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Math 48B, Lesson 6: Zeros of Polynomials, Part 2

In Math 48B Lessons 3, 4, 5, and 6, we study techniques to find the zeros of a polynomial by factoring that polynomial into a form:

$$\underbrace{\boxed{a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x^1 + a_0 x^0}}_{\substack{\text{Standard Form of an nth Degree Polynomial} \\ \text{(written in descending order)}}} = \underbrace{\boxed{a(x - c_1) \cdot (x - c_2) \cdots (x - c_n)}}_{\substack{\text{Complete Zero Factorization Form} \\ \text{(Factored completely into linear factors)}}}$$

In this lesson, we continue our exploration from Lesson 4.

0. USE REMAINDERS TO FIND ZEROS

0A. Read the following theorem and "how to" sections:

THE REMAINDER THEOREM

If a polynomial $f(x)$ is divided by $x - k$, then the remainder is the value $f(k)$.

HOW TO

Given a polynomial function f, evaluate $f(x)$ at $x = k$ using the Remainder Theorem.

1. Use synthetic division to divide the polynomial by $x - k$.
2. The remainder is the value $f(k)$.

Look back at your work from Math 48A, Lesson 5. Now, translate these theorems into abuelita (intuitive) language. What, exactly, are is this saying about the zeros of a polynomial.

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0B. Read the following theorem and "how to" sections:

THE FACTOR THEOREM

According to the **Factor Theorem**, k is a zero of $f(x)$ if and only if $(x - k)$ is a factor of $f(x)$.

HOW TO

Given a factor and a third-degree polynomial, use the Factor Theorem to factor the polynomial.

1. Use long division to divide the polynomial by $(x - k)$.
2. Confirm that the remainder is 0.
3. Write the polynomial as the product of $(x - k)$ and the quadratic quotient.
4. If possible, factor the quadratic.
5. Write the polynomial as the product of factors.

How are these sections related to your work on Problem 0A above?

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1. CREATE A POLYNOMIAL WITH DESIRED ZERO VALUES

1A. Create a quadratic polynomial with two zero given by

$$x = \frac{2}{5} \qquad \text{and} \qquad x = -\frac{3}{4}$$

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1B. Put the quadratic function from Problem 1A above into standard from

$$P(x) = a_2 x^2 + a_1 x^1 + a_0 x^0.$$

List all possible factors of the coefficients a_2 and a_0.

1C. What relationship do you notice between the rational zeros of this polynomial from Problems 1AB and the factors of the coefficients a_2 and a_0.

2. USE DESMOS TO FIND THE ZEROS

2A. Use Desmos.com to find the zeros of the polynomial

$$P(x) = 2x^3 + x^2 - 4x + 1$$

If our goal is to find the zeros of this polynomial, how can we do that using the graph of our function.

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2B. Put the quadratic function from Problem 2A above into standard from

$$P(x) = a_3\, x^3 + \; a_2\, x^2 + \; a_1\, x^1 + a_0\, x^0.$$

List all possible factors of the coefficients a_2 and a_0.

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2C. What relationship do you notice between the rational zeros of this polynomial from Problems 2AB and the factors of the coefficients a_3 and a_0.

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3. INTERPRET THEOREMS ABOUT ZEROS

Read the theorem and "how to" section below.

THE RATIONAL ZERO THEOREM

The **Rational Zero Theorem** states that, if the polynomial $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$ has integer coefficients, then every rational zero of $f(x)$ has the form $\frac{p}{q}$ where p is a factor of the constant term a_0 and q is a factor of the leading coefficient a_n.

When the leading coefficient is 1, the possible rational zeros are the factors of the constant term.

HOW TO

Given a polynomial function $f(x)$, use the Rational Zero Theorem to find rational zeros.

1. Determine all factors of the constant term and all factors of the leading coefficient.
2. Determine all possible values of $\frac{p}{q}$, where p is a factor of the constant term and q is a factor of the leading coefficient. Be sure to include both positive and negative candidates.
3. Determine which possible zeros are actual zeros by evaluating each case of $f(\frac{p}{q})$.

Translate this theorem into abuelita language. What, exactly, is this theorem saying about the zeros of polynomials?