Name : _____ **Class Number**: _____

Spring 2021, Math 1C, Quiz 3
Due: Tuesday 06/08/2021 at 11:59pm (via CANVAS)

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1. Using the second partial derivative test, to find the point(s) on the cone $z^2 = x^2 + y^2$ nearest the point $P(-2, 2, 0)$. Show your work.

2. Find the point (x, y) so that the tangent plane to the surface

$$z = f(x, y) = x^2 - 5xy + y^2$$

at the point $\Big(x, y, f(x, y)\Big)$ is *parallel* to the plane $x + 8y + z = 4$.

3. Consider the function

$$f(x, y) = (x - 1)^2 + (y - 2)^2$$

Find the minimum and maximum values of $f(x, y)$ subject to the constraint that $x^2 + y^2 = 45$.

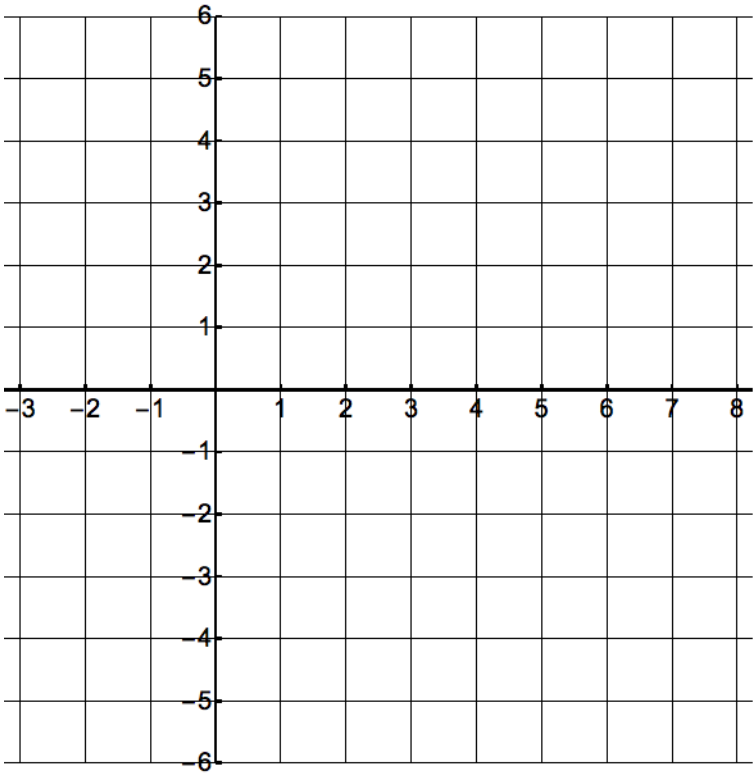
4. For problems 4 - 5, let $f(x, y) = x^2 + y^2 - 6x + 2y - 10$. Given this definition, find a vector-valued equation for the tangent line to the level curve

$$L_5(f) = \{(x, y) \, : \, f(x, y) = 5\}$$

at the point $(6, -5)$.

5. On the axes below, sketch the level curve $L_5(f)$ and it's the tangent line from problem 4 above. Also, sketch the vector $\mathbf{u} \in \mathbb{R}^2$ with tail at point $(6, -5)$ where $\mathbf{u}$ is the unit vector in the direction of the gradient vector $\nabla f(6, -5)$ given by

$$\mathbf{u} = \frac{\nabla f(6, -5)}{\|\nabla f(6, -5)\|_2}$$



Now, use full sentences to explain how your graph above relates your knowledge about the shape of the surface $f(x, y)$ and your solution to problem 6 above.

6. Use the second partial derivative test to find the point on the plane $x + y + z = 1$ nearest the point $P(2, 0, -3)$. Show your work in detail.

7. Find a point(s) on the surface $x^2 + y^2 + z^2 = 1$ with a normal vector to the corresponding tangent plane that is parallel to vector $(1, 1, 1)$.