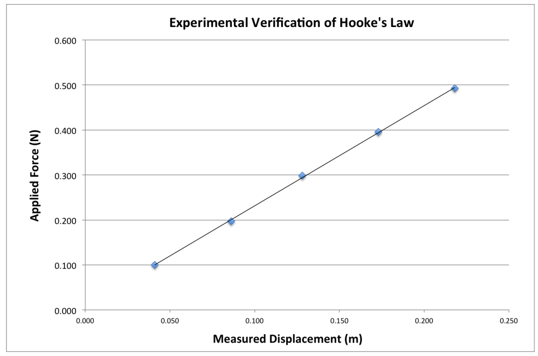
Name : ______________________ © Jeffrey A. Anderson **Class Number**: _____

Spring 2021, Math 1C, Quiz 1
Due: Tuesday 4/20/2021 at 1:30pm (via CANVAS)

Hooke's Law is a principle of physics stating that the force required to stretch a spring u units from the equilibrium position is given by $F(u) = k \cdot u$, where the positive spring constant k measures the stiffness of the spring. Recall from class that we can set up an experiment to verify Hooke's law using a spring, masses of various size, a scale, and a measuring stick. Below are five collected data points relating to Hooks Law. This data is plotted on a graph in the figure next to the table below. Although the data do not exactly lie on a straight line, we can create a linear model to fit this data.

Displacement u in Meters (m)	Applied force f in Newtons (N)
0.041	0.100
0.086	0.197
0.128	0.298
0.173	0.395
0.218	0.492



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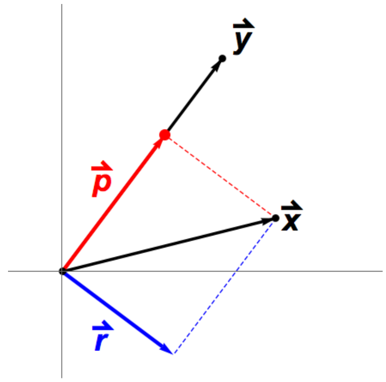
1. Set up a model for the error e_i between the ith data point (u_i, f_i) and any associated linear model

$$f(u) = b + k \cdot u$$

In this case, the parameters $b, k \in \mathbb{R}$ are unknown and the linear function $f(u)$ is the modeled internal force of the spring in Newtons corresponding to a measured displacement u in meters. Then, using the model for the errors, set up the least-squares problem for this input data. In particular, create a two variable function $E(b, k)$ that you can use to solve create the "best-fit" model for this data. Explain in detail the choices that you made to construct your error function E.

2. Suppose that $\mathbf{v}$ is a vector in that starts a point $A(1, 0, -1)$ and ends at point $B(-5, 6, -4)$. Find a vector of length 6 that is in the same direction but opposite orientation of the vector $\mathbf{v}$. You can assume that the new vector you create has an initial point at the origin $O(0, 0, 0)$.

3. (8 points) Let $\mathbf{x}, \mathbf{y} \in \mathbb{R}^3$ and consider the diagram below



Derive an equation for the projection of vector $\mathbf{x}$ onto $\mathbf{y}$. Be sure to specifically DEFINE the vector $\mathbf{p}$ and the vector $\mathbf{r}$ from the diagram above. Please explain your work. For top scores, please demonstrate multiple dimensions of your concept image associated with this derivation.

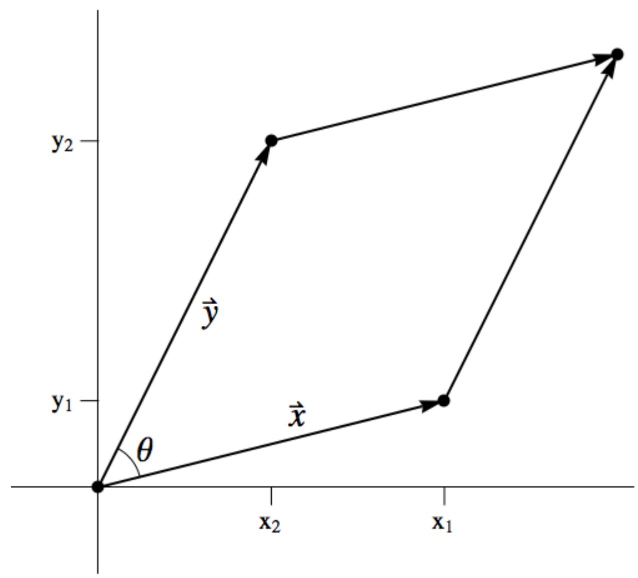
4. Let $\mathbf{a} = \langle 1, 0, 1 \rangle$ and $\mathbf{b} = \langle 0, 1, -1 \rangle$. Express $\mathbf{b}$ as the sum

$$\mathbf{b} = \mathbf{p} + \mathbf{r}$$

where $\mathbf{p}$ is parallel to $\mathbf{a}$ and $\mathbf{r}$ is orthogonal to $\mathbf{a}$. Then, use the cosine formula for the dot product to show that $\mathbf{r}$ is orthogonal to $\mathbf{a}$. Explain your work. Why does this make sense?

Below, please explain your understanding of the cross product between two vectors in $\mathbb{R}^3$ by answering each of the questions 5, 6, and 7 below.

5. Let $\mathbf{x} = \langle x_1, y_1 \rangle$ and $\mathbf{y} = \langle x_2, y_2 \rangle$ be two vectors in $\mathbb{R}^2$. Using the diagram below, derive an equation for the area of the parallelogram formed by vectors $\mathbf{x}$ and $\mathbf{y}$ based only on the components of these vectors (note: this equation should NOT be based on the angle θ between these vectors). Please explain your answer and specifically identify the steps you took to arrive at your final answer.



6. (4 points) Under the same assumptions in problem 3 above, suppose that the variable θ denotes the angle between the vectors $\mathbf{x}, \mathbf{y} \in \mathbb{R}^2$. Derive a formula for the area of the parallelogram parallelogram formed by vectors $\mathbf{x}$ and $\mathbf{y}$ as a function of θ and the two norms of these vectors. Please explain your answer and specifically identify the steps you took to arrive at your final answer.

7. Explain how we can use our work on problems 3 and 4 above to derive the component form of the cross product between the vectors $\mathbf{a}, \mathbf{b} \in \mathbb{R}^3$ where

$$\mathbf{x} = \langle x_1, y_1, z_1 \rangle, \qquad \mathbf{y} = \langle x_2, y_2, z_2 \rangle,$$

Make sure to explicitly state the component form of the cross product in your explanation. Please explain your work. For top scores, please demonstrate multiple dimensions of your concept image associated with this derivation.