Name : ____________________ © Jeffrey A. Anderson **Class Number**: ______

Exam 2, Version 5A

Math 1C: Calculus III

What are the rules of this exam?

- PLEASE DO NOT TURN THIS PAGE UNTIL TOLD TO DO SO!
- It is a violation of the Foothill Academic Integrity Code to, in any way, assist another person in the completion of this exam. Please keep your own work covered up as much as possible during the exam so that others will not be tempted or distracted. Thank you for your cooperation.
- No notes (other than your note card), books, or classmates can be used as resources for this exam.
- Please turn off your cell phones during this exam. No cell phones will be allowed on your desk.

How long is this exam?

- This exam is scheduled for a 110 minute class period.
- Make sure you have 5 sheets of paper (10 pages front and back) including this cover page.
- There are a total of 7 separate questions (50 points) on this exam including:
  - 6 Free-Response Questions (50 points)
  - 1 Optional, Extra Credit Challenge Problem (5 points)

What can I use on this exam?

- You may use up to three note sheets that are no larger then 11 inches by 8.5 inches. You may write on both sides of these note sheets. PLEASE SUBMIT ALL OF YOUR NOTE SHEETS WITH YOUR EXAM.
- You are allowed to use calculators for this exam. Examples of acceptable calculators include TI 83, TI 84, and TI 86 calculators. You are not allowed to use any calculator with a Computer Algebra System including TI 89 and TI NSpire. If you have a question, please ask your instructor about this.

How will I be graded on the Free-Response Questions?

- Read the directions carefully. Show all your work for full credit. In most cases, a correct answer with no supporting work will NOT receive full credit. What you write down and how you write it are the most important means of getting a good score on this exam. Neatness and organization are IMPORTANT!
- You will be graded on proper use of vector notation.

Free Response

1. (16 points) Let $f : D \subseteq \mathbb{R}^2 \longrightarrow \mathbb{R}$ be a two-variable function with explicit representation $z = f(x, y)$. Let $A\big(a, b, f(a, b)\big)$ be a point on the surface $z = f(x, y)$. Let $\mathbf{u} = \langle u_1, u_2 \rangle$ be a unit vector in the domain of function f.

   A. Using the 5 steps process to constructing a derivative that we discussed in our Lesson 11 videos, derive the limit definition of the directional derivative.

(Problem 1 continued on next page)

B. Use the multivariable chain rule with two intermediate variables and one independent variables to derive the dot product formula for the directional derivative.

C. Use the cosine formula for the dot product to explain which unit vector $\mathbf{u} = \langle u_1, u_2 \rangle$ gives the direction of steepest ascent on the surface. Please explain your reasoning.

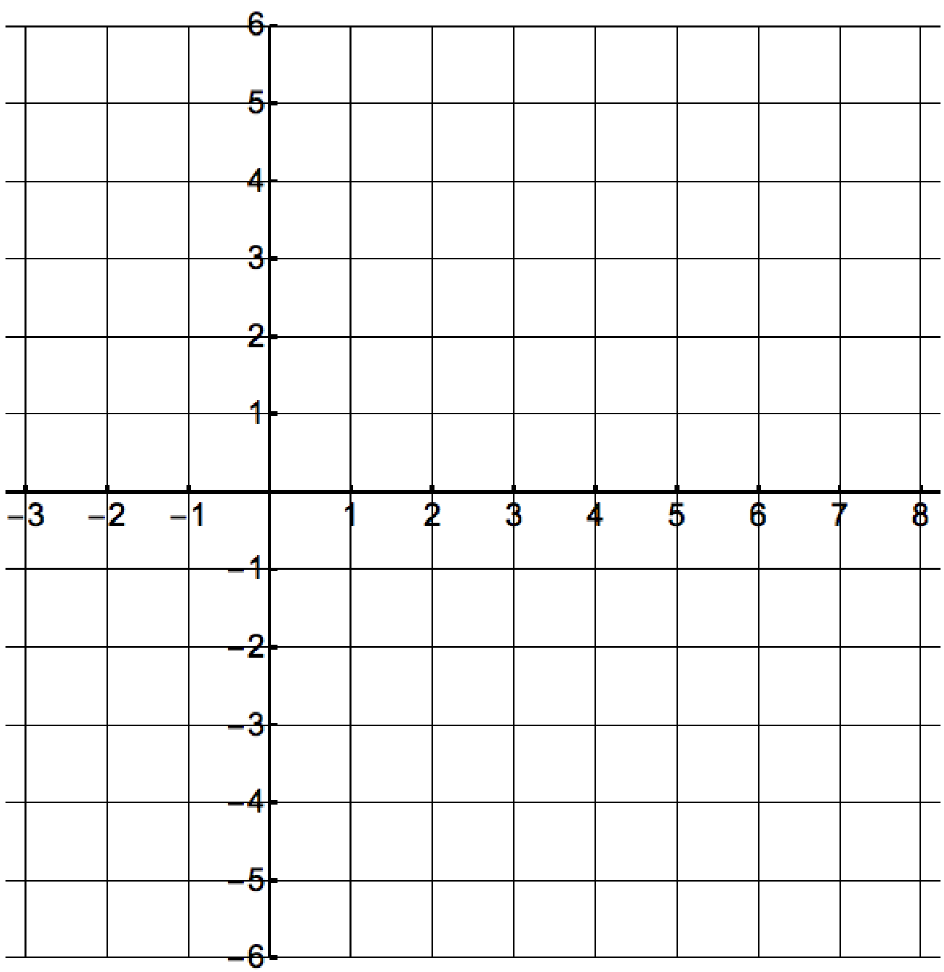
2. (12 points) Let $f(x, y) = x^2 + y^2 - 4x$.

   A. Find a vector-valued equaton for the tangent line to the level curve $L_1(f) = \{(x, y) : f(x, y) = 1\}$ at the point $(1, 2)$.

(Problem 2 continued on next page)

B. Show that the gradient $\nabla f(1,2)$ is orthogonal to the direction of the line you found in part A above.

C. Sketch the level curve, the tangent line, and the gradient vector from parts A. and B. on the axis below.



3. (12 points) Using the second partial derivative test, find the minimum distance from the point $P(1, -2, 4)$ to the plane $3x + 2y + 6z = 5$. Please explain your answer and specifically identify the steps you took to arrive at your final answer. NOTE: To earn full credit, you must use the second partial derivative test (NOT projections or any other method).

4. (10 points) Find the extreme values of the function $f(x,y) = x^2 + 2y^2$ on the circle $x^2 + y^2 = 1$. Please explain your answer and specifically identify the steps you took to arrive at your final answer.

Challenge Problem

5. (Optional, Extra Credit, Challenge Problem) Suppose that the general form of a tangent quadratic approximation to a function $f(x, y)$ at point (α, β) is given by

$$ax^2 + bxy + cy^2.$$

Using this information, explain each of the four conclusions of the second partial derivative test based on the behavior of the quadratic approximation. Make explicit connections to the scalar values of a, b, c and the geometric interpretations of these values based on the behavior of the corresponding quadratic surfaces.