Name: Solutions Class #:

Math 48B, Quiz 1

1. REVIEW : PROPERTIES OF POWERS

For each of the following statements, simplify the expression as much as possible and eliminate any negative exponents.

1A. $\frac{x^{14}}{x^{-7}}$

1B. $(-3x^3y)^2 \cdot (2x^3y^4)$

1C. $\left(\frac{4x^4y^{-2}}{2x^{-3}y^5}\right) \cdot (-2x^{-2}y^3)$

2. REVIEW : FACTOR THE FOLLOWING EXPRESSION COMPLETELY

Factor the following expressions completely. If you need a review of factoring, please take a look at this review of factoring. Remember: the goal of this is to help you understand... In fact, understanding is more important than getting the correct answer. Go slow and use any resources you need to fully understand the work you are writing.

2A. $2x^2 + 5x - 12$

2B. $14x^3 - 26x^2 - 4x$

3. REVIEW : GRAPHING

Use Desmos.com to graph each of the following two polynomials:

3A. $P(x) = (x-1)(x-2)(x+3)$

3B. $P(x) = x^4 - 3x^3 - 13x^2 + 15x$

Either make a sketch of each graph or take a screen shot and print the graph. On your figure, identify the following relevant features:

- ☐ x-intercepts (also known as the zeros of the graph)
- ☐ y-intercepts
- ☐ End behavior of output values of $P(x)$ as input $x \to -\infty$
- ☐ End behavior of output values of $P(x)$ as input $x \to +\infty$
- ☐ Local minimum values
- ☐ Local maximum values

Problem 1A) Simplify as much as possible

$$\frac{x^{14}}{x^{-7}} = x^{14} \div x^{-7}$$

$$= \frac{x^{14}}{1} \div \frac{1}{x^{7}} \qquad x^{-n} = \frac{1}{x^n}$$

$$= \frac{x^{14}}{1} \cdot \frac{x^{7}}{1} \qquad \frac{A}{B} \div \frac{C}{D} = \frac{A}{B} \cdot \frac{D}{C}$$

$$= \frac{x^{14} \cdot x^{7}}{1 \cdot 1} \qquad \frac{A}{B} \cdot \frac{C}{D} = \frac{AC}{BD}$$

$$= \frac{x^{14+7}}{1} \qquad x^m \cdot x^n = x^{m+n}$$

$$= \boxed{x^{21}}$$

Problem 1B) Simplify as much as possible

$$(-3 \cdot x^3 y)^2 \cdot (2x^3 y^4)$$

$$= (-3 \cdot x^3 \cdot y) \cdot (-3 \cdot x^3 \cdot y) \cdot (2 \cdot x^3 \cdot y^4)$$

$$= -3 \cdot -3 \cdot 2 \cdot x^3 \cdot x^3 \cdot x^3 \cdot y \cdot y \cdot y^4$$

$$= \boxed{18 \cdot x^9 \cdot y^6}$$

Problem 1c) Simplify as much as possible

$$\frac{4x^4 \cdot y^{-2}}{2 \cdot x^{-3}\, y^5} \cdot -2 \cdot x^{-2} \cdot y^3 = \frac{4}{2} \cdot \frac{x^4}{x^{-3}} \cdot \frac{y^{-2}}{y^5} \cdot -2 \cdot x^{-2} \cdot y^3$$

$$= 2 \cdot x^{4-(-3)} \cdot y^{-2-5} \cdot -2 \cdot x^{-2} \cdot y^3$$

$$= 2 \cdot -2 \cdot x^7 \cdot y^{-7} \cdot x^{-2} \cdot y^3$$

$$= -4\,(x^7 \cdot x^{-2})\,(y^{-7} \cdot y^3)$$

$$= -4 \cdot x^{7+-2} \cdot y^{-7+3}$$

$$= -4 \cdot x^5 \cdot y^{-4}$$

$$= \boxed{\frac{-4x^5}{y^4}}$$

Problem 2A) Factor completely

$$2x^2 + 5x - 12 = 2x^2 + 5x + -12$$

$a = 2$, $b = 5$, $c = -12$

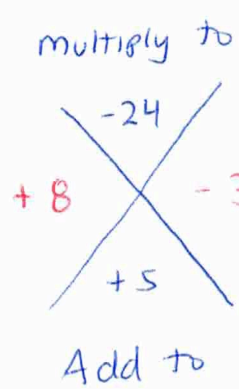
Let's split the middle term using the AC method.

To do that, we want to find two numbers

that ☐ multiply to $a \cdot c = 2 \cdot -12 = -24$

☐ add to $b = +5$

We'll capture this using the diamond below



see next page for more information

Problem 2A, continued...

Let's take a look at factors of product -24:

want to add factors to get b

$$-24 = -1 \cdot 24 \qquad \begin{cases} -1 + 24 = +23 \leftarrow \text{NO} \\ 1 + -24 = -23 \leftarrow \text{NO} \end{cases}$$

$$= -2 \cdot 12 \qquad \begin{cases} -2 + 12 = +10 \leftarrow \text{NO} \\ 2 + -12 = -10 \leftarrow \text{NO} \end{cases}$$

$$= -3 \cdot 8 \qquad -3 + 8 = \underbrace{+5} \leftarrow \text{yes}$$

this is the value of our coefficient in the middle term

Now we can split the middle term

$$5 = -3 + 8$$

$$\Rightarrow 5x = (-3 + 8)x$$

$$\Rightarrow 5x = -3x + 8x = 8x - 3x$$

$$\Rightarrow \quad 2x^2 + 5x - 12 = \underbrace{2x^2 + 8x}_{\text{group 1}} \underbrace{- 3x - 12}_{\text{group 2}}$$

split the middle term

factor by grouping

$$= \overbrace{2x(x+4)}\,\overbrace{-3(x+4)}$$

$$= (2x-3)\cdot(x+4)$$

$$\Rightarrow \quad \boxed{2x^2 + 5x - 12 = (2x-3)\cdot(x+4)}$$

Problem 2B: Factor Completely

We're going to factor the polynomial

$$14x^3 - 26x^2 - 4x$$

completely using our chosen AC method.

We begin our work by searching for a greatest common factor (GCF) which in this case looks like $2x$:

$$14x^3 - 26x^2 - 4x = \underbrace{2x}_{\text{GCF}} \cdot (\underbrace{7x^2 - 13x - 2}_{\text{Remaining quadratic trinomial}})$$

Next, we're going to split the middle term (using the ac-method) of the remaining quadratic trinomial.

Problem 2B, continued...

Notice the quadratic expression that results from factoring out our gcf is

$$a x^2 + b x + c$$

$$\underbrace{7x^2 \underbrace{- 13x}_{\text{middle term}} - 2}_{} = 7x^2 + -13x + -2$$

$a = 7$, $b = -13$, $c = -2$

Let's split the middle term using the AC method by finding two numbers that satisfy the following condition

Condition 1: multiply to $a \cdot c = 7 \cdot -2 = -14$

Condition 2: Add to $b = -13$

We'll start with condition 1 and look at factors of -14

$-14 = -1 \cdot 14$

want to add to get b

$-1 + 14 = +13$ ← NO

$+1 - 14 = \underbrace{-13}_{}$ ← YES

this is the value of coefficient b

Problem 2b, continued...

split middle term

AC method

Multiply To

-14

$\boxed{-14}$ $\boxed{+1}$

-13

Add to

Now we can split the middle term:

$$-13 = -14 + 1$$

$$\Rightarrow -13x = (-14+1) \cdot x$$

$$\Rightarrow -13x = -14x + x$$

$$\Rightarrow 7x^2 - 13x - 2 = \underbrace{7x^2 - 14x}_{\text{group 1}} + \underbrace{x - 2}_{\text{group 2}}$$

factor by grouping

$$= \overbrace{7x(x-2)} + \overbrace{1(x-2)}$$

$$= (7x+1)(x-2)$$

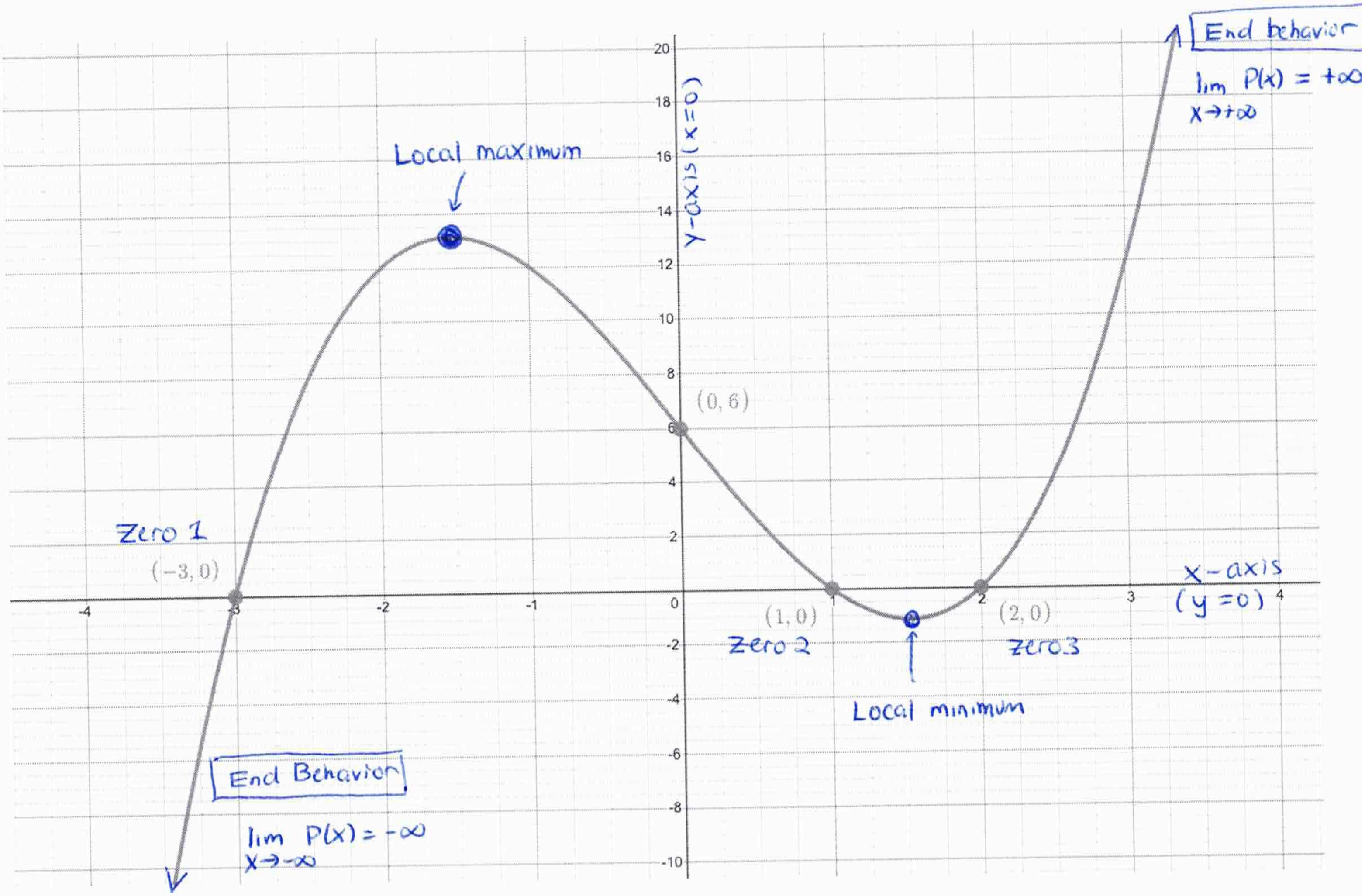
$$\Rightarrow \boxed{\underbrace{14x^3 - 26x^2 - 4x}_{\text{standard form}} = \underbrace{2x \cdot (7x+1) \cdot (x-2)}_{\text{complete zero factorization}}}$$

Problem 3A: Create a the graph of given polynomials and identify relevant features

Below is a screen shot of our desired graph. Using this graph we can identify all the features we desire

☐ $\underbrace{\text{y-intercept}}$ at point (0, 6)

↳ the point where curve crosses the y-axis



☐ x-intercepts (also known as zeros of polynomials) occur at three seperate locations.

Zero 1 at point (-3, 0) $\Leftrightarrow$ $x = -3$

Zero 2 at point (1, 0) $\Leftrightarrow$ $x = 1$

Zero 3 at point (2, 0) $\Leftrightarrow$ $x = 2$

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Problem 3A, continued...

- Recall that the x-intercepts are the points where curve touches horizontal x-axis. In equation form, these are the x-values such that

$$P(x) = (x-1)\cdot(x-2)\cdot(x+3) = \underbrace{0}$$

recall that the height of the x-axis is 0
(In other words, the equation $y=0$ represents the horizontal line called the x-axis)

- Each x-intercept can be written in at least two forms to capture that information:

	Point form	"Zero" form	
Zero 1:	$(-3, 0)$	$x = -3$	← the x-value for zero 1: notice $P(x)\big\rvert_{x=-3} = 0$
Zero 2:	$(1, 0)$	$x = 1$	← the x-value for zero 2
Zero 3:	$(2, 0)$	$x = 2$	← the x-value for zero 3

all y-values for x-intercepts are equal to zero (since these are all on x-axis where $y=0$)

Problem 3A, continued...

Side note: Find standard form

$$P(x) = [(x-1)(x-2)]\cdot(x+3)$$

$$= (x^2 - 2x - x + 2)\cdot(x+3)$$

$$= (x^2 - 3x + 2)\cdot(x+3)$$

$$= (x^2-3x+2)\cdot x + (x^2-3x+2)\cdot 3$$

$$= x^3 - 3x^2 + 2x + 3x^2 - 9x + 6$$

$$= x^3 - 7x + 6$$

We notice from our work above, we have two different forms of our polynomial:

$$P(x) = \underbrace{x^3 - 7x + 6}_{\text{standard form}} = \underbrace{(x-1)\cdot(x-2)\cdot(x+3)}_{\text{complete zero factorization}}$$

Moreover the zeros we found come directly from the complete zero factorization

$$P(x) = (x-1)\cdot(x-2)\cdot(x+3) = 0$$

$$\Rightarrow \quad x-1=0 \quad \text{OR} \quad x-2=0 \quad \text{OR} \quad x+3=0$$

$$\Rightarrow \quad x=1 \quad \text{or} \quad x=2 \quad \text{OR} \quad x=-3$$

Problem 3A, continued...

- The last features of our graph we want to explore is the end behavior as $x \to +\infty$ and also the end behavior as $x \to -\infty$

- Let's begin with the end behavior as $x \to +\infty$. we see that as x gets larger and larger, P(x) grows without bound. We write this as

Symbolic Notation → $$\lim_{x \to +\infty} P(x) = +\infty$$

Nerdy language "the limit of P(x) as x goes to infinity is positive infinity"

Abuelita Language the curve for P(x) goes higher and higher as we move further to the right on the graph.

- Next we check end behavior on left-hand side as $x \to -\infty$. we see that as x gets smaller and smaller, the graph of P(x) decreases toward negative infinity. We write

symbolic notation → $$\lim_{x \to -\infty} P(x) = -\infty$$

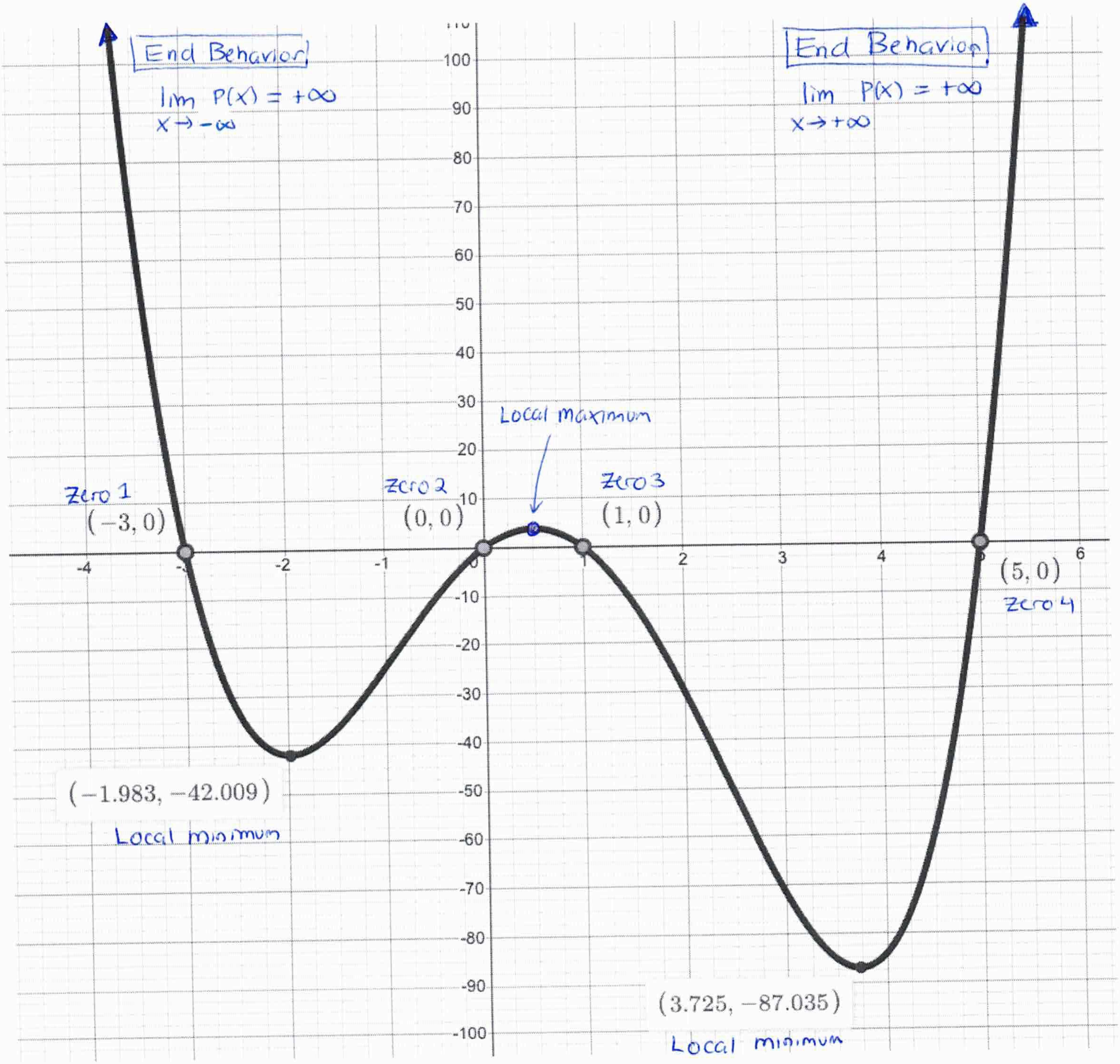
Nerdy language → "the limit as x goes to negative infinity of P of x is negative infinity

Abuelita Language → the curve for P(x) goes lower and lower (in negative direction) as we move further to left on the graph.

Problem 3B: Create the graph of a given polynomial and identify relevant features.

Below is a graph of the function

$$P(x) = x^4 - 3x^3 - 13x^2 + 15x$$



Let's explore all the relevant features of this graph mentioned in the problem statement.

Problem 3b, continued ...

☐ x-intercepts: we see there are four points where the curve for $P(x)$ crosses the x-axis.

	Point form	"Zero" form
Zero 1	$(-3, 0)$	$x = -3 \iff x+3 = 0$
Zero 2	$(0, 0)$	$x = 0 \iff x = 0$
Zero 3	$(1, 0)$	$x = 1 \iff x-1 = 0$
Zero 4	$(5, 0)$	$x = 5 \iff x-5 = 0$

the 2nd coordinate (y-value) for each x-intercept is zero since $y=0$ for all points on x-axis

We can factor our polynomial completely by looking at the "zero" form:

$$(x+3)\cdot x \cdot (x-1)\cdot(x-5) = 0$$

$$\Rightarrow \quad P(x) = x^4 - 3x^3 - 13x^2 + 15x$$ ← standard form

$$= x\,(x+3)(x-1)\cdot(x-5)$$ ← complete zero factorization form