Name : ______________________ © Jeffrey A. Anderson **Class Number**: ______

Spring 2021, Math 1C, Quiz 2

Due: Tuesday 05/25/2021 at 11:59pm (via CANVAS)

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1. Consider the two lines

$$\mathbf{r}_1(t) = \langle 1+t, 1-t, 2t\rangle \qquad \text{and} \qquad \mathbf{r}_2(s) = \langle -s+2, s, 2\rangle$$

   Let point $\mathbf{r}_0$ be the point of intersection between $\mathbf{r}_1(t)$ and $\mathbf{r}_2(s)$. Find the equation for a line $L(\tau)$ that passes $\mathbf{r}_0$ and is is orthogonal to both lines. Make sure to draw a diagram and explain your work.

2. Recall that for a circle centered at the point (h, k) with radius $r > 0$, we can describe this circle using parametric equation

$$\langle h + r\cos(\theta), k + r\sin(\theta)\rangle$$

Starting with the pythagorean theorem and the definition of a circle as all points r units from the center (h, k), derive the parametric equation for the circle from first principle.

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3. Given three points $P(1,-1,0)$, $Q(0,1,2)$, and $R(-1,-1,1)$, find the distance from the point P to the line passing through points Q and R.

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4. Find the area of a triangle whose vertices are $A(1, -1, 2)$, $B(4, 0, 1)$, and $C(2, -1, 2)$.

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For problems 5 - 8, let $f : D \subseteq \mathbb{R}^2 \longrightarrow \mathbb{R}$ be a two-variable function with explicit representation $z = f(x, y)$. Let $A\big(a, b, f(a, b)\big)$ be a point on the surface

$$S_f = \{(x, y, z) : (x, y) \in D \text{ and } z = f(x, y)\}.$$

Let $\mathbf{u} = \langle u_1, u_2 \rangle$ be a unit vector in the domain of function f.

5. Please derive the limit definition of the directional derivative from first principles. If you're confused where to start, please follow the 5 steps process to constructing a derivative that we discussed in our Lesson 11 videos.

6. Using the limit definition for the directional derivative of f in the direction of $\mathbf{u}$ at the point (a, b) that you derived in problem 5 above, show how to construct a composite function $g(t)$. This single variable function should have the property that the derivative $g'(t)$ is the same value as the limit we constructed to compute the directional derivative in problem 5.

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7. Derive the dot product formula for the directional derivative. Be sure to specifically refer to the the function $g(t)$ from problem 6 above along with the multivariable chain rule with two intermediate variables and one independent variables. When appropriate, please explicitly state and use the multivariable chain rule in your work. Also, make sure to explain the value of t that you use to take the ordinary derivative in this derivation.

8. Using your work in problem 7, explain which unit vectors $\mathbf{u} = \langle u_1, u_2 \rangle$ in the domain D give

   A. the direction of steepest ascent on the surface.

   B. the direction of no change on the surface.

   C. the direction of steepest descent on the surface.

   Please provide evidence that your concept images associated with these directions incorporate multiple categories of knowledge including verbal, graphical, and symbolic representations of these ideas. To earn top scores, your solution should combine the work you did in problem 7 with the cosine formula for the dot product. Also, please make specific connections to between your explanations of each direction and your knowledge of the extreme values of the cosine function.

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For problems 9 - 10, let $f(x,y) = 15 - x^2 - 4y^2 + 2x - 40y$.

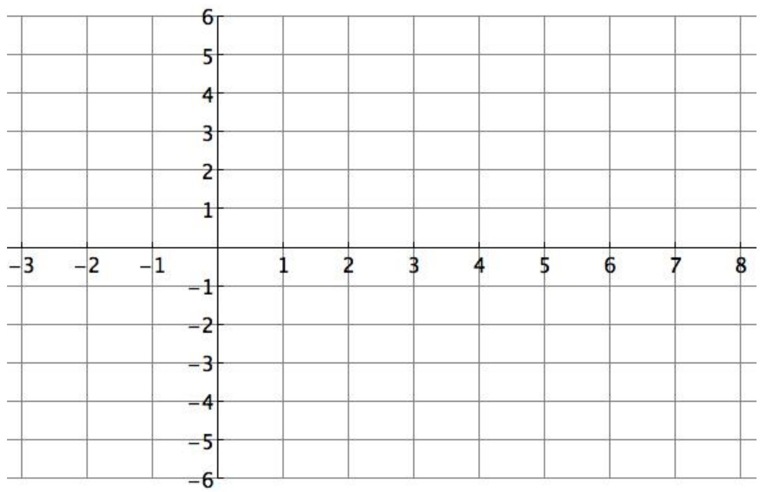
9. Find a vector-valued equation for the tangent line to the level curve

$$L_{100}(f) = \{(x,y) \,:\, f(x,y) = 100\}$$

at the point $(-3,-5)$.

10. On the axes below, sketch the level curve $L_{100}(f)$ and it's the tangent line from problem 5 above. Also, sketch the vector $\mathbf{u} \in \mathbb{R}^2$ with tail at point $(-3, -5)$ where $\mathbf{u}$ is the unit vector in the direction of the gradient vector $\nabla f(-3, -5)$ given by

$$\mathbf{u} = \frac{\nabla f(-3, -5)}{\|\nabla f(-3, -5)\|_2}$$



Now, use full sentences to explain how your graph above relates your knowledge about the shape of the surface $f(x, y)$ and your solution to problem 8 above.