Name: Solutions

V : visual
V : verbal
S : symbol

Class #:______

Math 48B, Lesson 4: Dividing Polynomials, Part 2

In Math 48B Lessons 3, 4, 5, and 6, we are going to learn how to find the zeros of a polynomial by factoring that polynomial into a form:

zero 1 zero 2 zero n

$$\boxed{a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x^1 + a_0 x^0} = \boxed{a(x - c_1)\cdot(x - c_2)\cdots(x - c_n)}$$

Standard Form of an nth Degree Polynomial (written in descending order)

Complete Zero Factorization Form (Factored completely into linear factors)

In this lesson, we continue our exploration from Lesson 3.

1. WHAT IS MULTIPLICATION?

Please multiply and simplify the following polynomial expression:

Acronym FOIL

F : First
O : Outer
I : Inner
L : Last

$$P(x) = (6x - 2)(x - 4) + 4$$

$$P(x) = (6x - 2)\cdot(x - 4) + 4$$

$$= \underset{F}{6x^2} \underset{O}{- 24x} \underset{I}{- 2x} \underset{L}{+ 8} + 4$$

combine like terms

$$= 6x^2 - 26x + 12$$

Note: where does FOIL come from:

$\boxed{\text{Conquer Math Class}}$

- ☐ Minimize the number of things I have to remember
- ☐ memorize key ideas from which I can create other knowledge

$\boxed{\text{Distributivity}}$

Little Distributivity: $A \cdot (B \pm C) = A \cdot B \pm A \cdot C$

Right Distributivity: $(A \pm B) \cdot C = A \cdot C \pm BC$

$$\underset{(A\ -\ B)\ \cdot}{(6x - 2)} \cdot \underset{C}{\boxed{(x - 4)}} = 6x(x-4) - 2 \cdot (x - 4)$$

$$= \underset{F}{6x^2} - \underset{O}{24x} - \underset{I}{2x} + \underset{L}{8}$$

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2. WHAT IS DIVISION?

Use the same idea of the long-division algorithm from our previous life in mathematics to execute the following division problem:

numerator

quotient

$$\frac{\boxed{N}}{\boxed{D}} = \frac{6x^2 - 26x + 12}{(x-4)} = \boxed{q}\,\text{R}\,\boxed{r}$$

denominator

remainder (left over)

" we never really understand math. we just get used it"
- John von Neumann

Polynomial Long division

what do I need to multiply x by to get $6x^2$ (match the term exactly)

What do I need to multiply x by to match the term exactly

$$\begin{array}{r l}
 & (\boxed{6x} - 2) \\
(x-4) & \overline{)\,6x^2 - 26x + 12} \\
 & -\,6x^2 + 24x \\
\hline
 & 0 - 2x + 12 \\
 & +\,2x - 8 \\
\hline
 & 0 + 4
\end{array}$$

$-2(x-4)$
$= -2x + 8$

Remainder notation

numerator

$$\Rightarrow \frac{\boxed{6x^2 - 26x + 12}}{\boxed{(x-4)}} = \boxed{(6x-2)}\ \text{R}\ \boxed{4}$$

denom. quotient remainder

Division Problem

numerator

$$\frac{\boxed{6x^2 - 26x + 12}}{\boxed{(x-4)}} = (6x-2) + \frac{4}{(x-4)}$$ ← addition notation

denominator

$$= (6x-2)\frac{4}{(x-4)}$$ ← mixed-number notation

$$= \underbrace{\boxed{(6x-2)}}_{\text{quotient}}\ R\ \underbrace{\boxed{4}}_{\text{remainder}}$$ ← remainder notation

Multiplication Problem

Numerator = Denominator • quotient + remainder

$$6x^2 - 26x + 12 = (x-4)\cdot(6x-2) + 4$$

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3. PRACTICE WITH POLYNOMIAL DIVISION

3A. Please multiply and simplify the polynomial expression given below. Identify the locations of the zeros of this polynomial. Then specifically identify where these zeros show up in either of the two forms of this polynomial (complete factorization and standard form).

$$P(x) = (x-1)(x-2)(x+3)$$

Let's consider :

$$P(x) = \left[\underbrace{(x-1)\cdot(x-2)}\right]\cdot(x+3)$$

Side note:

$$(x-1)\cdot(x-2) = x\cdot(x-2) - 1(x-2)$$
$$= x^2 - 2x - x + 2$$
$$= x^2 - 3x + 2$$

$$= \overbrace{(x^2 - 3x + 2)}\cdot(x+3)$$

$$= (x^2 - 3x + 2)\cdot x + (x^2 - 3x + 2)\cdot(+3)$$

$$= \boxed{x^3} - \cancel{3x^2} + 2x + \cancel{3x^2} - 9x + 6$$

$$= \boxed{x^3 - 7x + 6}$$

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3B. Practice the polynomial long-division algorithm we explored above on the problem given below:

$$\frac{N}{D} = \frac{x^3 - 7x + 6}{(x-2)} = q \text{ R } r$$

$\boxed{\text{Polynomial Long Division}}$

$$
\begin{array}{r l}
 & \quad x^2 + 2x - 3 \\
(x-2) & \overline{)\, x^3 - 7x + 6} \\
 & -\ x^3 + 2x^2 \\
\hline
 & 0 + 2x^2 \\
 & \quad -\ 2x^2 + 4x \\
\hline
 & \quad 0 - 3x \\
 & \qquad + 3x - 6 \\
\hline
 & \qquad 0 + \boxed{0}
\end{array}
$$

Remainder

$$\Rightarrow \frac{x^3 - 7x + 6}{(x-2)} = \boxed{(x^2 + 2x - 3)} \text{ R } \boxed{0}$$

quotient

remainder

remainder notation

$$\Rightarrow \quad x^3 - 7x + 6 = (x-2)\cdot(\underbrace{x^2 + 2x - 3}_{\text{we can factor this using AC method}})$$

$$\Rightarrow \quad \underbrace{x^3 - 7x + 6}_{\text{standard form}} = \underbrace{(x-2)(x+3)\cdot(x-1)}_{\text{complete zero factorization}}$$

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4. MORE PRACTICE WITH POLYNOMIAL DIVISION

4A. Construct a polynomial $P(x)$ that has four zeros at $-3, 0, +1$, and $+5$. Create both forms of this polynomial: the complete factorization form and also the standard form. When looking at the standard form, make sure that the degree three term x^3 has a coefficient of $a_3 = -6$. Using Desmos.com, create a graph to confirm that your polynomial has the desired zero points.

We know that we have zeros:

$x = -3$ or $x = 0$ or $x = +1$ or $x = +5$

$\Rightarrow x + 3 = 0$ or $x = 0$ or $x - 1 = 0$ or $x - 5 = 0$

* complete zero factorization form *

$\Rightarrow \boxed{(x+3) \cdot x \cdot (x-1) \cdot (x-5)} = 0$

$\Rightarrow (x^2 + 3x) \cdot (x^2 - 6x + 5) = 0$

$\Rightarrow x^2 \cdot (x^2 - 6x + 5) + 3x \cdot (x^2 - 6x + 5) = 0$

$\Rightarrow x^4 - 6x^3 + 5x^2 + 3x^3 - 18x^2 + 15x = 0$

$\Rightarrow x^4 - 3x^3 - 13x^2 + 15x = 0$

$\Rightarrow 2 \cdot (x^4 - 3x^3 - 13x + 15x) = 2 \cdot 0$

$\Rightarrow \boxed{2x^4 - 6x^3 - 26x + 30x} = 0$

standard form

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Notice on this problem, we have

$$\underbrace{2 \cdot x \cdot (x-5) \cdot (x-1) \cdot (x+3)}_{\text{complete zero factorization}} = \underbrace{2x^4 - 6x^3 - 26x + 30x}_{\text{standard form}}$$

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4B. Practice the polynomial long-division algorithm we explored above on the problem given below:

$$\frac{N}{D} = \frac{2x^4 - 6x^3 - 26x^2 + 30x}{(x+3)} = q \; \mathrm{R} \; r$$

CAREFUL: we made a silly algebra

NO

$$\boxed{2x^3} \qquad \boxed{\cancel{-26x}} \; \cancel{+108}$$

$$\boxed{(\boxed{x} + 3)} \;\Big|\; \boxed{2x^4} - 6x^3 - \cancel{26x^2 + 30x} \quad \text{NO}$$

$$-(2x^4 \boxed{+6x^3})$$

NO

$$0 \; + \; -(-\cancel{26x^2 - 78x})$$

$$0 + 108x$$

side note

$2x^3(x+3) = 2x^4 + 6x^3$

$-26x(x+3) = -26x^2 - 78$

$$\begin{aligned} -26 \cdot 3 &= -(20+6)\cdot 3 \\ &= -(10\cdot 2 + 6)\cdot 3 \\ &= -(10\cdot 6 + 18) \\ &= -(60+18) \\ &= -78 \end{aligned}$$

careful: I should have put a negative here (the results are sewed)

$$
\begin{array}{r|l}
 & 2x^3 - 12x^2 + 10x \\
\boxed{(x+3)} & \boxed{2x^4} - 6x^3 - 26x^2 + 30x \\
 & -\ 2x^4 - 6x^3 \\
 & 0 \ \boxed{-12x^3} \\
 & \quad +12x^3 + 36x^2 \\
 & \quad 0 \ +10x^2 \\
 & \qquad -10x^2 - 30x \\
 & \qquad 0 \ + 0
\end{array}
$$

$\boxed{\text{math work flow}}$

side note

$2x^3(x+3) = 2x^4 + 6x^3$

$-12x^2(x+3) = -12x^3 - 36x^2$

$10x\ (x+3) = 10x^2 + 30x$

Numerator quotient remainder

$$\frac{\boxed{2x^4 - 6x^3 - 26x^2 + 30x}}{\boxed{(x+3)}} = \boxed{(2x^3 - 12x^2 + 10x)} \, R \, \boxed{0}$$

Denominator

Remainder notation

$$= \boxed{2x^3 - 12x^2 + 10x + \frac{0}{x+3}}$$

add notation

$$= \boxed{(2x^3 - 12x^2 + 10x) \frac{0}{x+3}}$$

mixed notation

Check this out:

$$2x^3 - 12x^2 + 10x = 2x\,(\underbrace{x^2 - 6x + 5})$$

$$= 2x\,(x-5)(x-1)$$

quotient · denominator + remainder

Numerator

$$\Rightarrow \boxed{2x^4 - 6x^3 - 26x + 30x} = \underbrace{(2x^3 - 12x^2 + 10x)}\cdot(x+3) + 0$$

standard form

$$= \underbrace{2x\cdot(x-5)\cdot(x-1)\cdot(x+3)}$$

complete zero factorization form

$\Rightarrow$ zeros are at $\boxed{x=0,\ x=5,\ x=1,\ x=-3}$ ✓