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Math 48A, Exam 2
Lessons 4, 5, 6, 7, and 8

1.	IDENTIFY LINEAR FUNCTIONS USING EQUATIONS

Which of the following equations represent linear functions?

a. $3y = 5x - 2$ ✓ linear function ____ not a linear function

b. $y = x^2 - 7$ ____ linear function ✓ not a linear function

c. $y = \frac{2}{x} - 5$ ____ linear function ✓ not a linear function

d. $y^2 = 4x - 7$ ____ linear function ✓ not a linear function

Recall that the slope-intercept form of the equation for a line is given as

$$y = mx + b$$

where m is the slope and the point $(0, b)$ is the y-intercept. For these problems, we want to put these into this form.

Problem 1a) Lets start with the given equation

$$3y = 5x - 2$$

$$\Rightarrow \quad \frac{\cancel{3}y}{\cancel{3}_{1}} = \frac{5x - 2}{3}$$

$$\Rightarrow \quad y = \frac{5x - 2}{3}$$

Note: $\frac{A \pm B}{C} = \frac{A}{C} \pm \frac{B}{C}$

$$\Rightarrow \quad y = \frac{5x}{3} - \frac{2}{3}$$

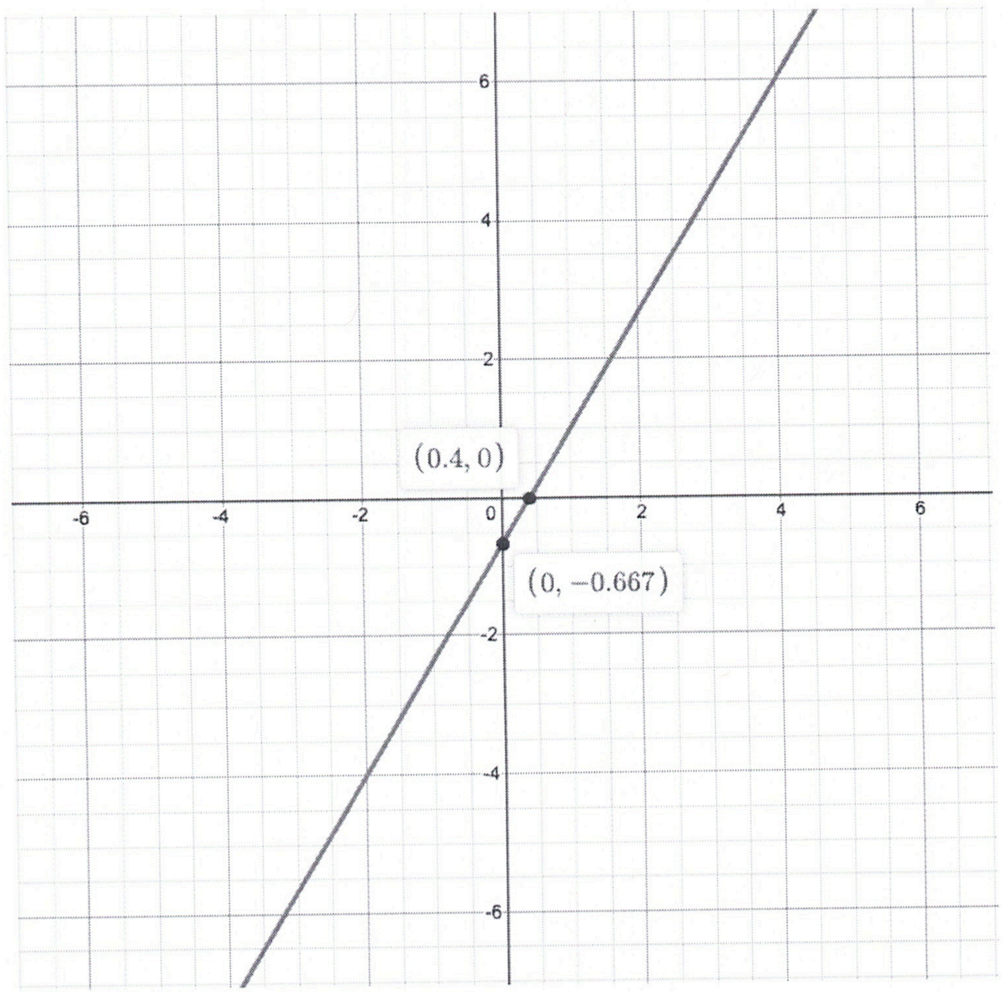
$$\Rightarrow \quad y = \underbrace{\boxed{\frac{5}{3}}}_{\text{slope}} x + \underbrace{\boxed{\frac{-2}{3}}}_{\text{y-intercept}}$$

Notice this is a linear function. We confirm this intuition by looking at a graph of our function on next page and see this is indeed a line

Problem 1A

Below, we graph the equation $3y = 5x - 2$ using Desmos.com. We see this graph is a line as claimed.

$y = \frac{5}{3}x - \frac{2}{3}$



Problem 1b Consider the function

$$y = x^2 - 7$$

This function has an x^2 term and we can categorize this as a quadratic function in the form

$$y = \boxed{ax^2 + bx + c}$$

standard form of quadratic function

$$= x^2 - 7$$

$$= 1 \cdot x^2 + 0 \cdot x + -7$$

$$(a=1, b=0, c=-7)$$

Problem 1c For the function $y = \frac{2}{x} - 5$, we see the term $\frac{2}{x}$ which is known as a rational function. This is not at all linear since we cannot write it in the form $y = mx + b$

(see next pages for graphs to confirm this intuition about the equations here)

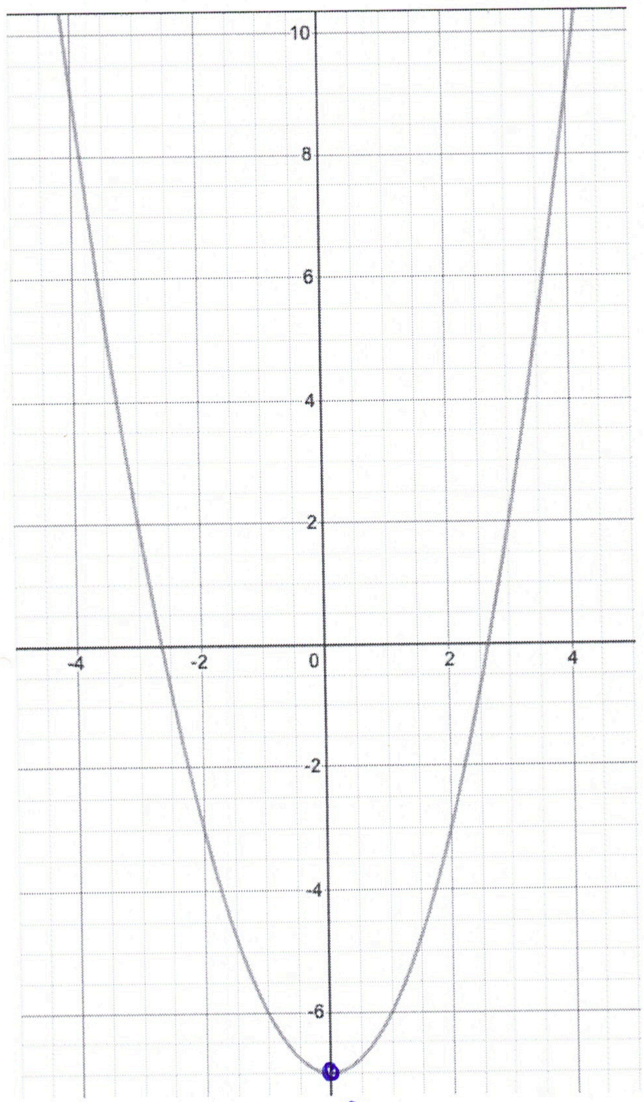
Problem 1B

Below is the graph of the function

$$f(x) = x^2 - 7$$

which is a parabola, not a line

$$y = x^2 - 7$$



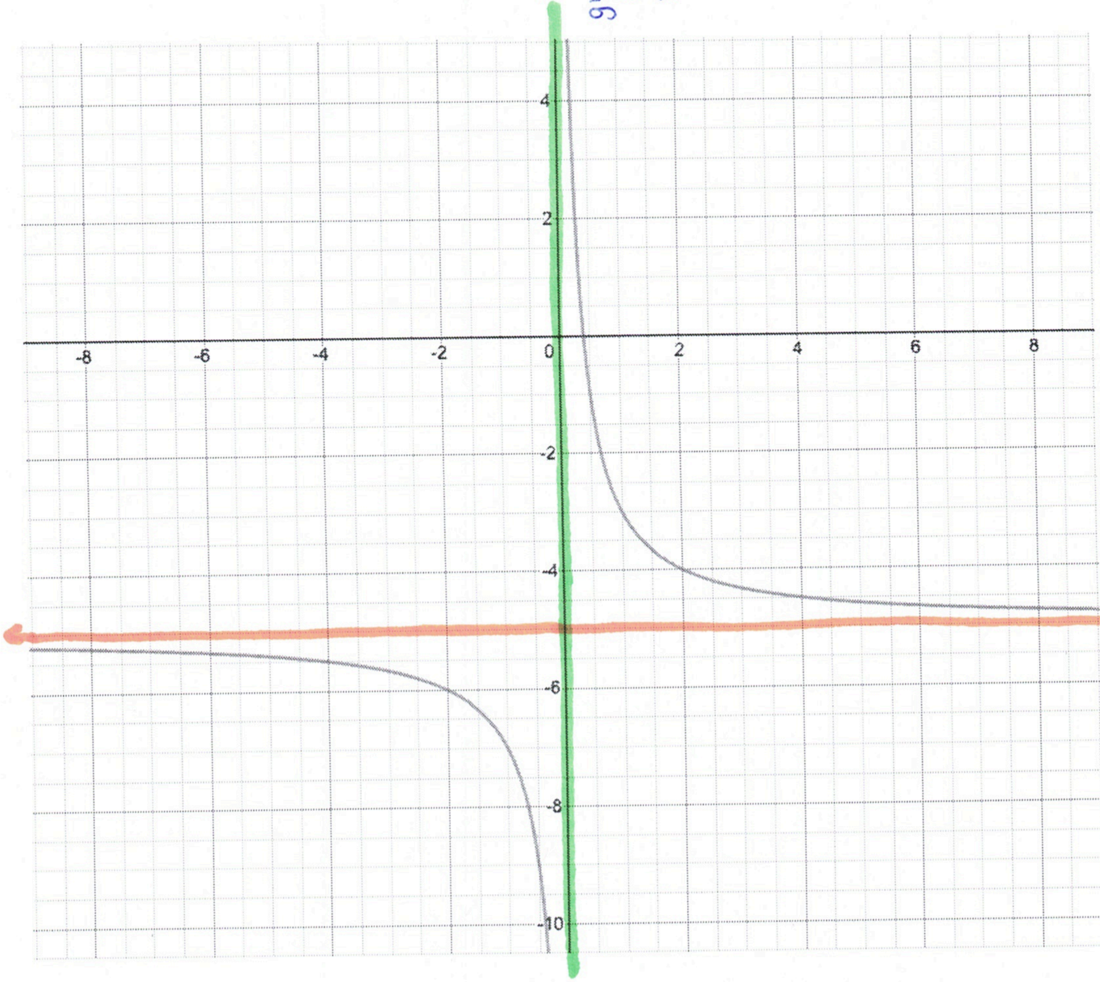
vertex of parabola is at point $(0, -7)$

$\boxed{\text{Problem 1C}}$ Below is the graph of the function

$$f(x) = \frac{2}{x} - 5$$

This graph is called a hyperbola and definitely not a line.

y-axis given by equation x = 0



x-axis (given by equation y = 0

this is called a horizontal asymptote

y = -5
Horizontal line drawn in orange

x = 0
vertical line drawn in green

This is called a vertical asymptote

Problem 1d

Consider the implicit relation

$$y^2 = \boxed{4x - 7}$$

While the RHS looks very similar to the

$$y = mx + b$$

format, the LHS includes a term y^2 rather than y.

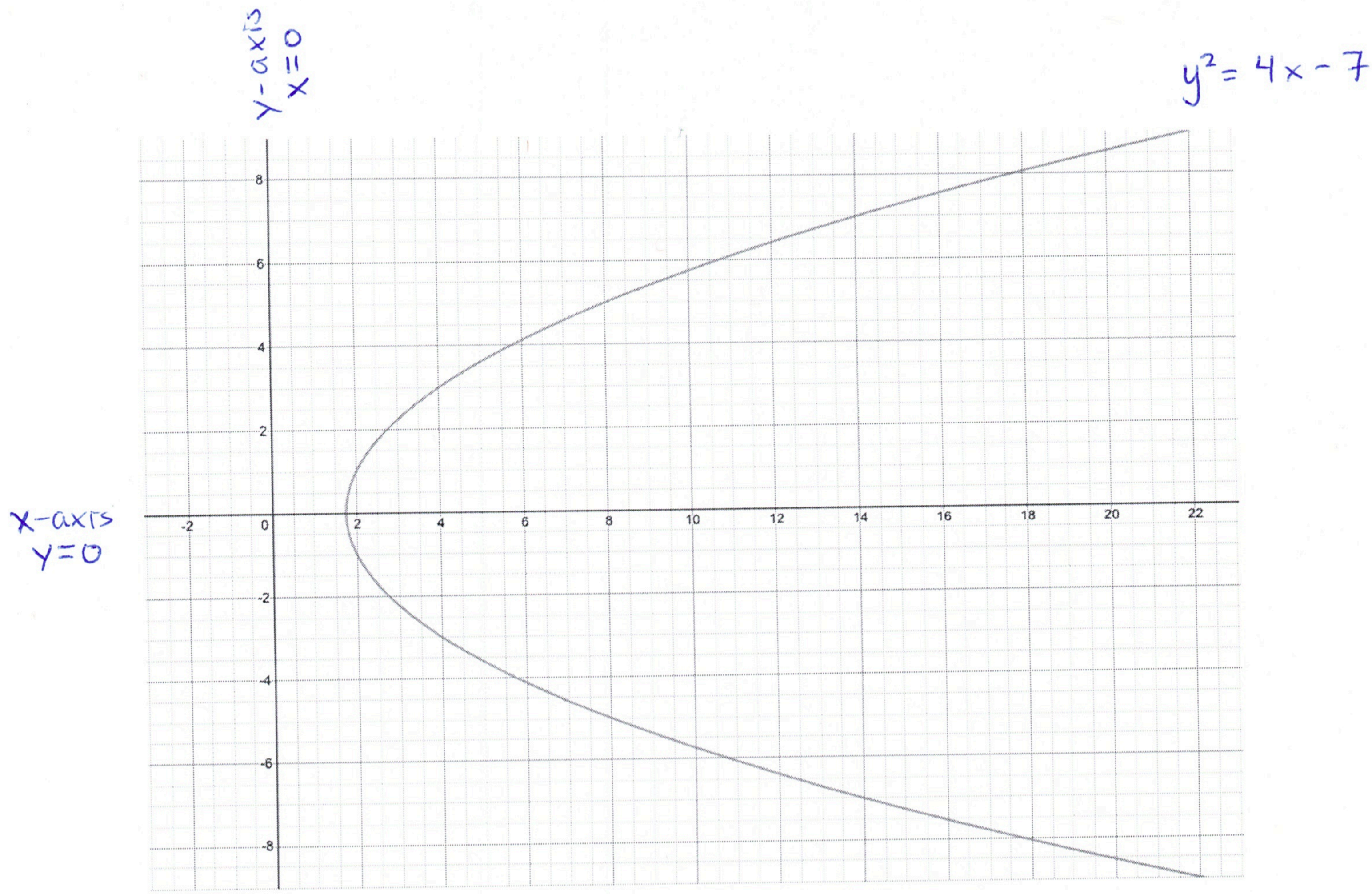
There is no easy way to eliminate the square and so we conclude this is not a linear relationship.

Problem 1d

Below is the graph of the relation

$$y^2 = 4x - 7$$

which is a "parabola" turned on it's side



This graph fails the vertical line test and is neither a function nor a line.

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2. IDENTIFY LINEAR FUNCTIONS USING A TABLE

Below are tables for two different functions. One of these tables has points from a linear function, and the other does not.

x	$f(x)$
- 8	2
- 4	4
-1	6
1	8
2	10

x	$g(x)$
- 6	14
- 4	10
- 1	4
1	0
2	-2

← This one represents a line

2A. Which function is linear? Explain your reasoning.

The function g(x) is a line since the output values change consistently. One way to see this is to graph the points using Desmos.com. See nxt page We can also confirm this algebraically.

2B. Write an equation for the linear function. Justify your answer.

Let's write the equation for line using point-slope form

$$y - y_1 = m(x - x_1)$$

where m = slope and point (x_1, y_1) is on the line. In our case let's define our point to be $(x_1, y_1) = (1, 0)$ so that $x_1 = 1$ and $y_1 = 0$.

Problem 2B

To find the slope m, we start with two points on the line

$$(x_1, y_1) = (1, 0) \qquad (x_2, y_2) = (2, -2)$$

and calculate the slope

rise

change in y

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$$

run

change in x

$$\Rightarrow \quad m = \frac{-2 - 0}{2 - 1}$$

$$\Rightarrow \quad m = \frac{-2}{1}$$

$$\Rightarrow \quad \boxed{m = -2}$$

Problem 2B   Note $m = -2$, $x_1 = 1$, $y_1 = 0$

Then, we can substitute the values back into our point-slope form for our equation and find

$$y - y_1 = m(x - x_1)$$

$$\Rightarrow y - 0 = -2 \cdot (x - 1)$$

$$\Rightarrow y = -2x + 2$$

$$\Rightarrow y = mx + b$$

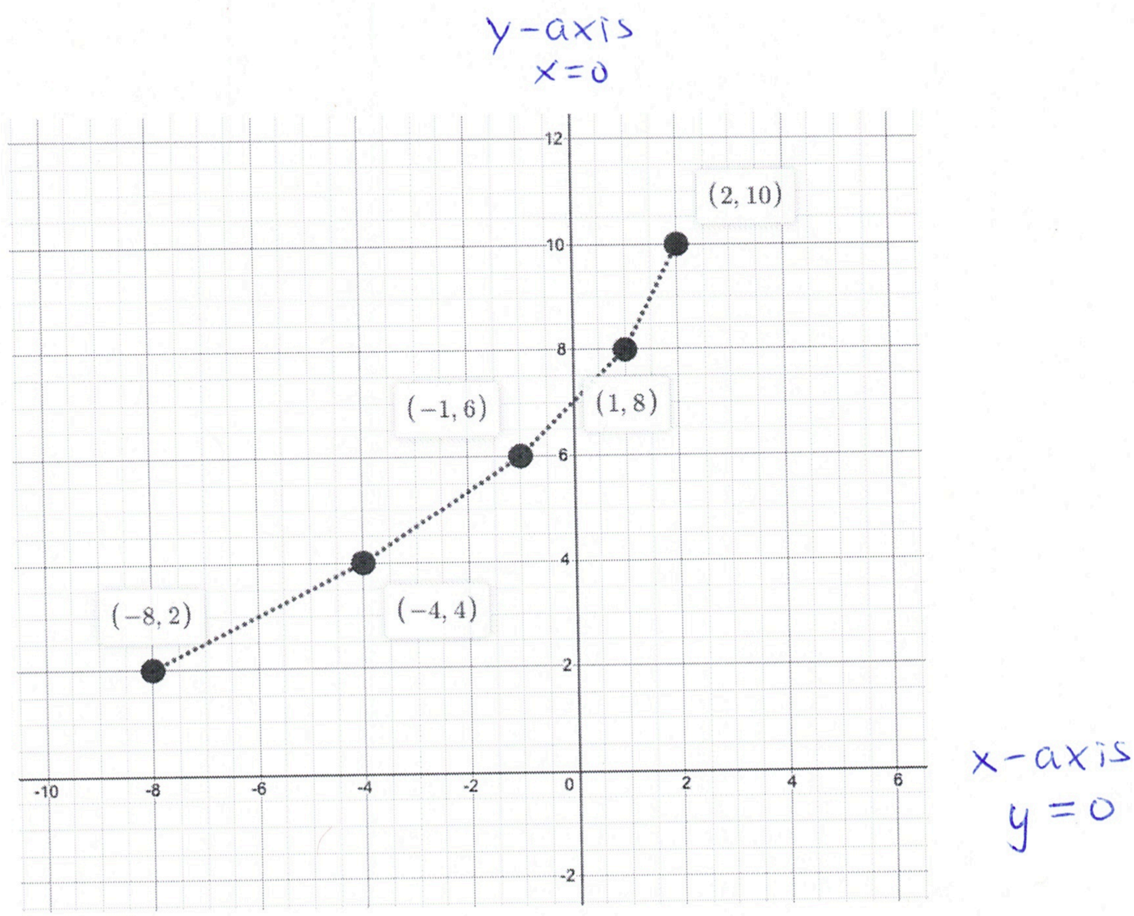
y-intercept at point $(0,2)$

We confirm that $g(x)$ has points from a linear function here and on following pages.

Problem 2

Below is a graph of the points for f(x) given in our table.

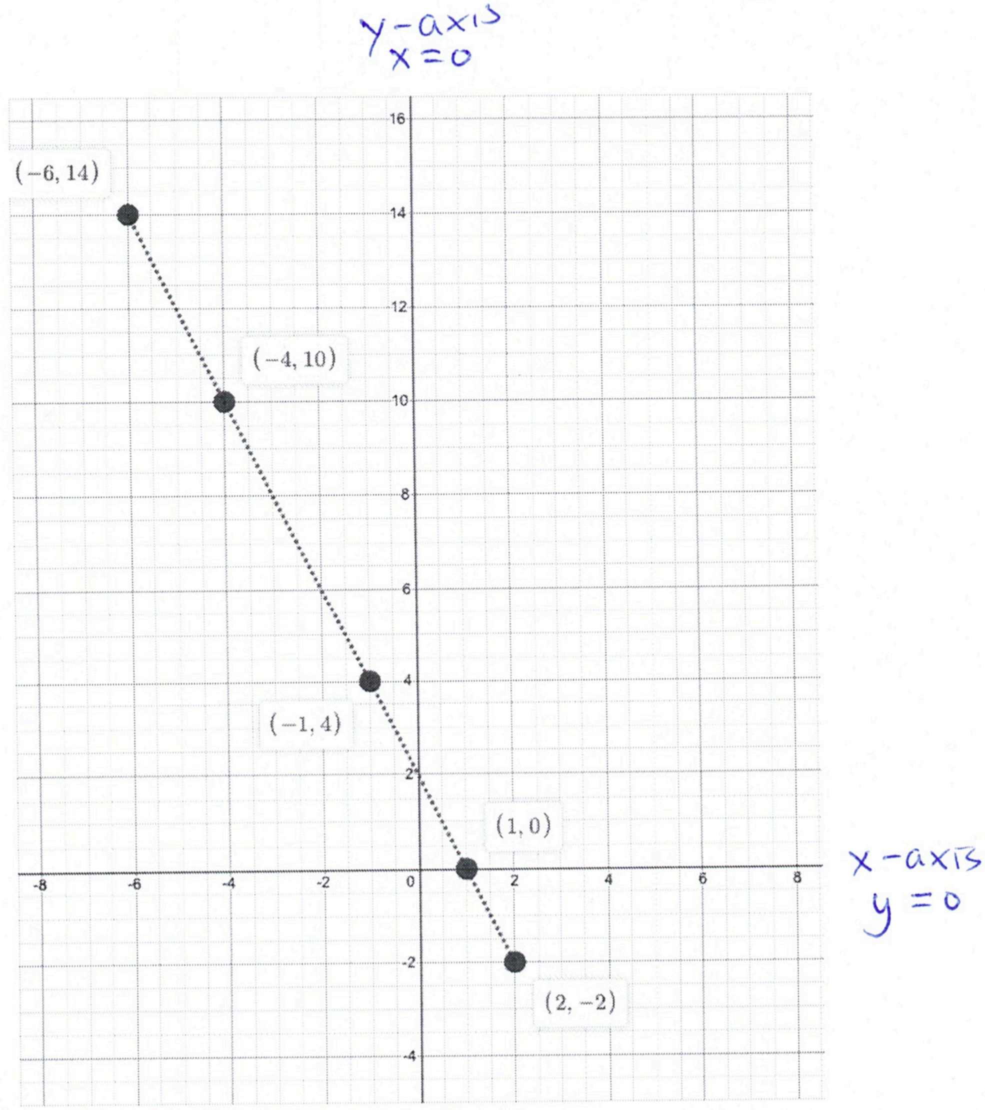
In this graph, we connect the dots.



Notice that the points do not all lie on the same line.

Problem 2

Below is a graph of the points of g(x) given in our table with interpolating lines (connecting the dots)



Notice these points all lie on the same line.

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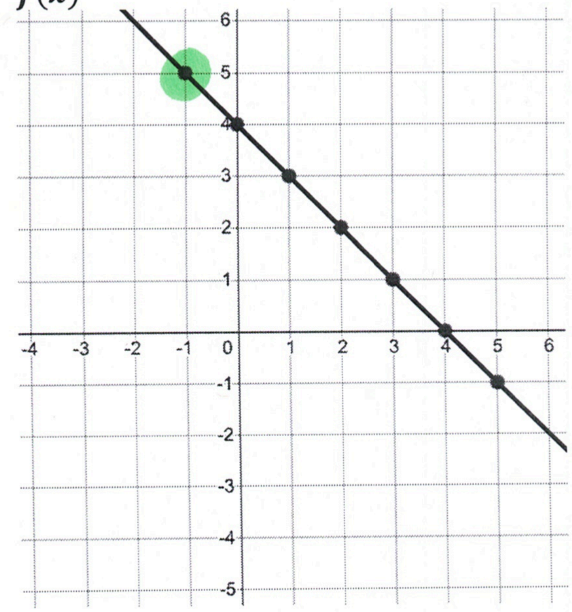
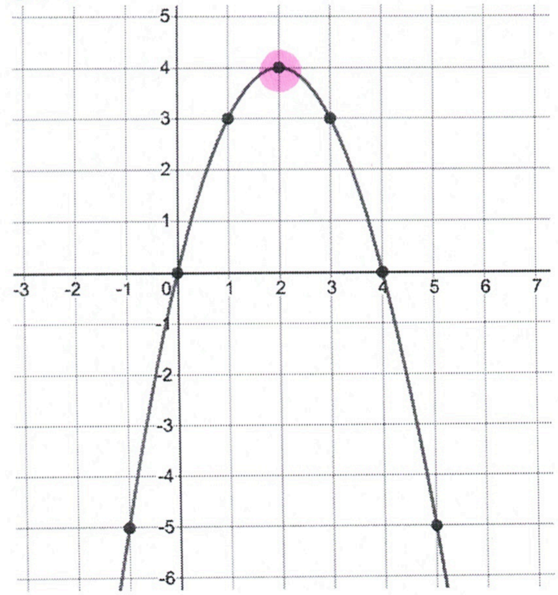
3. EVALUATE FUNCTIONS USING GRAPHS

Below are graphs of functions $f(x)$ and $g(x)$.

$\boldsymbol{f(x)}$



$\boldsymbol{g(x)}$



Use the graphs above to evaluate each of the following:

$\boxed{f(-1)}$ input: $x = -1$

when $x = -1$, we see y-value on point $(-1, 5)$ is given as

$$\boxed{f(-1) = 5}$$

$\boxed{g(2)}$ input: $x = 2$

when $x = 2$, we see the y-value of point $(2, 4)$ is

$$\boxed{g(2) = 4}$$

Challenge Problems: Use the graphs to evaluate each of the following:

$f(g(0))$ input $x = 0$

Notice we can break this up into two problems:

$f(g(0)) = f(y)$ where $y = g(0)$

$= f(0)$ since $y = g(0) = 0$

$= \boxed{4}$ since $f(0) = 4$

$g(f(5))$ input $x = 5$

we can break this into two problems:

$g(f(5)) = g(y)$ where $y = f(5)$

$= g(-1)$ since $f(5) = -1$

$= \boxed{-5}$ since $g(-1) = -5$

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4. SOLVE ABSOLUTE VALUE EQUATIONS ALGEBRAICALLY

Consider the following absolute value equation:

$$|2x - 4| - 5 = 1$$

Solve this equation using an algebraic method (not graphically). Hint: you might check your work by solving problem 5A below and looking back at this problem.

We start at the beginning:

$$|2x - 4| - 5 = 1$$

$$+5 \qquad +5$$

0

← Get absolute value expression by itself on left-hand side

$$\Rightarrow \quad |2x - 4| = 6$$

← Get rid of absolute value bars

Now, we want to get rid of the absolute value bars. To do this, we take the expression inside the absolute value bars (in this case: $2x-4$) and set this equal to both the positive and negative value of the right-hand side.

$\Rightarrow$ Equation 1 : $2x - 4 = -6$

Equation 2 : $2x - 4 = 6$

Let's solve each of these.

Equation 1 :

$$2x \cancel{-4} = -6$$
$$\quad +4 \to 0 \qquad +4$$

$$\Rightarrow \quad 2x = -2$$

$$\Rightarrow \quad \frac{\cancel{2}x}{\cancel{2}_{\,1}} = \frac{-2}{2}$$

$$\Rightarrow \quad \boxed{x = -1} \ \checkmark$$

$\boxed{\text{Equation 2:}}$

$$2x \cancel{-4} = 6$$
$$\quad \cancel{+4}_{0} \qquad +4$$

$$\Rightarrow \quad 2x = 10$$

$$\Rightarrow \quad \frac{\cancel{2}x}{\cancel{2}_{1}} = \frac{10}{2}$$

$$\Rightarrow \quad \boxed{x = 5} \checkmark$$

We see we have two candidates for solutions to this equation:

$x = -1$ or $x = 5$

Let's check each one:

$x = -1$: $|2x - 4| - 5 = |2 \cdot (-1) - 4| - 5$

$= |-2 - 4| - 5$

$= |-6| - 5$

$= 1$ ✓

Here, we confirm the LHS is identical to RHS at $x = -1$

$x = 5$: $|2x - 4| - 5 = |2 \cdot 5 - 4| - 5$

$= |10 - 4| - 5$

$= |6| - 5$

$= 1$ ✓

Here, we confirm that the LHS is identical to RHS at input $x = 5$

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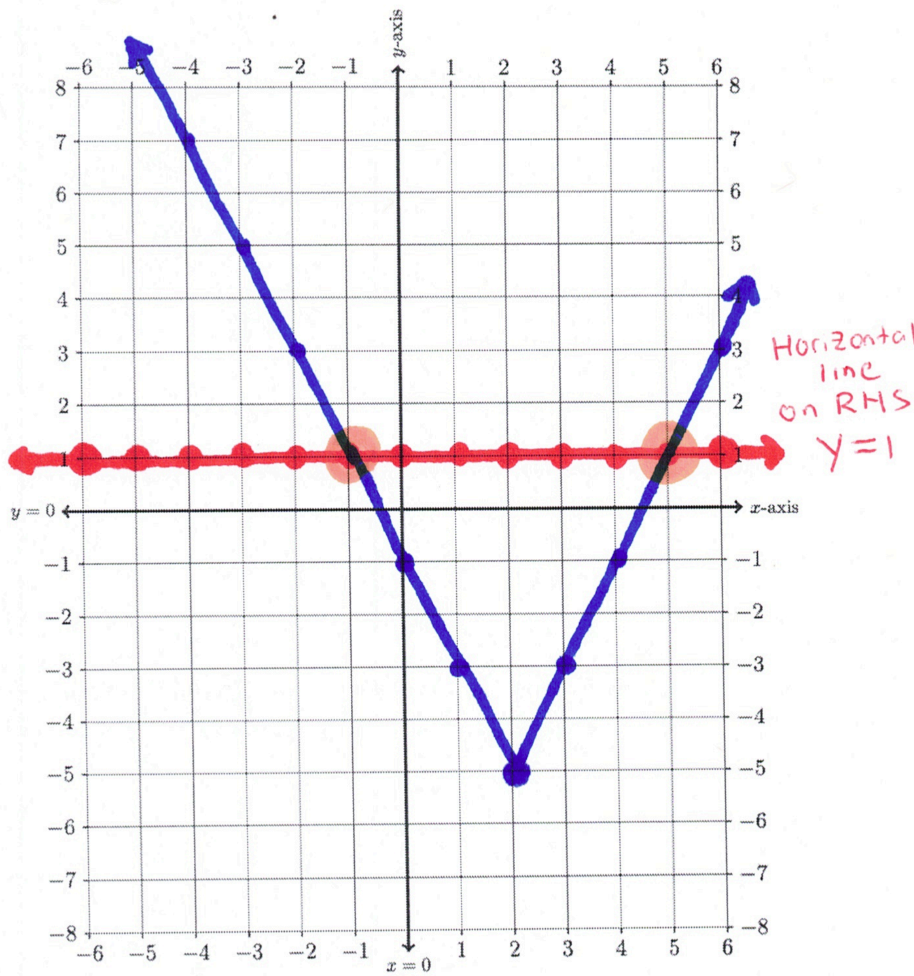
5. SOLVE ABSOLUTE VALUE EQUATIONS GRAPHICALLY

5A. Consider the following absolute value equation:

$$|2x - 4| - 5 = 1$$

Use the left-hand side (LHS) and right-hand side (RHS) of this equation to a table of values and draw the resulting graph on the axes below. Then, solve this equation using the information in your graph.

x	LHS $\lvert 2x-4\rvert -5$	RHS 1
-6	11	1
-5	9	1
-4	7	1
-3	5	1
-2	3	1
-1	1	1
0	-1	1
$\frac{4}{5}$	-2.6	1
1	-3	1
2	-5	1
3	-3	1
4	-1	1
5	1	1
6	3	1



Points of Intersection

Left Point $(-1, 1)$

Right Point $(5, 1)$

Solutions to equation

$\boxed{x = -1}$ or $\boxed{x = 5}$

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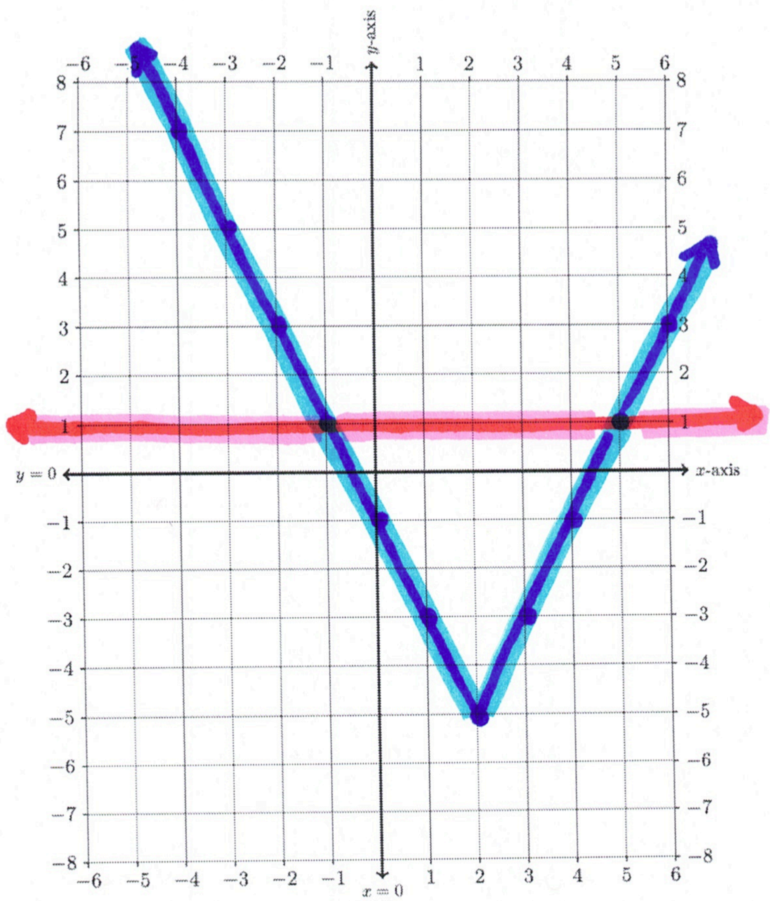
5B. Redraw your graph from problem 5A in the axes below. Then consider the absolute value inequality:

$$\boxed{|2x-4|-5} \geq \boxed{1}$$

Using your graph, identify all x-values that solve this equation.



We want to know where the blue curve is above (or touching) the pink line. Notice this happens on two intervals:

$x \leq -1$ or $x \geq 5$

$\Rightarrow$ $(-\infty, -1]$ or $[5, \infty)$

open parenthesis   closed bracket   closed bracket   open parenthesis

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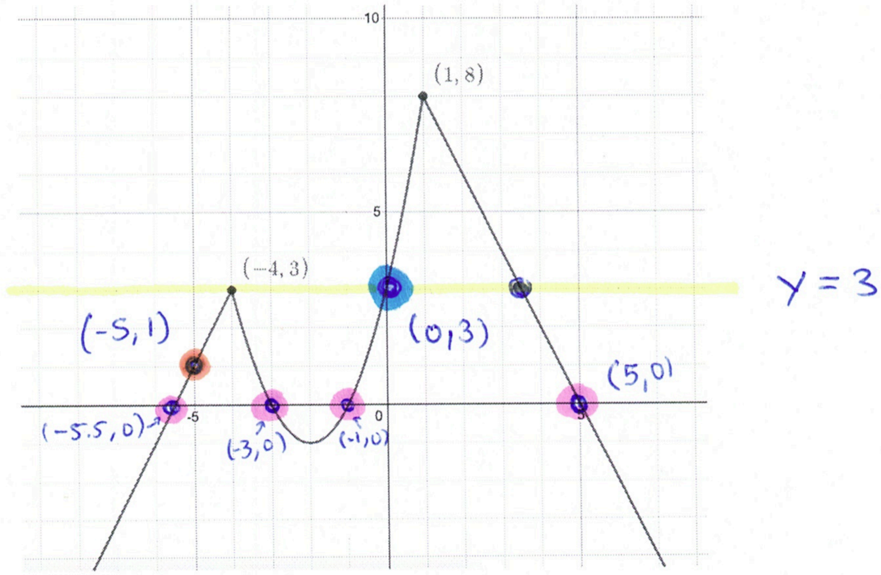
6. ANALYZE THE GRAPH OF A FUNCTION

Below is a graph of a function $f(x)$.



Use the graph to answer each of the following questions about the function f. For some of your answers you may need to approximate the value. Please give a decimal approximation using your best judgment based on the graph.

6A. What is $f(-5)$?

$\boxed{f(-5) = 1}$

associated with ordered pair $(-5, 1)$ on graph

6B. What is $f(0)$?

$\boxed{f(0) = 3}$

associated with ordered pair $(0, 3)$ on graph

6C. Find the x values for which $f(x) = 0$.

We know $f(x) = 0$ when graph of $f(x)$ crosses the horizontal line $y = 0$. These are the x-intercepts (aka the "roots" of $f(x)$) and are highlighted in pink on graph above: $(-5.5, 0)$, $(-3, 0)$, $(-1, 0)$, $(5, 0)$:

$\Rightarrow f(x) = 0 \iff x = -5.5$ or $x = -3$ or $x = -1$ or $x = 5$

6D. Find the x values for which $f(x) \leq 3$.

We want to find the locations where $f(x)$ is below (or touching) horizontal line $y = 3$ (highlighted in yellow):

$(-\infty, 0]$ OR $[3.5, \infty)$

6E. Find the x values for which $f(x) > 9$

* No solution:

we see $f(x) \leq 9$ for all $x \in \mathbb{R}$

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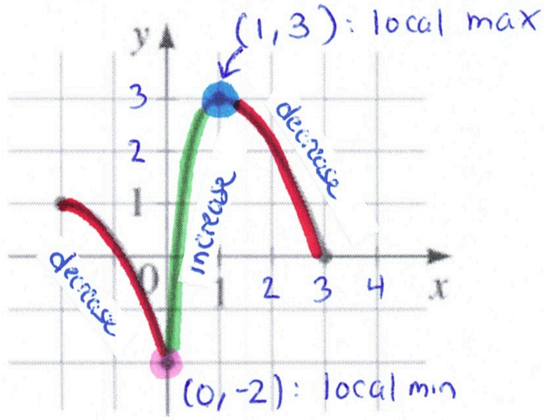
7. ANALYZE THE GRAPH OF A FUNCTION

Below is a graph of a function $k(x)$. Use the graph to answer the questions about the function.



7A. At what point(s) does $k(x)$ have a local maximum?

We see the "highest point" is at the top of graph highlighted in blue with coordinates (1,3)

7B. On what interval(s) is $k(x)$ decreasing.

We see K(x) is decreasing on two intervals

$[-2, 0]$ and $[1, 3]$

Two points on secant line

☐ $(x_1, y_1) = (0, -2)$

☐ $(x_2, y_2) = (3, 0)$

7C. Find the average rate of change of $k(x)$ from $x = 0$ to $x = 3$.

Avg RoC = slope of secant line $= \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{0 - (-2)}{3 - 0}$

$= \boxed{\dfrac{2}{3}}$

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8. EVALUATE FUNCTIONS

For all problems below, let $f(x) = x^2 - x + 3$.

8A. Evaluate $f(3)$ Let $x = 3$. Then consider

$$f(3) = f(x)\Big|_{x=3} = x^2 - x + 3\Big|_{x=3}$$

$$= (3)^2 - 3 + 3$$

$$= 9$$

$$\Rightarrow \boxed{f(3) = 9}$$

8B. Evaluate $f(-5)$

Let $x = -5$ and consider:

$$f(-5) = f(x)\Big|_{x=-5}$$

$$= x^2 - x + 3\Big|_{x=-5}$$

$$= (-5)^2 - (-5) + 3$$

$$= 25 + 5 + 3 \Rightarrow \boxed{f(-5) = 33}$$

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8C. If $f(x) = x^2 - x + 3$, then evaluate $f(2a)$

$$f(2a) = (2a)^2 - 2a + 3$$

$$\Rightarrow \boxed{f(2a) = 4a^2 - 2a + 3}$$

8D. If $f(x) = x^2 - x + 3$, then evaluate $f(a + h)$

$$f(a+h) = (a+h)^2 - (a+h) + 3$$

$$= (a+h)\cdot(a+h) - (a+h) + 3$$

$$= a^2 + a\cdot h + a\cdot h + h^2 - a - h + 3$$

$$= a^2 + 2ah + h^2 - a - h + 3$$

$$= \boxed{a^2 - a + 3 + h^2 + 2ah - h}$$

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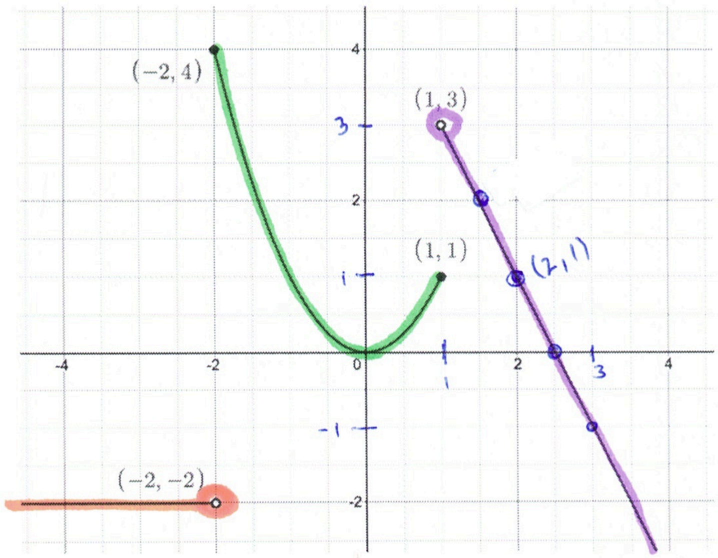
9.	FIND PIECEWISE FUNCTIONS (CHALLENGE PROBLEM)

The following is a graph of a piecewise defined function $g(x)$. Find the formula (rule) for each part of the function and the x-values for which it applies. Explain your reasoning.



$$g(x) = \begin{cases} 2 & \text{if } x < -2 \\ x^2 & \text{if } -2 \le x < 1 \\ -2x + 5 & \text{if } 1 < x \end{cases}$$

Piece 1: orange line at $y = -2$

Piece 2: part of green parabola

Piece 3: Purple line through points $(2, 1)$ and $(3, -1)$

slope $m = \dfrac{-1 - 1}{3 - 2} = \dfrac{-2}{1} = -2$

point slope form: $y - y_1 = m(x - x_1)$

$\Rightarrow y - 1 = -2(x - 2)$

$\Rightarrow y = -2x + 4 + 1$

$\Rightarrow y = -2x + 5$