Name: Solutions Class #:

3. PRACTICE FUNCTION NOTATION

Let $f(x) = x^2 - 3x + 4$.

3A. Find the value of $f(2)$.
3B. Find the value of $f(2+h)$.
3C. Find the average rate of change between points $(2, f(2))$ and $(2+h, f(2+h))$
3D. For any constant a, find the average rate of change over interval $(a, f(a))$ and $(a+h, f(a+h))$

Let $f(x) = x^2 - 3x + 4$.

3A. To find the value of $f(2)$ we note that input $x = 2$ and then find

$$f(2) = f(x)\Big|_{x=2}$$

$$= (x^2 - 3x + 4)\Big|_{x=2}$$

$$= (2)^2 - 3\cdot(2) + 4$$

$$= 4 - 6 + 4$$

$$= 2 \Rightarrow \boxed{f(2) = 2} \checkmark$$

3B. Now let's find the value of $f(2+h)$.

We notice in this notation

$$\underbrace{f(x)}_{\text{input value is inside the parenthesis}} = f(\underbrace{2+h}_{\text{input value is inside the parenthesis}})$$

the input value is given by $x = 2+h$.

In other words, everywhere we see an x

we substitute the "value" of $2+h$. Thus

$$f(2+h) = f(x)\Big|_{x=2+h}$$

$$= (x^2 - 3x + 4)\Big|_{x=2+h}$$

3B continued ...

$$\Rightarrow f(2+h) = (2+h)^2 - 3(2+h) + 4$$

Side note:

$$(2+h)^2 = (2+h)\cdot(2+h)$$
$$= 2\cdot(2+h) + h\cdot(2+h)$$
$$= 4 + 2h + 2h + h^2$$
$$= 4 + 4h + h^2$$

$$= 4 + 4h + h^2 - 6 - 3h + 4$$

$$= (4+4-6) + (4h - 3h) + h^2$$ combine like terms

$$= 2 + h + h^2$$

$$\Rightarrow \boxed{f(2+h) = h^2 + h + 2}$$

3c. Recall from Lesson 6 of Math 48A,

to find the average rate of change between

the points $(2, f(2))$ and $(2+h, f(2+h))$,

we find the slope of the secant line

through those two points.

Side Note:	Math Memories make Money To solve problems, we have to remember stuff from class

- ☐ The average rate of change is the slope of a secant line through points $(a, f(a))$ and $(b, f(b))$
- ☐ A secant line is a line passing through two points on the graph of a function
- ☐ To calculate the average rate of change, we find

$$m = \frac{f(b) - f(a)}{b - a} = \frac{\text{change in } y}{\text{change in } x}$$

3C continued...

We see we can find our desired rate of change between the points $(2, f(2))$ and $(2+h, f(2+h))$ using the formula:

$$m = \frac{f(2+h) - f(2)}{2+h-2}$$

$$= \frac{h^2+h+2-2}{h}$$

$$= \frac{h^2+h}{h}$$

$$= \boxed{\frac{h(h+1)}{h}} \leftarrow \text{unsimplified}$$

$$= \frac{h}{h}\cdot(h+1) = \boxed{h+1 \quad \text{if } h \neq 0} \leftarrow \text{simplified}$$

3D. Let a be any constant. For $f(x) = x^2 - 3x + 4$ we'll find the average rate of change between the points $(a, f(a))$ and $(a+h, f(a+h))$.

We'll begin by finding $f(a)$ and $f(a+h)$:

$$f(a) = f(x)\Big|_{x=a}$$

$$= (x^2 - 3x + 4)\Big|_{x=a}$$

$$= a^2 - 3a + 4$$

3D, continued ...

To find $f(a+h)$ note :

$$f(a+h) = f(x) \Big|_{x=a+h}$$

$$= (x^2 - 3x + 4) \Big|_{x=a+h}$$

$$= (a+h)^2 - 3(a+h) + 4$$

Side Note:

$$(a+h)^2 = (a+h)\cdot(a+h)$$

$$= a(a+h) + h(a+h)$$

$$= a^2 + ah + ah + h^2$$

$$= a^2 + 2ah + h^2$$

$$= a^2 + 2ah + h^2 - 3a - 3h + 4$$

$$= (a^2 - 3a + 4) + h^2 - 3h + 2ah$$

3D, continued ...

To find our desired average rate of change, we write

$$m = \frac{f(a+h) - f(a)}{a+h - a}$$

$$= \frac{a^2 - 3a + 4 + h^2 - 3h + 2ah - (a^2 - 3a + 4)}{h}$$

$$= \frac{a^2 - 3a + 4 + h^2 + 2ah - 3h - a^2 + 3a - 4}{h}$$

$$= \frac{(\cancel{a^2 - a^2})^{0} + (\cancel{3a - 3a})^{0} + (\cancel{4 - 4})^{0} + h^2 + 2ah - 3h}{h}$$

$$= \frac{h^2 + 2ah - 3h}{h}$$

3D, continued ...

$$\Rightarrow \underbrace{m}_{\text{average rate of change}} = \frac{h^2 + 2ah - 3h}{h}$$

$$= \frac{h \cdot (h + 2a - 3)}{h}$$

$$= \underbrace{\cancel{\frac{h}{h}}}_{1 \text{ if } h \neq 0} \cdot (h + 2a - 3)$$

Remember that $\frac{h}{h} = 1$

as long as the denominator

$h \neq 0$

$$\Rightarrow \boxed{m = h + 2a - 3} \checkmark$$

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4. PRACTICE FUNCTION NOTATION

Let $g(x) = 2^x$.

4A. Find the value of $g(0)$.
4B. Find the value of $g(0+h)$.
4C. Find the average rate of change between points ~~$(0, f(0))$~~ and ~~$(0+h, f(0+h))$~~

$(0, g(0))$ & $(0+h, g(0+h))$

mistake on exam

Let $g(x) = 2^x$.

4A. Let's find the value of $g(0)$. We notice the input value inside the parenthesis is given as $x=0$. Then we find

$$g(0) = g(x)\Big|_{x=0}$$

$$= (2^x)\Big|_{x=0}$$

$$= 2^0$$

$$= 1 \Rightarrow \boxed{g(0) = 1}$$ ✓

4B. To find $g(0+h)$, we notice the input value inside the parenthesis is given as $x = 0+h$.

$$\Rightarrow \quad g(0+h) = g(x)\Big|_{x=0+h}$$

$$= (2^x)\Big|_{x=0+h}$$

Math memories make money

Recall: sum rule for exponent powers

$$b^{m+n} = b^m \cdot b^n$$

$$= 2^{0+h}$$

$$= 2^0 \cdot 2^h$$

Recall: $b^0 = 1$

$$= 1 \cdot 2^h$$

$$= 2^h$$

$$\Rightarrow \quad \boxed{g(0+h) = 2^h}$$

4C. To find the average rate of change between points $(0, g(0))$ and $(0+h, g(0+h))$ we use our AROC formula from Lesson 6:

$$m = \frac{g(0+h) - g(0)}{0+h - 0}$$

$$= \boxed{\frac{2^h - 1}{h}}$$

This is the slope of the secant line through points $(0, g(0))$ and $(0+h, g(0+h))$.

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5. GRAPHING PIECEWISE FUNCTIONS

Consider the piecewise function below:

$$f(x) = \begin{cases} 16 - 2x & \text{if} \quad x < -1 \\ x^2 - 4 & \text{if} \quad -1 \le x < 3 \\ 6 & \text{if} \quad 3 \le x \end{cases}$$

← piece #1: $x < -1$

← piece #2: $-1 \le x < 3$

← piece #3: $3 \le x$

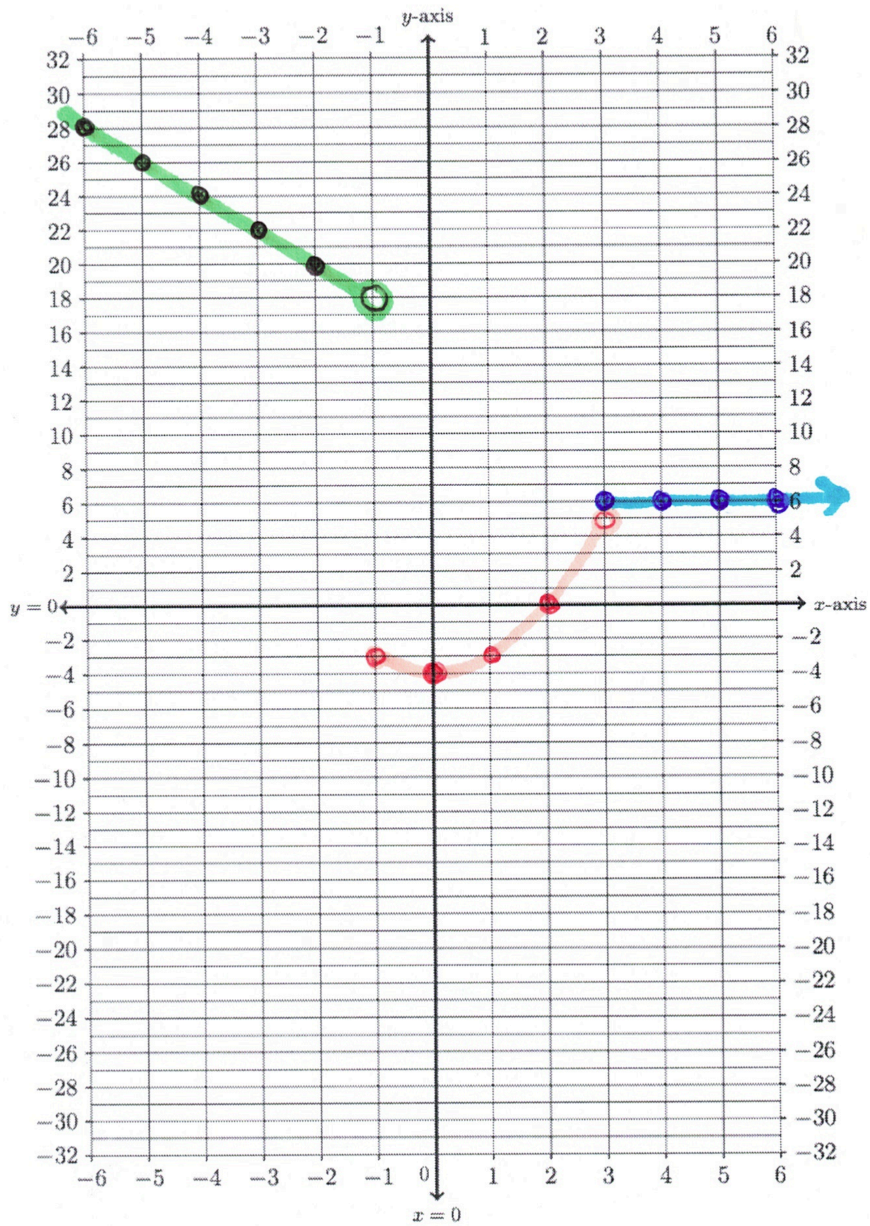
Using this description, fill out the table of values below and then graph the function

Piece #1 Input x-values strictly less than −1

Piece #2 $-1 \le x < 3$

Piece #3 $x \ge 3$

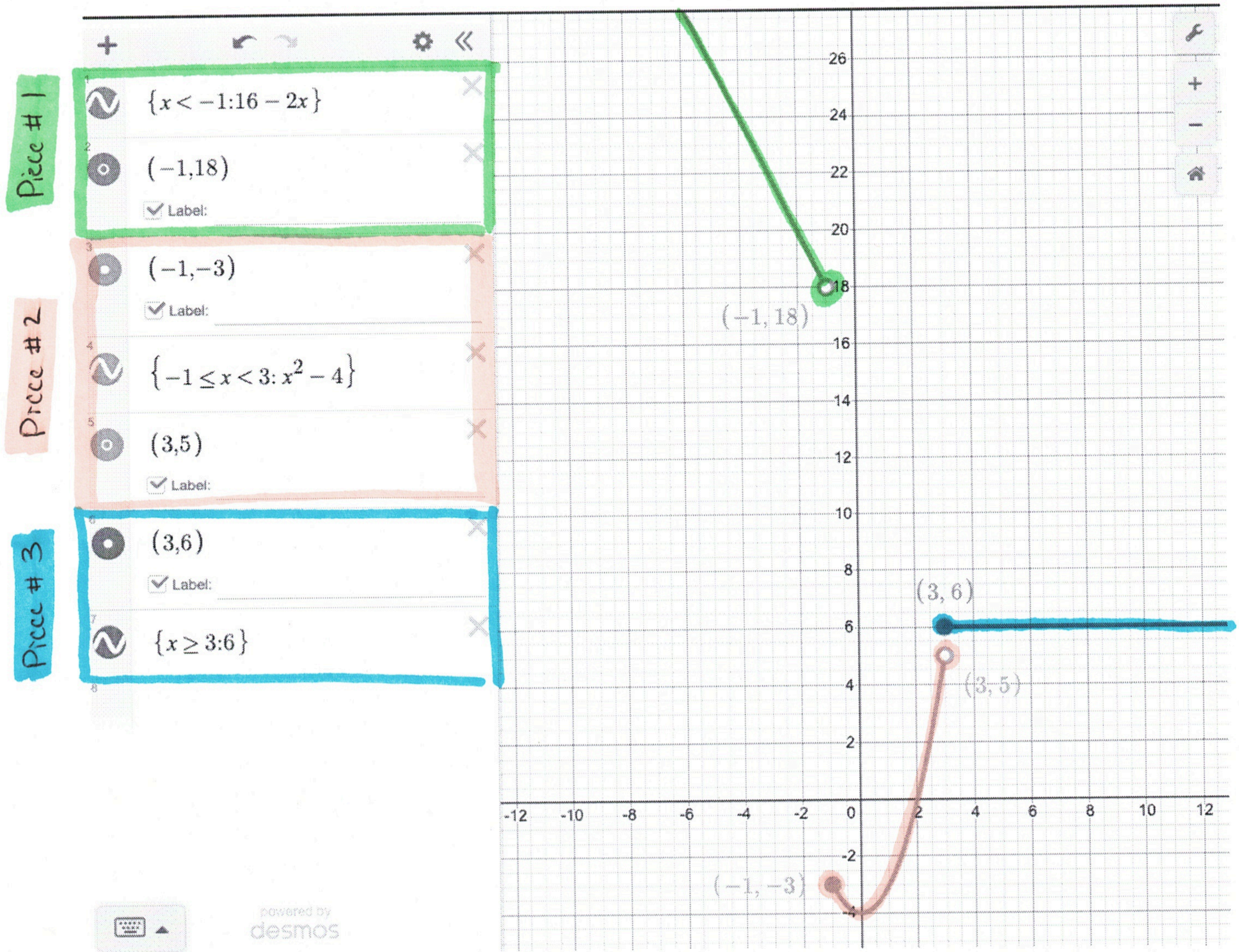
Input	*Output*
x	$f(x)$
−6	28
−5	26
−4	24
−3	22
−2	20
−1	−3
0	−4
1	−3
2	0
3	6
4	6
5	6
6	6



On the next page, we view the Desmos.com version of the same graph.

1:35pm – 1:43pm

Problem 5: Graphing Piecewise Function



1:44pm - 1:52pm

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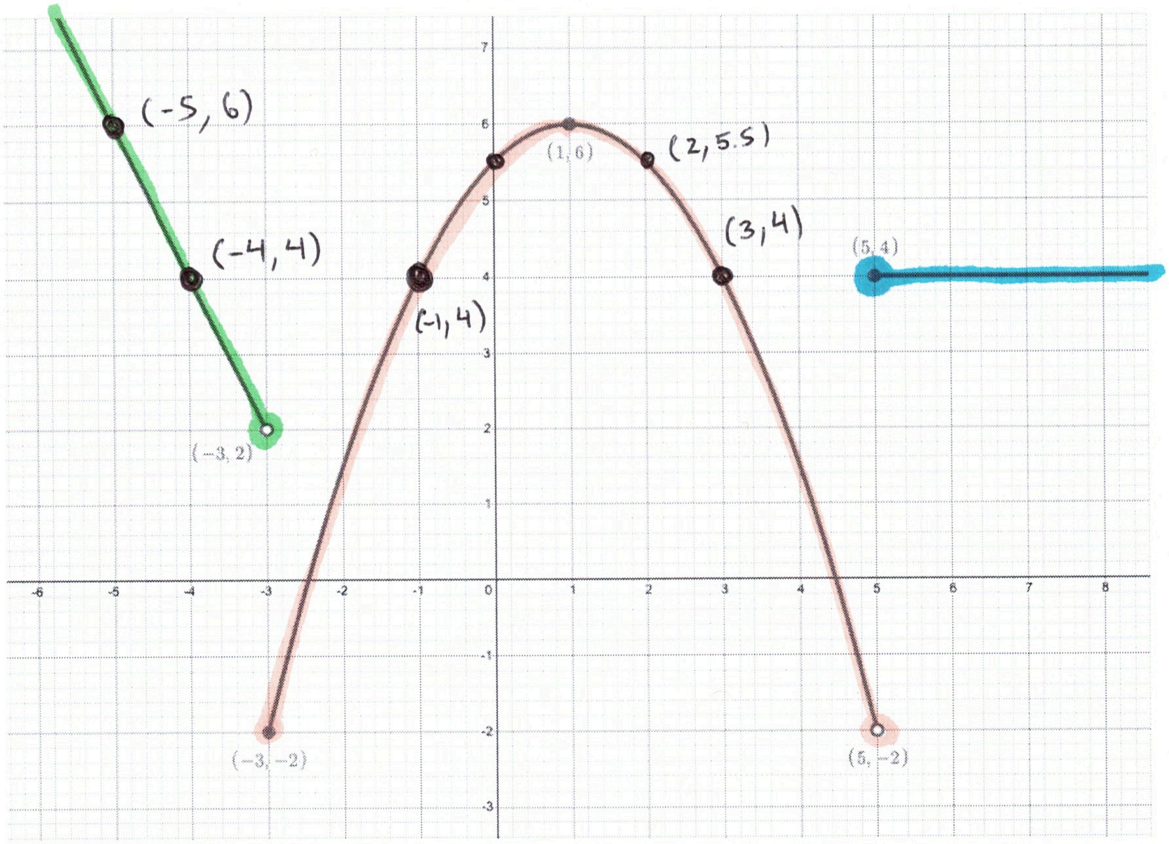
6.	ANALYZE THE GRAPH OF A PIECEWISE FUNCTIONS

The following is a graph of a piecewise defined function $g(x)$.



Find the formula (rule) for each part of the function and the x-values for which it applies. Explain your reasoning.

$$g(x) = \begin{cases} -2x - 4 & \text{if } x < -3 \quad \leftarrow \text{piece \#1} \\ 6 - \frac{1}{2}(x-1)^2 & \text{if } -3 \le x < 5 \quad \leftarrow \text{piece \#2} \\ 4 & \text{if } x \ge 5 \quad \leftarrow \text{piece \#3} \end{cases}$$

1:52pm – 2:08pm

Piece # 1

We notice that piece # 1 is a line

through the points $(-4, 4)$ and $(-5, 6)$.

We can find the slope

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 4}{-5 - (-4)} = \frac{2}{-1} = -2$$

Our point-slope form is given by:

$$y - y_1 = m(x - x_1)$$

$$\Rightarrow y - 4 = -2(x - -4)$$

$$\Rightarrow y - 4 = -2(x + 4)$$

$$\Rightarrow y = 4 + -2x - 8 = \boxed{-2x - 4}$$

Piece #2

We notice that piece #2 is a parabola with vertex at $(h, k) = (1, 6)$.

We recall from Lesson 16 that the vertex form of our quadratic function is given as

$$a(x-h)^2 + k$$

In our case we know $h = 1$, $k = 6$

The last question we have is what is our value of a. To answer that question we note point $(-3, -2)$ is on parabola

$$\Rightarrow \quad a(x-h)^2 + k = f(x)$$

$$\Rightarrow \quad a(x-1)^2 + 6 = f(x)$$

At point $(-3, -2)$ we see $x = -3$ and $f(x) = -2$

$$\Rightarrow \quad a(-3-1)^2 + 6 = -2$$

$$\Rightarrow \quad a(-4)^2 + 6 = -2$$

$$\Rightarrow \quad a \cdot 16 + 6 = -2$$
$$-6 \qquad -6$$

$$\Rightarrow \quad 16a = -8$$

$$\Rightarrow \quad a = \frac{-8}{16} = -\frac{1}{2} \Rightarrow$$

Piece #2

$$-\frac{1}{2}(x-1)^2 + 6$$

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7. PRACTICE FACTORING USING THE AC-METHOD

Use the AC-Method to factor each of the polynomials given below. For a reminder about using the AC Method to factor a quadratic polynomial, check out the Factoring Review Sheet.

7A. $12x^2 + 5x - 2$
7B. $9x^4 + 6x^3 - 3x^2$

To factor using the AC Method, we will use the standard form of our quadratic expression

$$ax^2 + bx + c$$

Problem 7A

Let's find the values of a,b,c for

$$12x^2 + 5x - 2$$

$$= 12x^2 + 5x + -2$$

$$ax^2 + bx + c$$

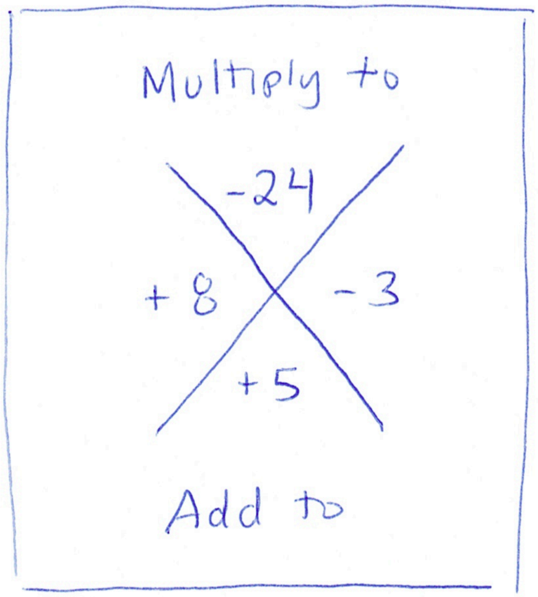
We see for this quadratic expression, we have

$$a = 12, \quad b = 5, \quad c = -2$$

2:09pm – 2:12pm

Problem 7A

Just like we did in problem 4 on this exam, we will use the AC method to factor our quadratic expression:



Multiply to: -24

Add to: $+5 = +8 - 3$

$$-24 = -1 \cdot 24 \leftarrow \text{NO}$$
$$= -2 \cdot 12 \leftarrow \text{NO}$$
$$= -3 \cdot 8 \leftarrow \text{Yes}$$

Now we split the middle term:

$$+5 = +8 - 3$$

$$\Rightarrow \quad 5x = (8-3)x = 8x - 3x$$

$$\Rightarrow \quad 12x^2 + 5x - 2 = 12x^2 + 8x - 3x - 2$$

2:12pm – 2:17pm

Problem 7A, continued

Now we factor by grouping

$$\underbrace{12x^2 + 8x}_{\text{group 1}} \underbrace{- 3x - 2}_{\text{group 2}}$$

$$= 4x(3x+2) - 1\cdot(3x+2)$$

$$= (4x-1)\cdot(3x+2)$$

$$\Rightarrow \boxed{12x^2 + 5x - 2 = (4x-1)\cdot(3x+2)}$$

2:18pm – 2:19pm

Problem 7B

Lets find the values of a, b, c for

$$9x^4 + 6x^3 - 3x^2$$

$$ax^2 + bx + c$$

We notice a problem: the two expressions are not in the same form. So let's begin by looking for the greatest common factor

$$9x^4 + 6x^3 - 3x^2 = 3x^2(3x^2 + 2x - 1)$$

Now we can focus on

$$3x^2 + 2x + -1$$

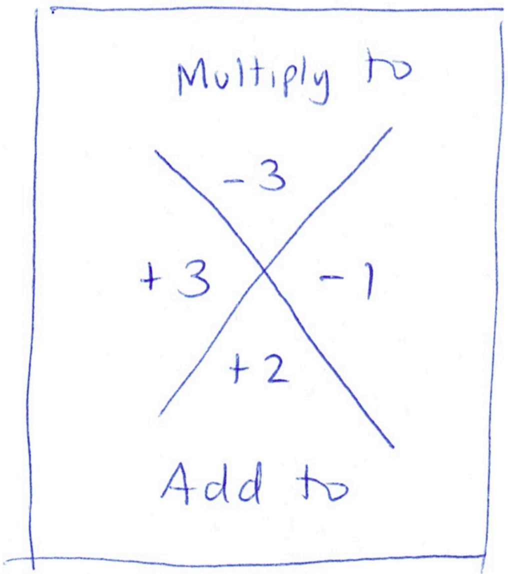
$$ax^2 + bx + c$$

$a = 3$, $b = 2$, $c = -1$

2:20pm - 2:22pm

Problem 7B, continued...

Let's repeat the AC method:



Add to: $+2$

Multiply to: $-3 = -1 \cdot 3$

Now let's split the middle term

$$+2 = 3 - 1$$

$$\Rightarrow \quad 2x = (3-1)x$$

$$\Rightarrow \quad 2x = 3x - x$$

$$\Rightarrow \quad 3x^2 + 2x - 1 = 3x^2 + 3x - x - 1$$

2:23pm – 2:24pm

Now we can factor by grouping

$$3x^2 + 2x - 1 = \underbrace{3x^2 + 3x}_{\text{group 1}} \underbrace{- x - 1}_{\text{group 2}}$$

$$= \overbrace{3x(x+1)}^{\text{group 1}} \overbrace{-1(x+1)}^{\text{group 2}}$$

$$= (3x-1)\cdot(x+1)$$

We go back to original expression to find

$$9x^4 + 6x^3 - 3x^2 = \boxed{3x^2(3x-1)(x+1)}$$

2:24pm - 2:25pm

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8. PRACTICE FACTORING DIFFERENCES OF SQUARES

Remember the difference of square formula is given by:

$$(a^2 - b^2) = (a - b)(a + b)$$

Factor each of the polynomials given below. Use the difference of squares formula.

8A. $y^2 - 64$
8B. $4m^2 - 9$
8C. $9x^2 - 16$

Let's take a look at the difference of squares formula

subtraction

$$\underbrace{a}_{\text{perfect square}}^2 - \underbrace{b}_{\text{perfect square}}^2 = (a - b)\cdot(a + b)$$

Problem 8A

Let's transform the given expression into general form using the perfect square format:

$$y^2 - 64 = \underbrace{y^2}_{a^2} - \underbrace{8^2}_{b^2}$$

$$\boxed{\begin{aligned} a &= y \\ b &= 8 \end{aligned}}$$

$$\Rightarrow y^2 - 64 = \boxed{\underbrace{(y - 8)\cdot(y + 8)}_{(a - b)\cdot(a + b)}} \checkmark$$

4:35p – 4:39pm

$\boxed{\text{Problem 8b}}$

Let's transform the given expression into the general form. Notice

- $4m^2 = 2 \cdot 2 \cdot m \cdot m = 2 \cdot m \cdot 2 \cdot m = \boxed{(2m)^2}$ ← perfect square
- $9 = 3 \cdot 3 = \boxed{3^2}$ ← perfect square

We see that we can write this

$$4m^2 - 9 = (2m)^2 - 3^2$$

$$a^2 - b^2$$

$$\boxed{\begin{aligned} a &= 2m \\ b &= 9 \end{aligned}}$$

$$\Rightarrow 4m^2 - 9 = \boxed{(2m - 3) \cdot (2m + 3)}$$

$$(a - b) \cdot (a + b)$$

Problem 8c

Let's transform the given statement

$$9x^2 - 16$$

into general form. To do so, we note

- $9x^2 = 3 \cdot 3 \cdot x \cdot x = 3 \cdot x \cdot 3 \cdot x = \boxed{(3x)^2}$ ← perfect square
- $16 = 4 \cdot 4 = \boxed{4^2}$ ← perfect square

Now we can write this:

$$9x^2 - 16 = \underbrace{(3x)^2 - 4^2}_{a^2 - b^2} \qquad \boxed{\begin{array}{l} a = 3x \\ b = 4 \end{array}}$$

$$\Rightarrow 9x^2 - 16 = \underbrace{(3x - 4) \cdot (3x + 4)}_{(a - b) \cdot (a + b)}$$