Name : ______________ © Jeffrey A. Anderson **Class Number**: ______

Spring 2021, Math 1C, Quiz 4

Due: Thursday 06/24/2021 at 11:59pm (via CANVAS)

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1. Let $a_k = a \cdot r^{k-1}$ for scalars $a, r \in \mathbb{R}$, where $k \in \mathbb{N}$ with $a \neq 0$ and $r \neq 1$.

   A. Show that $S_n = \sum_{k=1}^{n} a_k = a\frac{1-r^n}{1-r}$.

B. Use your result from part A. to derive the geometric series test.

2. Use the geometric series test to express the repeating decimal $2.451141414\overline{14}...$ as a fraction.

3. Use the integral test to verify that the infinite series

$$\sum_{k=1}^{\infty} \frac{1}{(2k+1)^6}$$

converges. In your solution, I expect you to do more than just quote the p-series test. I want to see that you can apply the integral test by showing all work associated with this approach.

4. Using the remainder theorem for the integral test, find how many terms of the series $\sum_{k=1}^{\infty} \frac{1}{(2k+1)^6}$ are required to ensure that the approximate is accurate to within $\epsilon = \frac{5}{10^6}$

5. Determine if the series

$$\sum_{k=1}^{\infty} \frac{k!}{(2k+1)!}$$

converges or diverges. Give reasons for your answer and explain which test(s) you relied on to form your conclusion.

Optional Challenge Problems (Ungraded)

6. Find the points $\left(x, y, z\right)$ on the hyperboloid defined by the implicit equation

$$x^2 + 4y^2 - z^2 = 4$$

where the tangent plane is *parallel* to the plane $2x + 2y + z = 5$.

7. Find the extreme values of the function

$$f(x, y) = x^2 + y^2$$

subject to the constraint that $x^2 - 2x + y^2 - 4y = 0$.

8. Using the second partial derivative test, find the point on the surface defined by the explicit function

$$f(x, y) = x^2 + y^2 + 10$$

that is nearest to the plane $x + 2y - z = 0$. Please explain your answer and specifically identify the steps you took to arrive at your final answer. NOTE: To earn full credit, you must use the second partial derivative test (NOT projections or Lagrange multipliers or any other method. However, you can check your work using a second method or earn partial credit using a different approach).

9. Let $f : D \subseteq \mathbb{R}^2 \longrightarrow \mathbb{R}$ be a two-variable function with continuous second-order partial derivatives on the domain D and suppose $(a, b) \in D$. Write the general formula for the tangent quadratic approximation $T_2(x, y)$ to a function $f(x, y)$ at point $(a, b, f(a, b))$. Assume $\nabla f(a, b) = \mathbf{0}$. Show that we can write

$$T(x, y) = ax^2 + bxy + cy^2 + k_0.$$

Make sure to give formulas for coefficients $a, b,$ and c in terms of the second-order partial derivatives $f_{xx}(a, b)$, $f_{xy}(a, b)$, and $f_{yy}(a, b)$. Using this information, explain each of the four conclusions of the second partial derivative test based on the behavior of the quadratic approximation. Make explicit connections to the scalar values of a, b, c and the geometric interpretations of these values based on the behavior of the corresponding quadratic surfaces.

Use for Scratch Work

