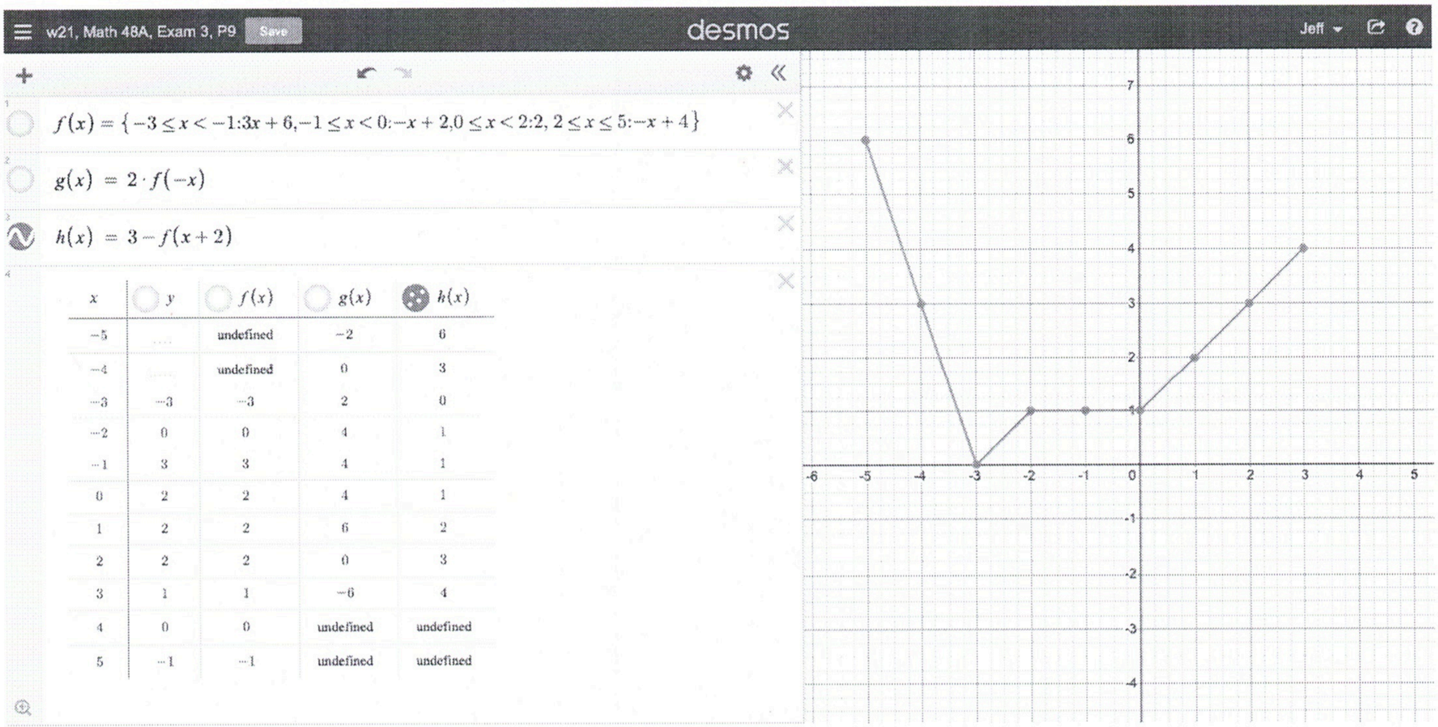
5:20am – 5:28am

Problem 9c, continued ...

Below we see the graph of $h(x) = 3 - f(x+2)$

combined with associated table.

We've hidden all other data that is not related to $h(x)$.



x	y	$f(x)$	$g(x)$	$h(x)$
−5		undefined	−2	6
−4		undefined	0	3
−3	−3	−3	2	0
−2	0	0	4	1
−1	3	3	4	1
0	2	2	4	1
1	2	2	6	2
2	2	2	0	3
3	1	1	−6	4
4	0	0	undefined	undefined
5	−1	−1	undefined	undefined

Notice that $h(x) = 3 - f(x+2)$ has three translation

1. Horizontal shift to the left by 2 units: $f(x) \longrightarrow f(x+2)$
2. Vertical reflection about x-axis: $f(x+2) \longrightarrow -f(x+2)$
3. Vertical shift upward by 3 units: $-f(x+2) \rightarrow 3 - f(x+2)$

5:29am - 5:31am

Name:______________________________ Class #:______

10. FIND COMPOSITION OF FUNCTIONS (Lesson 13)

Let $f(x) = x^2 - 3$ and $h(x) = 2x + 1$. Find each of the following:

10A. $(h \circ f)(3)$

10B. $f(h(x))$

$\boxed{\text{Problem 10 A}}$

Recall that $h \circ f(3) = h(f(x)) \Big|_{x=3}$

To start, let's consider

$$h(f(x)) = h(y) \quad \text{where} \quad y = f(x) = x^2 - 3$$

$$= 2y + 1 \Big|_{y = f(x)}$$

$$= 2y + 1 \Big|_{y = x^2 - 3}$$

$$= 2(x^2 - 3) + 1$$

$$= 2x^2 - 6 + 1$$

$$= 2x^2 - 5$$

Problem 10 A, continued...

$$\Rightarrow \quad h \circ f(3) = h(f(x))\Big|_{x=3}$$

$$= 2x^2 - 5\Big|_{x=3}$$

$$= 2(3)^2 - 5$$

$$= 2 \cdot 9 - 5$$

$$= 18 - 5$$

$$= 13$$

$$\Rightarrow \quad \boxed{h \circ f(3) = 13} \checkmark$$

Check: $f(3) = 3^2 - 3 = 9 - 3 = 6$

$h(6) = 2 \cdot 6 + 1 = 12 + 1 = 13 \checkmark$

12:30pm – 12:34pm.

Problem 10B

We begin by noticing:

$$f(h(x)) = f(y) \quad \text{where} \quad y = h(x) = 2x+1$$

$$= y^2 - 3 \Big|_{y=2x+1}$$

$$= (2x+1)^2 - 3$$

Sidenote:

$$(2x+1)^2 = (2x+1)\cdot(2x+1)$$

$$= 2x(2x+1) + 1(2x+$$

$$= 4x^2 + 2x + 2x + 1$$

$$= 4x^2 + 4x + 1$$

$$= 4x^2 + 4x + 1 - 3$$

$$\Rightarrow \boxed{f(h(x)) = 4x^2 + 4x - 2 = (2x+1)^2 - 3}$$

12:35pm – 12:39pm

Name:____________________________ Class #:______

11.	SOLVE EQUATIONS ALGEBRAICALLY

Solve each of the following equations. Show your work.

11A. $2 + 3|x - 2| = 11$

11B. $1 + \sqrt{3x - 5} = x$

Problem 11A

Let's solve the absolute-value equation

$$2 + 3|x - 2| = 11$$

Using the techniques from Lessons 8 - 9:

$$2 + 3 \cdot |x - 2| = 11$$
$$-2 \qquad\qquad\qquad -2$$

$$\Rightarrow \quad \frac{3 \cdot |x - 2|}{3} = \frac{9}{3}$$

$$\Rightarrow \quad |x - 2| = 3$$

We can break this equation into two parts:

$\Rightarrow$ $x - 2 = -3$ or $x - 2 = +3$

$+2 \quad +2$ $\qquad$ $+2 \quad +2$

$\Rightarrow$ $\boxed{x = -1}$ ✓ or $\boxed{x = 5}$ ✓

$\boxed{\text{Check:}}$ $x = -1$

Left-hand side: $2 + 3|x-2| = 2 + 3|-1-2|$

$= 2 + 3 \cdot |-3|$

$= 2 + 3 \cdot 3$

$= 2 + 9$

$= 11$ ← right-hand side

$\boxed{\text{Check:}}$ $x = 5$

left-hand side: $2 + 3|x-2| = 2 + 3|5-2|$

$= 2 + 3 \cdot |3|$

$= 2 + 3 \cdot 3$

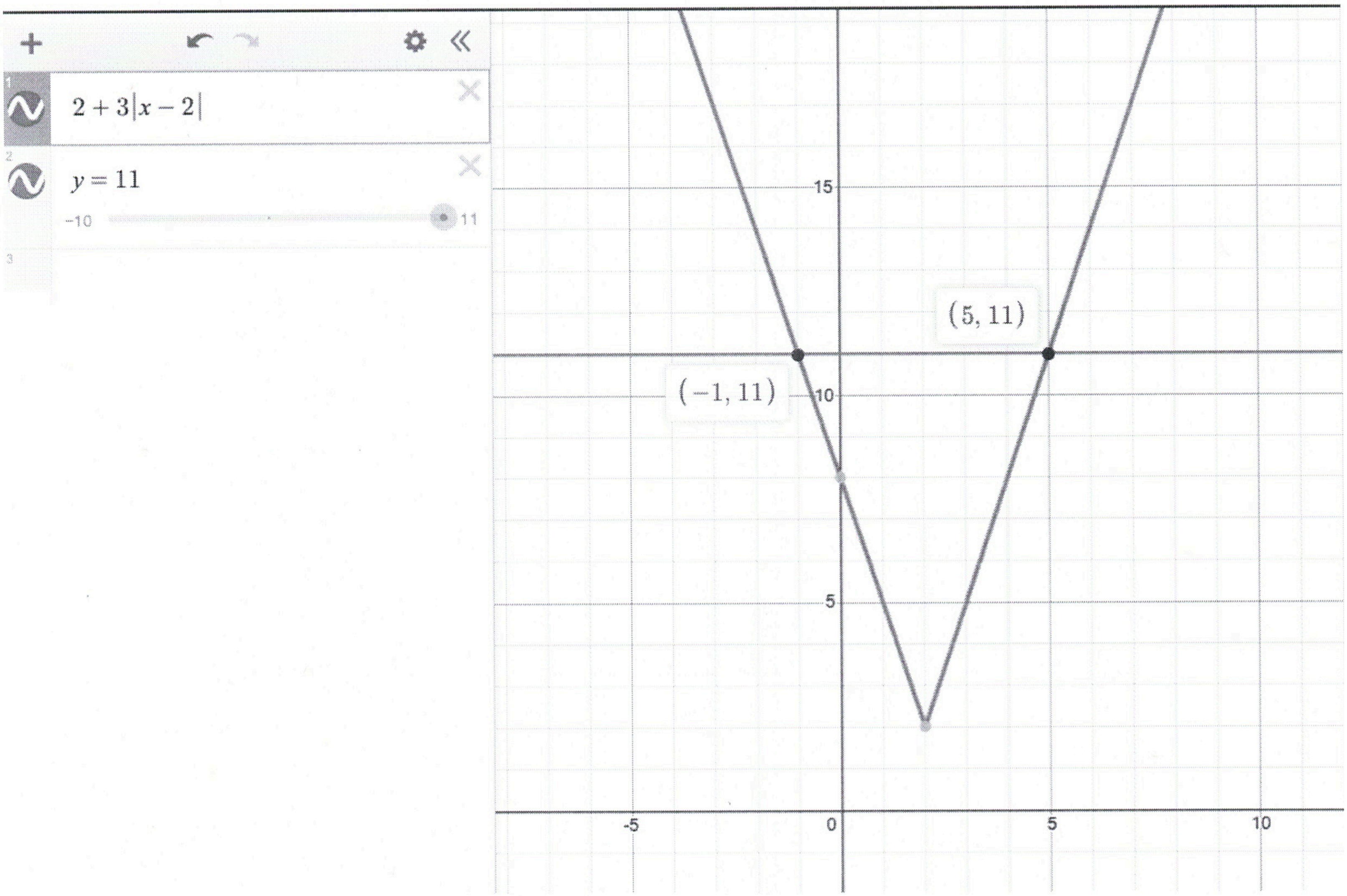
$= 2 + 9 = 11$ ← right-hand side

Problem 11A, continued...

Let's check our work using Desmos.com.

We see two points of intersection

(-1, 11) and (5, 11)



$\Rightarrow$ we have two solutions at

$\boxed{x = -1}$ and $\boxed{x = 5}$

Problem 11B

Let's solve the radical equation

$$1 + \sqrt{3x-5} = x$$

using techniques from Lesson 10:

$$1 + \sqrt[2]{3x-5} = x$$

$$-1 \qquad\qquad -1$$

$$\Rightarrow \sqrt[2]{3x-5} = x-1$$

$$\Rightarrow \left(\sqrt[2]{3x-5}\right)^2 = (x-1)^2$$

$$\Rightarrow 3x-5 = (x-1)\cdot(x-1)$$

5:29pm – 5:30pm

Problem 11B, continued ...

$$\Rightarrow \quad 3x - 5 = x \cdot (x-1) - 1 \cdot (x-1)$$

$$\Rightarrow \quad 3x - 5 = x^2 - x - x + 1$$

$$\Rightarrow \quad 3x - 5 = x^2 - 2x + 1$$

$$-3x \quad +5 \qquad\qquad -3x \quad +5$$

$$\Rightarrow \quad 0 = x^2 - 5x + 6$$

$a = 1$, $b = -5$, $c = 6$

Multiply to

6

-2 $\quad$ -3

-5

add to

$$\Rightarrow \quad 0 = (x - 2)(x - 3)$$

$$\Rightarrow \quad x - 2 = 0 \quad \text{or} \quad x - 3 = 0$$

5:31pm - 5:33pm

$\boxed{\text{Problem 11B, continued...}}$

$$\Rightarrow \boxed{x = 2} \quad \text{or} \quad \boxed{x = 3}$$

$\boxed{\text{Check: } x = 2}$ $\underline{\checkmark}$

Left-hand side: $1 + \sqrt{3x-5} = 1 + \sqrt{3 \cdot 2 - 5}$

$$= 1 + \sqrt{6-5}$$

$$= 1 + \sqrt{1}$$

$$= 1 + 1 = 2 \leftarrow \text{right-hand side}$$

$\boxed{\text{Check: } x = 3}$ $\underline{\checkmark}$

Left-hand side: $1 + \sqrt{3x-5} = 1 + \sqrt{3 \cdot 3 - 5}$

$$= 1 + \sqrt{9-5}$$

$$= 1 + \sqrt{4}$$

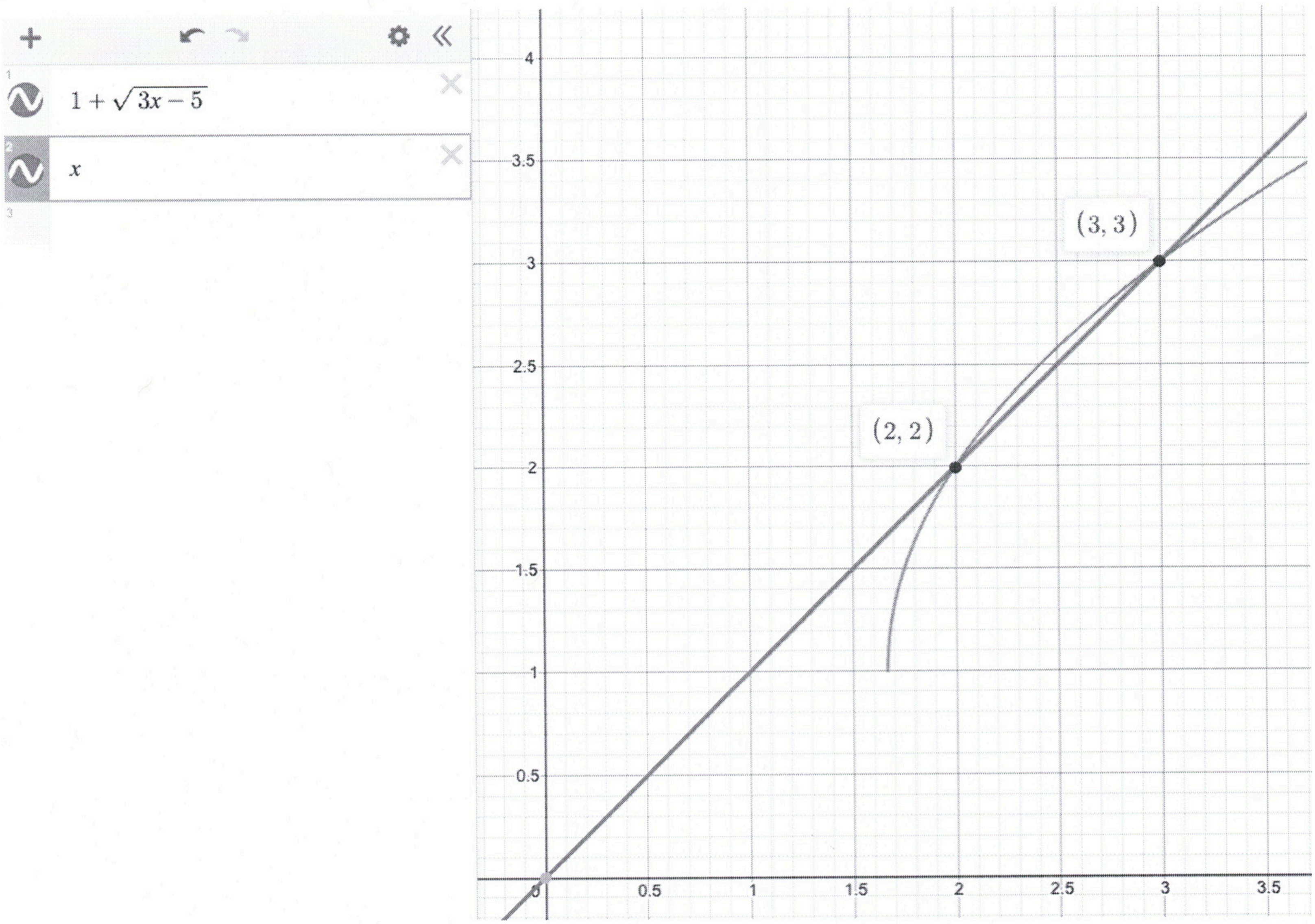
$$= 1 + 2$$

$$= 3 = x \leftarrow \text{right-hand side}$$

5:34 pm – 5:37 pm

Problem 11 B, continued...

We can also check our work graphically using Desmos.com. We see two points of intersection at (2, 2) and (3, 3)



$\Rightarrow$ two solutions at

$x = 2$ and $x = 3$.

Name:______________________________ Class #:________

12. FIND THE INVERSE FUNCTION (Lesson 14)

Use algebra to find the inverse of the following functions.

12A. $f(x) = \frac{4x-2}{3x+1}$

12B. $g(x) = \frac{6-x}{5}$

To find the desired inverse functions, we switch the x- and y-valu

and then isolate y in terms of x.

Problem 12A

We start with original problem

$$y = \frac{4x-2}{3x+1}$$

and then switch the x- and y-variables

$$x = \frac{4y-2}{3y+1}$$

$$\Rightarrow \quad x(3y+1) = 4y-2$$

12:39pm – 12:42pm

Problem 12 A, continued ...

$$\Rightarrow \quad 3xy + x = 4y - 2$$
$$-3xy \qquad +2 \qquad\qquad +2 \quad -3xy$$

$$\Rightarrow \quad x + 2 = 4y - 3xy$$

$$\Rightarrow \quad \frac{x+2}{4-3x} = \frac{y(4-3x)}{(4-3x)}$$

$$\Rightarrow \quad y = \frac{x+2}{4-3x} = f^{-1}(x)$$

To check our work, we will check to see if

$$f \circ f^{-1}(x) = x = f^{-1} \circ f(x)$$

12:43 pm - 12:45 pm

$\boxed{\text{Problem 12A, continued...}}$

$$f \circ f^{-1}(x) = f(f^{-1}(x))$$

$$= f(y) \quad \text{where} \quad y = f^{-1}(x) = \frac{x+2}{4-3x}$$

$$= \left.\frac{4y-2}{3y+1}\right|_{y = \frac{x+2}{4-3x}}$$

$$= \frac{\left[\frac{4\cdot[x+2]}{4-3x} - 2\right]}{\left[\frac{3\cdot[x+2]}{4-3x} + 1\right]}$$

$$= \frac{\left[\frac{4x+8}{4-3x} - \frac{2[4-3x]}{[4-3x]}\right]}{\left[\frac{3x+6}{4-3x} + \frac{[4-3x]}{[4-3x]}\right]}$$

12:46pm – 12:49pm

Problem 12A, continued...

$$\Rightarrow f \circ f^{-1}(x) = \left[\frac{4x+8-8+6x}{4-3x}\right] \div \left[\frac{3x+6+4-3x}{4-3x}\right]$$

$$= \frac{10x}{\cancel{[4-3x]}} \cdot \frac{\cancel{[4-3x]}}{10}$$

$$= \frac{10x}{10}$$

$$= x$$

$$\Rightarrow f \circ f^{-1}(x) = x$$

$$\Rightarrow \boxed{f^{-1}(x) = \frac{x+2}{4-3x}}$$ ✓

12:50pm - 12:56pm

Problem 12 B

Let $g(x) = \dfrac{6-x}{5} = y$. To find inverse, we switch the x- and y- variables and solve for y:

$$\Rightarrow \frac{6-y}{5} = x$$

$$\Rightarrow \frac{\cancel{5}}{1} \cdot \frac{(6-y)}{\cancel{5}} = 5 \cdot x$$

$$\Rightarrow 6 - y = 5x$$

$$-5x \quad +y \qquad -5x \quad +y$$

$$\Rightarrow \boxed{6 - 5x = y = f^{-1}(x)} \checkmark$$

Let's check: $f^{-1} \circ f(x) = f^{-1}(f(x)) = f^{-1}(y)$ where $y = \dfrac{6-x}{5}$

$$= 6 - 5y \Big|_{y = \frac{6-x}{5}}$$

$$= 6 - 5 \cdot \left(\frac{6-x}{5}\right)$$

$$= 6 - (6-x) = x \checkmark$$

12:57pm – 1pm

Name:______________________________ Class #:________

13. GRAPH USING VERTEX FORM OF QUADRATIC

Consider the standard form for a quadratic function:

$$f(x) = x^2 - 8x + 8$$

13A. Convert $f(x)$ into vertex form.

Math Memories Make Money

- [ ] Recall the vertex form of a quadratic function is given as

$$f(x) = a(x-h)^2 + k$$

- [ ] To transform into vertex form, we can use one of two techniques:
  1. complete the square
  2. formulas

Method 1: Complete the square

Let $f(x) = x^2 - 8x + 8$

$= 1x^2 + -8x + 8$

$ax^2 + bx + c$

$a = 1$
$b = -8$
$c = 8$

Problem 13A, continued ...

$\Rightarrow \quad f(x) = x^2 - 8x + 8$

$b = -8$

Side note: complete the square

$x^2 - 8x + 16 = (x-4)$

$b = -8 \quad c = d^2 \quad d = \frac{b}{2} = \frac{-8}{2}$

$= (-4)^2$

$= x^2 - 8x + 0 + 8$

$$= \underbrace{x^2 - 8x + 16}_{\text{perfect square trinomial}} - 16 + 8$$

$\Rightarrow \boxed{f(x) = (x-4)^2 - 8}$

vertex form

$a = 1$

$h = 4$

$k = -8$

1:01pm – 1:05pm

Problem 12 A, continued...

Let's use method 2 based on a formula:

$$f(x) = x^2 - 8x + 8$$ where $a = 1, b = -8, c = 8$

$$= a(x-h)^2 + k$$

Side note: Formulas for vertex

$a = 1$

$$h = \frac{-b}{2a} = \frac{-(-8)}{2 \cdot 1} = \frac{+8}{2} = +4$$

$$k = c - \frac{b^2}{4a} = 8 - \frac{(-8)^2}{4 \cdot 1}$$

$$= 8 - \frac{64}{4}$$

$$= 8 - 16 = -8$$

$$\Rightarrow \boxed{f(x) = (x-4)^2 - 8}$$ ✓

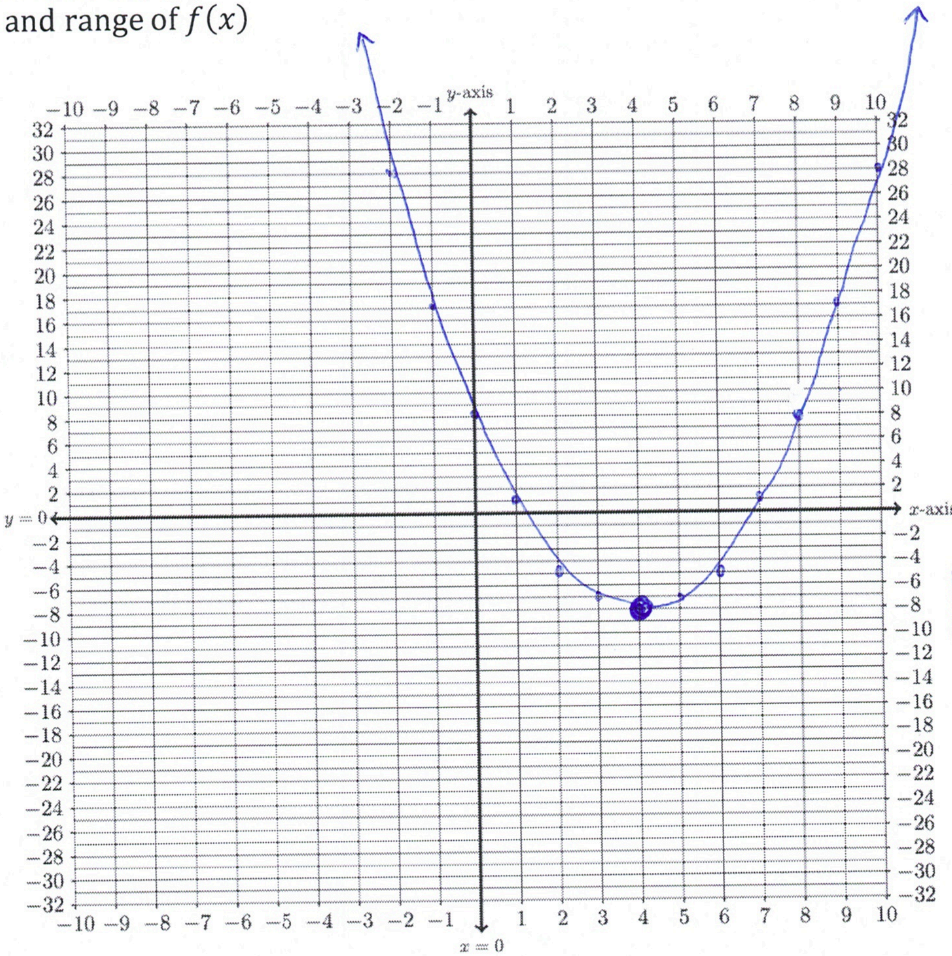
Name:____________________________ Class #:______

For $f(x) = x^2 - 8x + 8$ from problem 13A above:

13B. Sketch a graph of $f(x)$ using the vertex form.

13C. Find the min/max value of $f(x)$

13D. Find the domain and range of $f(x)$



vertex at $(4, -8)$

2C. The minimum value of $f(x)$ happens at $x = 4$ and is given by $f(4) = -8$

12D. The domain of $f(x)$ is $\mathbb{R}$

The range of $f(x)$ is $[-8, \infty)$.