Name: Solutions Class #:______

3. GRAPH A RATIONAL FUNCTION

Use Desmos.com to graph the rational function

$$g(x) = \frac{3}{x+2}. = 3 \cdot \frac{1}{x+2}$$

✓ ✓ ✓ ✓

Identify the vertical and horizontal asymptotes of this graph. Write your results using the notation we are developing and explain what this notation means. Then use transformations to connect function $g(x)$ to the function $f(x) = \frac{1}{x}$. Also, ✓ find the domain and range for this function.

Let's do some analysis:

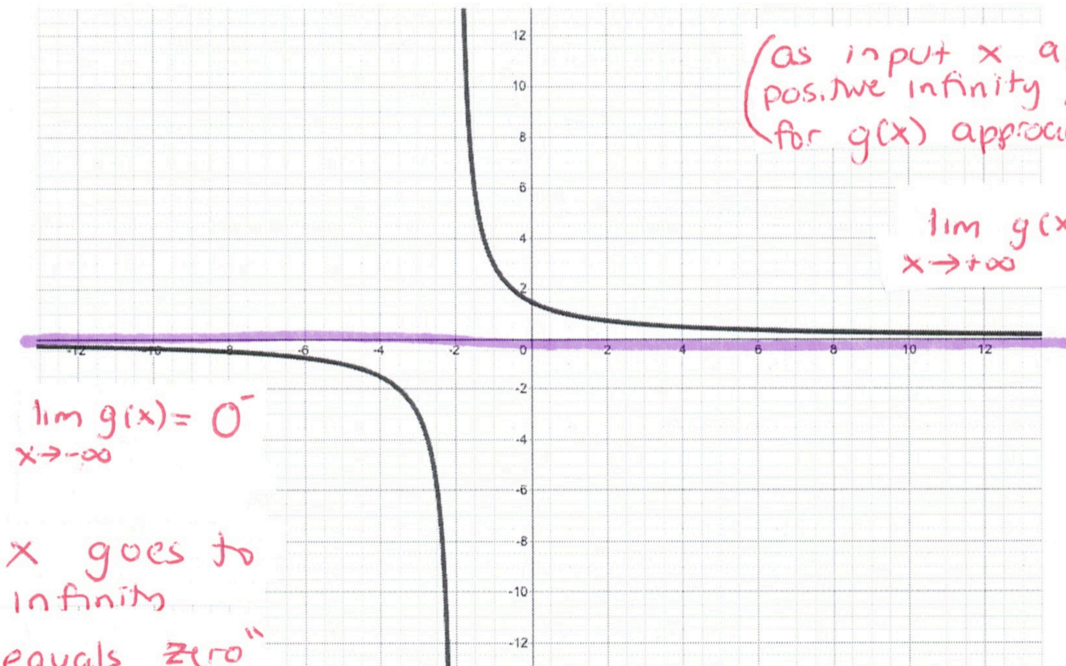
Horizontal asymptote: find the end behavior by making x go out really far to left $(-\infty)$ or right $(+\infty)$

Vertical asymptote: spaces where the denominator is zero

Lesson 9, Problem 3, continued...

- The horizontal asymptote is the line we are approaching as $x \to +\infty$ or $x \to -\infty$.

- In this case, we see that asymptote on the x-axis. For all points on x-axis $y = 0$



(as input x approaches positive infinity, the output for g(x) approach 0 from abo

$$\lim_{x \to +\infty} g(x) = 0^+$$

horizontal asymptote $\boxed{y = 0}$

$$\lim_{x \to -\infty} g(x) = 0^-$$

"limit as x goes to negative infinity of g(x) equals zero"

- We can use this to write the equation for our horizontal asymptote is $\boxed{y = 0}$

- To find horizontal asymptote, we remember we're looking behavior of $g(x)$ as $x \to \infty$.

Remember: $\dfrac{1}{\text{HUGE NUMBER}}$ = tiny number (vanishes to zero)

$\boxed{\text{Quick trick:}}$

$$\frac{3}{x+2} = \frac{3}{(x+2)} \cdot \boxed{\frac{\frac{1}{x}}{\frac{1}{x}}} \qquad x \neq 0$$

$$= \frac{3/x}{1 + 2/x} \approx \frac{0}{1} = 0$$

$\Rightarrow \quad x \to \infty \quad$ we see $\quad 3/x \to 0$

$\qquad\qquad\qquad\qquad\qquad\qquad 2/x \to 0$

Lesson 9, Problem 3, continued ...

Remember:

$$\frac{1}{\text{tiny number}} = \text{HUGE NUMBER}$$

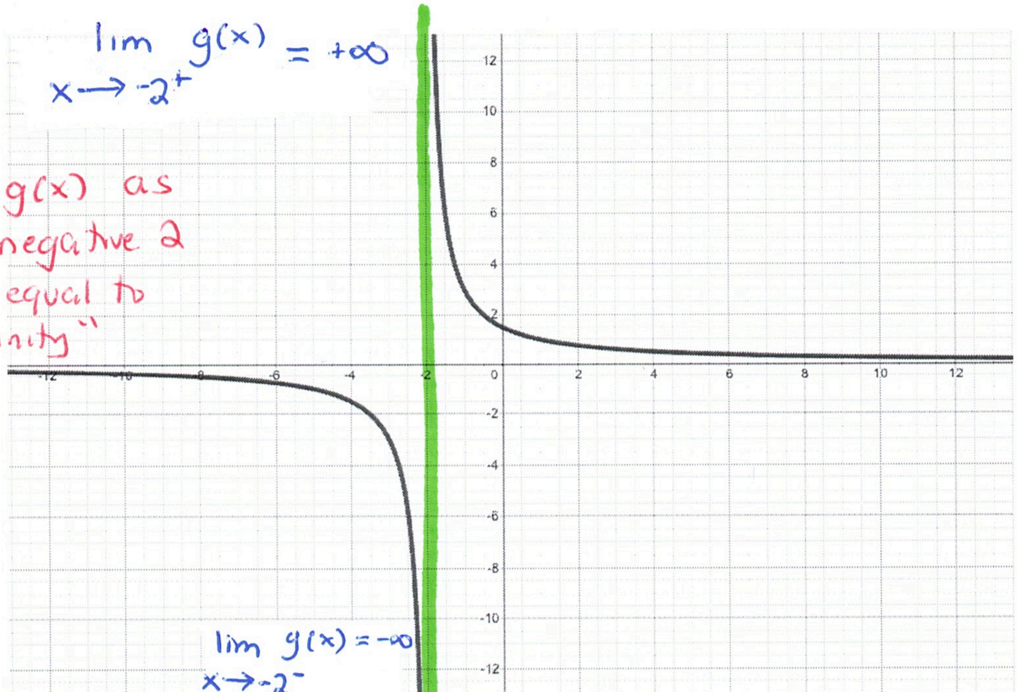
If $g(x) = \dfrac{3}{x+2}$, we know denominator can't be zero

$$\Rightarrow \quad x + 2 \neq 0$$

$$\Rightarrow \quad x \neq -2$$ ← at $x = -2$, we have some interesting behavior

$$\lim_{x \to -2^+} g(x) = +\infty$$

"the limit of g(x) as x approaches negative 2 from above is equal to positive infinity"



$$\lim_{x \to -2^-} g(x) = -\infty$$

"the limit of g(x) as x approaches negative 2 from the left is negative infinity"

- ☐ Notice from previous work $x \to -2^-$, we see that output $g(x) \to -\infty$

- ☐ As we approach -2 from positive side, our $g(x)$ will go to positive infinity

- ☐ Here we have a vertical asymptote $\boxed{x = -2}$

Example:

$$\frac{1}{\text{tiny number}}$$

Recall:

$$\frac{A}{B} \div \frac{C}{D} = \frac{A}{B} \cdot \frac{D}{C}$$

eg 1 :

$$\frac{1}{1} \div \frac{1}{1000} = \frac{1}{1/1000} \qquad \frac{1}{}$$

← tiny number

$$= \frac{1}{1} \cdot \frac{1000}{1}$$

$$= 1000$$

← Big number

eg2:

$$1 \div \frac{1}{10{,}000{,}000} = \frac{1}{1} \div \frac{1}{10^7}$$

$$= \frac{1}{1} \cdot \frac{10^7}{1}$$

$$= 10^7$$

$$\Rightarrow \frac{1}{1/10^7} = \boxed{10^7}$$

← HUGE NUMBER

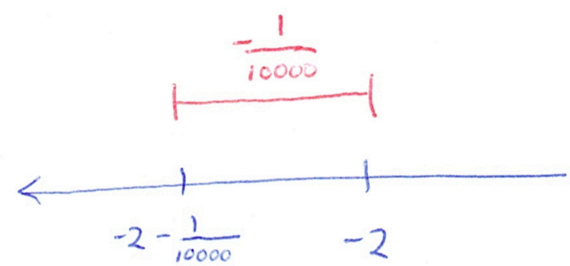
tiny number

We Know $x \neq -2$ since this results in a zero denom.

But, we can explore what happens as we "approach" input -2 from either side.

Let's approach $x \to -2^-$ (go toward -2 from left)

eg 1: $x = -2 - \frac{1}{10000}$



$$\Rightarrow g(x) = \left.\frac{3}{x+2}\right|_{x=-2-\frac{1}{10000}}$$

$$= \frac{3}{\cancel{-2} - \frac{1}{10000} + \cancel{2}}$$

$$= \frac{3}{-1/10000}$$

tiny number (close to zero)

$$= \frac{3}{1} \div \frac{-1}{10000}$$

$$= \frac{3}{1} \cdot \frac{-10000}{1} = \boxed{-30000}$$

HUGE Negative number

6

eg 2: $x = \underbrace{-2 - \frac{1}{10^n}}_{\text{somthing really close to } -2}$ where $n \gg 10$

(like $n = 70000$)

$$\Rightarrow g(x) = \left. \frac{3}{x+2} \right|_{x = -2 - \frac{1}{10^n}}$$

$$= \frac{3}{\cancel{-2} - \frac{1}{10^n} + \cancel{2}}$$

$$= \frac{3}{-1/10^n}$$

tiny number (close to zero)

$$= \frac{3}{1} \div \frac{-1}{10^n}$$

$$= \frac{3}{1} \cdot \frac{-10^n}{1}$$

$$= \boxed{-3 \cdot 10^n}$$ ← Huge number

Let's relate $g(x) = \frac{\boxed{3}}{x+2}$ to $f(x) = \frac{\boxed{1}}{x}$

Consider $g(x) = \frac{3}{x+2}$

$$= \frac{3 \cdot 1}{1 \cdot (x+2)}$$ factors out a 3

$$= \frac{3}{1} \cdot \frac{\boxed{1}}{(x+2)}$$ $\leftarrow$ $\frac{AC}{BD} = \frac{A}{B} \cdot \frac{C}{D}$

$$= 3 \cdot f(x+2)$$ since $f(x) = \frac{1}{x}$

$$= 3 \cdot f(x - -2) + 0$$

$$= a \cdot f(x - h) + k$$

vertical stretch

horizontal shift

vertical shift

$\boxed{a = 3 \quad h = -2 \quad k = 0}$

Finally, let's determine the domain and range:

Domain of $g(x)$: ▫ this is all valid input values for x

▫ in this case, $x \neq -2$

$\Rightarrow \text{dom}(g) = \{x : x \neq -2\}$

$\Rightarrow \text{dom}(g) = (-\infty, -2)$ OR $(-2, +\infty)$

(never touches -2)

Range of $g(x)$: ▫ this is all achieved output $y = g(x)$ values

▫ in this case we hit almost everything except $y = 0$

▫ $\text{Rng}(g) = \{y : y \neq 0\}$

▫ $\text{Rng}(g)$ is $(-\infty, 0)$ OR $(0, +\infty)$

open parenthesis

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4. GRAPH ANOTHER RATIONAL FUNCTION

To get good grades in math classes, read the whole problem

Use Desmos.com to graph the rational function

$$h(x) = \frac{2x+5}{x-3}$$

Identify the vertical and horizontal asymptotes of this graph. Write your results using the notation we are developing and explain what this notation means. Then use transformations to connect function $h(x)$ to the function $f(x) = \frac{1}{x}$. Also, find the domain and range for this function.

Let's start this problem with a transformation and find

$$h(x) = a \cdot f(x-h) + k$$

vertical shift

horizontal shift

vertical scale and reflection

In the form it's written, is not so easy to see this...

Let's try polynomial long division to identify our desired shifting constants

$$\begin{array}{r|ll} & 2 & \\ \hline x-3 & 2x & +5 \\ & -2x & +6 \\ \hline & 0 & +11 \end{array}$$

$$\Rightarrow \quad h(x) = \frac{2x+5}{x-3}$$

$$= 2 + \frac{11}{x-3}$$

$$= \frac{11}{x-3} + 2$$

$$= \frac{11}{1} \cdot \boxed{\frac{1}{\boxed{x-3}}} + 2$$

Note: the denominator can never be zero $(x \neq 3)$

$$= 11 \cdot f(\boxed{x-3}) + 2$$

vertical shift $k=2$ upward

$$= a \cdot f(x-h) + K$$

$a = 11$

$h = 3$

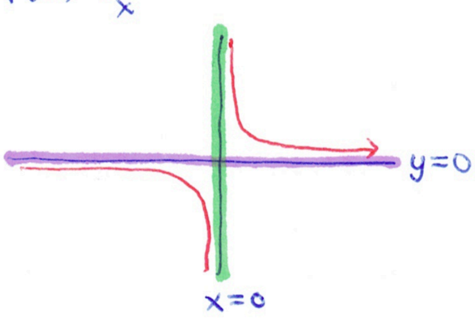
$k = 2$

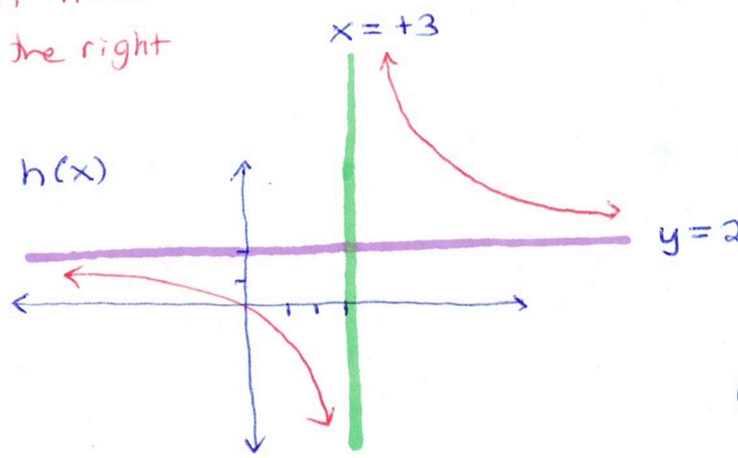
vertical stretch (No reflection)

horizontal shift $h=3$ to the right

Guess:

$f(x) = \frac{1}{x}$





Let's explore why: $f(x-3) = \frac{1}{x-3}$

$$f(x) = \frac{1}{x} \longleftrightarrow f(\boxed{\cdot}) = \frac{1}{\boxed{\cdot}}$$

eg1: $f(\boxed{2}) = \frac{1}{\boxed{2}}$

eg2: $f(\boxed{2+3}) = \frac{1}{\boxed{2+3}}$

$$f(\boxed{x-2}) = \frac{1}{\boxed{x-2}}$$

$$f(x) = \frac{1}{x} \qquad f(\boxed{-2}) = \frac{1}{-2}$$

- ☐ $\frac{1}{x-3}$ looks like a variation of the function $\boxed{\frac{1}{x}}$ parent graph to be transfo
- ☐ Since I recognize this, I can try to make $f(x) = \frac{1}{x}$ into form $\frac{1}{x-3}$ by changing input ...

input inside the parenthesis is identical to denom

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6. GRAPH ANOTHER RATIONAL FUNCTION

Use Desmos.com to graph the rational function

$$R(x) = \frac{2x^2 - 4x + 5}{x^2 - 2x + 1} = \frac{2x^2 - 4x + 5}{\boxed{(x-1)^2}}$$

CAN'T BE zer

Identify the vertical and horizontal asymptotes of this graph. Write your results using the notation we are developing and explain what this notation means. Explore methods to find the horizontal and vertical asymptotes using algebra and make connections between your findings and the graph you create. Then, find the domain and range for this function.

To find vertical asymptotes, remember:

- ☐ look for zeros in denominater
- ☐ $\frac{1}{\text{tiny number}}$ = Huge Number

$\Rightarrow$ $\boxed{x = 1 \text{ vertical asymptote}}$

To find Horizontal asymptotes,

- ☐ look for end behavior
- ☐ multiply by $\frac{1}{x^n}$

$\Rightarrow$ $y = 2$ is horizontal asympotote

To find horizontal asymptote: $x \to \pm\infty$

$$\frac{(2x^2 - 4x + 5)}{(x^2 - 2x + 1)} \cdot \underbrace{\frac{\left[\frac{1}{x^2}\right]}{\left[\frac{1}{x^2}\right]}}_{1 \text{ if } x \neq 0}$$

multiply by 1 in disguise

Recall: $\frac{A}{A} = 1$ as long as $A \neq 0$

$$= \frac{2x^2 \cdot \frac{1}{x^2} - 4x \cdot \frac{1}{x^2} + 5 \cdot \frac{1}{x^2}}{x^2 \cdot \frac{1}{x^2} - 2x \cdot \frac{1}{x^2} + 1 \cdot \frac{1}{x^2}}$$

$$\frac{x^1}{x^2} = \frac{x \cdot \boxed{1}}{x \cdot \boxed{x}} = \frac{1}{x}$$

$$= \frac{2 - 4 \cdot \frac{1}{x} + \frac{5}{x^2}}{1 - 2 \cdot \frac{1}{x} + 1/x^2}$$

goes toward zero as x grow

$$= \frac{2 - \boxed{4/x} + \boxed{5/x^2}}{1 - \boxed{2/x} + \boxed{1/x^2}}$$

$$\boxed{x \to +\infty}$$

$$\frac{1}{\text{Huge Number}} = \text{tiny number}$$

$x \to +\infty$

$$\approx \boxed{\frac{2 - 0 + 0}{1 - 0 + 0} \approx 2} \leftarrow \text{Horizontal asymptote}$$