Name: Solutions ______

V : visual
V : verbal
S : symbolic

Class #:______

Math 48B, Lesson 3: Dividing Polynomials, Part 1

In Math 48B Lessons 3, 4, 5, and 6, we are going to learn how to find the zeros of a polynomial by factoring that polynomial into a form:

$$\boxed{a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x^1 + a_0 x^0} = \boxed{a(x - \overset{\text{zero 1}}{c_1}) \cdot (x - \overset{\text{zero 2}}{c_2}) \cdots (x - \overset{\text{zero n}}{c_n})}$$

Standard Form of an nth Degree Polynomial
(written in descending order)

Complete Zero Factorization Form
(Factored completely into linear factors)

To start our exploration of this topic, we explore the topic of division and build visual, verbal, and symbolic representations for the division operation. The work we do in this lesson will support our work with polynomials in the coming lesson 4.

1. WHAT IS MULTIPLICATION? (See pages 2-6 for background)

1A. Use abuelita language (simple language that your grandma would understand) to describe what you see in multiplication problems below:

$3 + 3 + 3 + 3 + 3 = \boxed{5 \cdot 3} = 15$

product of two factors

(we take two things and multiply them)

$\boxed{5} \cdot \boxed{3} = \boxed{15}$

left factor

right factor

product

$5 + 5 + 5 = 3 \cdot 5 = 15$

"You have 3 rows with 5 boxes each how many boxes do you have total?"

Math Memories Make Money :
factor : a piece of a multiplication

Visual representation of multiplication

Claim: $\boxed{\text{every multiplication problem is a counting problem in disguise}}$

$\boxed{5} \cdot \boxed{3}$

left factor   right factor

- this is the same as three five times
- this product of five times three equals the right factor added to itself the amount of times as shown in right factor
- the product represent the total number of items/units that results from this count
- when we write $5 \cdot 3$ were counting the total number of units that result from adding groups of size 3 units 5 seperate times together. the product is the total number of units

Why are we exploring this? Let's return to our discussion of polynomials ...

Let's remember our work from last week:

degree n

$$\text{If } \underbrace{P(x)}_{\text{polynomial function}} = \underbrace{a_n x^{n} + \cdots + a_1 x^1 + a_0 x^0}_{\text{standard form of an nth degree polynomial}}$$

then a zero of P(x) is

- like an x-intercept

  zeros of P(x) is asking for what does x equal when the entire thing is set to zero:

$$P(x) = 0 = a_n x^n + \cdots + a_1 x^1 + a_0 x$$

- Remember that an x-intercept is a point where graph touches the x-axis. Each of those points, we can write as

1st coordinate (x-value)

2nd-coordinate: y-value

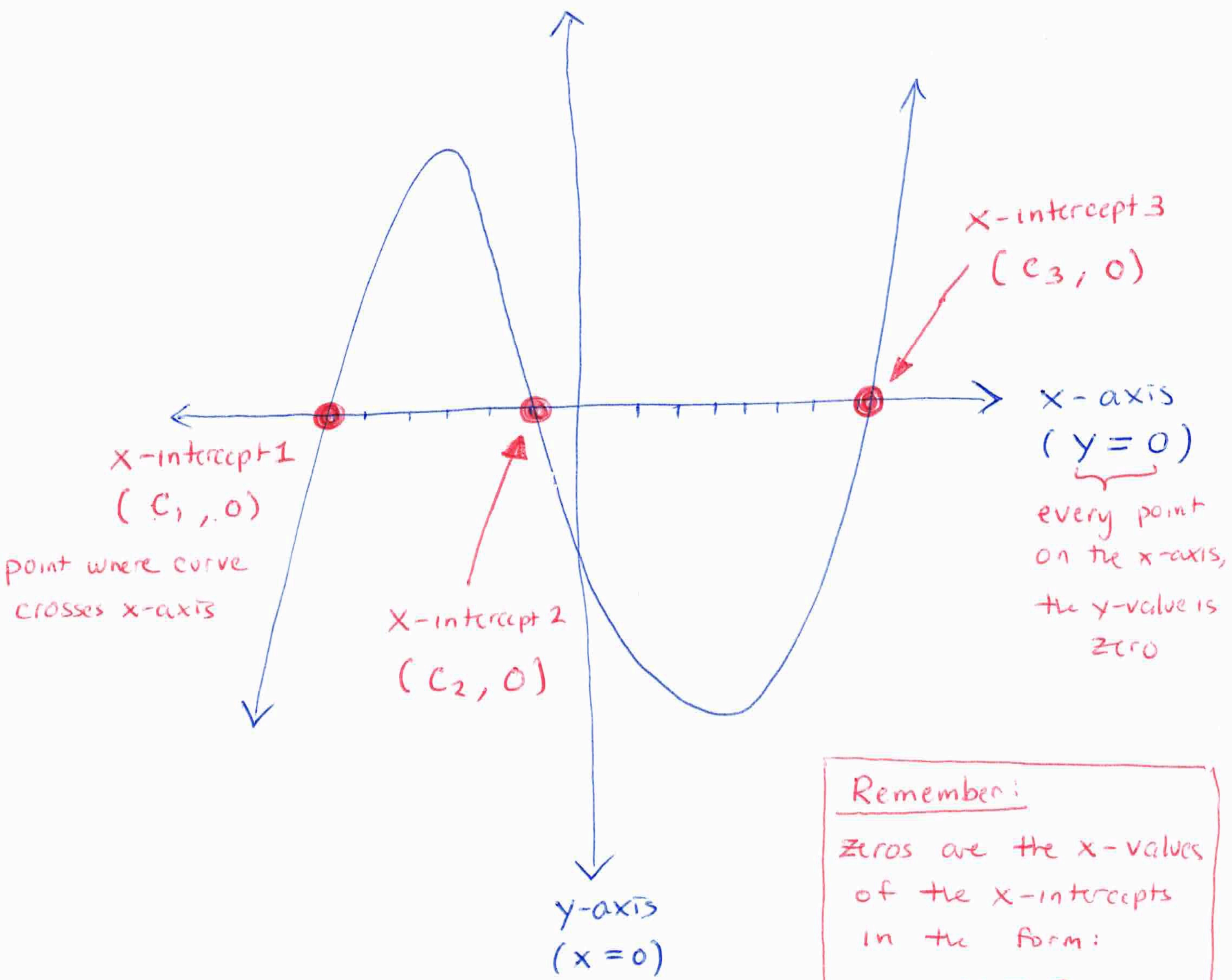
$$(c, 0)$$

y-value of x-intercept is zero since graph is touching x-axis (and height of x-axis is zero)

Visual representation:

We can write our zero as

$$x = c$$



Remember:

Zeros are the x-values of the x-intercepts in the form:

$$x = c$$

The <u>zeros</u> of our function are given as

$$x = c_1 \quad \text{or} \quad x = c_2 \quad \text{or} \quad x = c_3$$

$$\Rightarrow \quad x - c_1 = 0 \quad \text{or} \quad x - c_2 = 0 \quad \text{or} \quad x - c_3 = 0$$

$$\Rightarrow \quad (x - c_1)\cdot(x - c_2)\cdot(x - c_3) = 0 \cdot 0 \cdot 0 = 0$$

$$\Rightarrow \quad a \cdot (x - c_1)\cdot(x - c_2)\cdot(x - c_3) = a \cdot 0 = 0$$

$$\Rightarrow \quad \boxed{a \cdot \underbrace{\overset{\text{zero 1}}{(x - c_1)} \cdot \overset{\text{zero 2}}{(x - c_2)} \cdot \overset{\text{zero 3}}{(x - c_3)}} = 0}$$

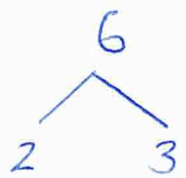
this is called the complete zero factorization form of our Polynomial

this highlights the location of all zeros

Analogy: Prime factorization of an integer

$$6 = 2 \cdot 3$$

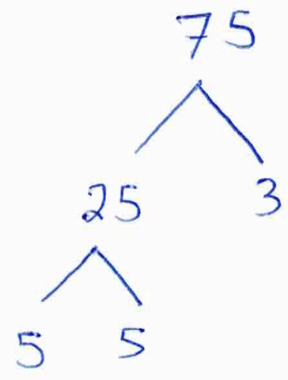
Factor Tree



Analogy : Prime factorization

- $$75 = 3 \cdot 5 \cdot 5$$

$$= 3^1 \cdot 5^2$$

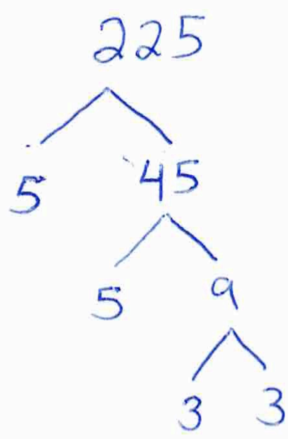


To translate standard form into a factored form we used division.

- $$\overbrace{225}^{\text{Standard form}} = \overbrace{3 \cdot 3 \cdot 5 \cdot 5}^{\text{Factored form}}$$

prime factors

(each part of a $\boxed{\text{product}}$ is called a $\boxed{\text{factor}}$)



$\boxed{\text{Long division}}$

$$\begin{array}{r|l} & \;\;45 \\ \hline 5 & 225 \\ & \underline{-20} \\ & \;\;\,25 \end{array}$$

Claim: - We break down an nth degree polynomial into n seperate factors, each one in a zero

- Remember: factors are the individual parts that when multiplied together form the product

We will learn to factor a polynomial

- Forward problem: Multiply seperate factors together to form a product

"In the forward, we start with both factors 1 & 2 and move forward to form a product"

factor 1 · factor 2 = Product

$$3 \cdot 5 = 15$$

forward towards product

$$\boxed{D} \cdot \boxed{q} = \boxed{N}$$

factor 1 · factor 2 = product

- Backward Problem:

"In the backward problem we start with one factor D and also the product N and work backward towards the other factor"

factor 1 (Known) · q = product

$$3 \cdot q = 15 \Rightarrow q = \frac{15}{3}$$

q: unknown & desired; quotient

$$D \cdot q = N \Leftrightarrow q = \frac{N}{D}$$

D: Known factor 1; q: unknown factor 2; N: Known product; q: quotient; N: numerator; D: denominator

- We're going to learn how to $\boxed{\text{divide}}$ polynomials in standard form to find the zero factors and hopefully create it's factored form.

- Let's do a refresher on division: that is Lesson 3.

Name:______________________________ Class #:______

Homework: VVS

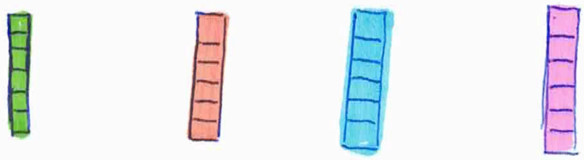
1B. Develop your own visual and verbal descriptions, like the one in problem 1A above, to describe BOTH of the multiplication problems below:

$$4 \cdot 6 = 6 \cdot 4$$

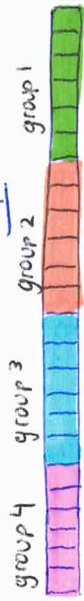
right factor counts the size of each group

addition is designed to count the total number of items

$\boxed{4} \cdot \boxed{6} = 6 + 6 + 6 + 6 = 24$



group 1 group 2 group 3 group 4



left factor counts the number of groups

multiplication problem is designed to count the total number of items: take four groups each of one has six units... How many total units do we have?

multiplication

product

When we write $\boxed{4} \cdot \boxed{6} = \boxed{24}$

left factor right factor

we want to know how many total items we have if we take four groups and each group has 6 units in it. This counting problem is identical to taking four copies of the number 6 and adding those together

$4 \cdot 6 = 6 + 6 + 6 + 6$

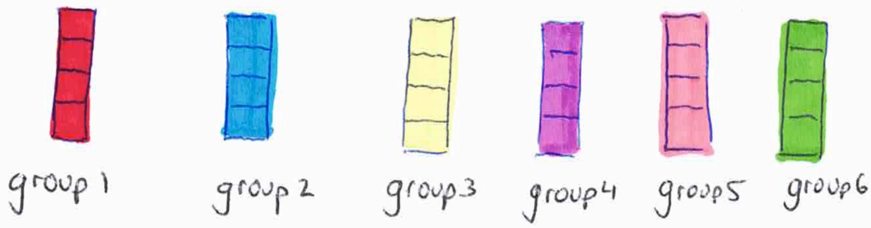
Let's take a look at the other direction:

left factor
ounts the
umber of groups

right factor
indicates the
size of each group

these addition operations
count the total number of units
represented in all summands

$$\boxed{6} \cdot \boxed{4} = 4 + 4 + 4 + 4 + 4 + 4 = 24$$



this multiplication problem
is designed to count the
total number of items
that results from taking
six groups (left factor), where
each group has four units (right factor)

When we write

multiplication

product

$$\underset{\text{left factor}}{\boxed{6}} \cdot \underset{\text{right factor}}{\boxed{4}} = \boxed{24}$$

we want to figure out (count) the total number of units that result from taking six groups where each group has four units in it. This count is equivalent to combining six copies of the number 4 and adding together:

$$6 \cdot 4 = 4 + 4 + 4 + 4 + 4 + 4$$

Name:______________________________ Class #:______

1C. Using your work in problems 1A and 1B above, develop a verbal and visual interpretations of the symbols written in the multiplication problems:

$$\underset{\text{left factor}}{\boxed{D}} \cdot \underset{\text{right factor}}{\boxed{q}} = \underset{\text{product}}{\boxed{N}}$$

$\boxed{\text{Intuition}}$

- ☐ we have D rows with q boxes each, how many total boxes do we have (which would be N).
- ☐ this is the same as taking "q-units" and adding those to itself by "D-times".

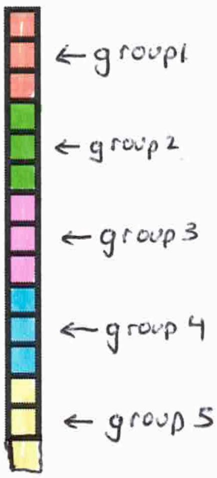
The product N is a count of the total number of things that result from adding q-thing D-times over.

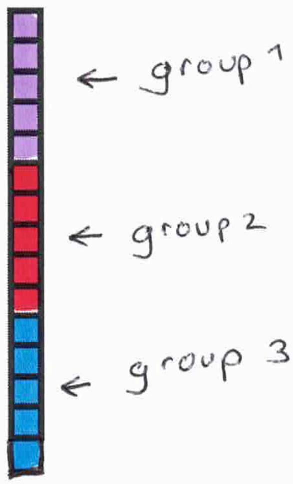
Name:______________________________ Class #:_______

2. WHAT IS DIVISION?

2A. Develop a visual representation for the division problems you see below. Then, describe what you've done in each problem using abuelita language. In this work, make explicit connections to Problem 1A above.

$$\frac{N}{D} = \frac{15}{3} = q = 5 \qquad \frac{N}{D} = \frac{15}{5} = q = 3$$





Numerator (product)

$$\frac{15}{3} = 5$$

Denominator (right factor)

quotient (left factor)

☐ when we divide 15 by 3, we take 15 and seperate into group of size three (groups that have 3 units) then count how many individual groups of size three we have

Name:________________________ Class #:______

Homework:

2B. Develop your own visual and verbal representations, like the work you did in problem 2A above, to describe BOTH of the division problems:

numerator (product)

$$\frac{\boxed{N}}{\boxed{D}} = \boxed{\frac{24}{6}} = 4 = \boxed{q}$$

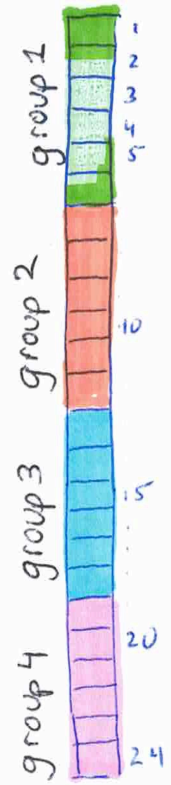
denominator (right factor)

quotient (left factor)

seperate 24 into groups of size 6:

How many groups do we gets?

we've got four seperate groups, each of size 6



4
5
6
9

Notice that we have a connection between our division and multiplication problems:

numerator

quotient

$$\frac{\boxed{24}}{\boxed{6}} = \boxed{4} \iff \boxed{24} = \boxed{4} \cdot \boxed{6}$$

denominator

product

left factor

right factor

☐ When we divide 24 by 6, we cut 24 items into seperate groups each of which has 6 units. The division problem asks us to figure out how many groups we have ...

Let's take a look at the other division

Numerator

$$\frac{24}{4} = \boxed{6}$$

quotient

Denominator

V : verbal
V : visual
S : symbolic

Name:______________________ Class #:______

Homework:

2C. Using your work in problems 2A and 2B above, develop verbal and visual interpretations of the symbols written in the division problem below. Then make explicit connections between this division problem and the corresponding multiplication problem you did in Problem 1C.

numerator → $\boxed{N}$
denominator → $\boxed{D}$

$$\frac{\boxed{N}}{\boxed{D}} = \boxed{q}$$

quotient

Note:
- factor : is a part of a multiplication problem

Division Problem

$$\frac{\boxed{N}}{\boxed{D}} = \boxed{q}$$

numerator (product)

denominator (left factor)

quotient (right term)

Multiplication Problem

$$\boxed{D} \cdot \boxed{q} = \boxed{N}$$

left factor (denominator)

right factor (quotient)

product (Numerator)

verbal Intuition

- seperate N units into groups of size D. after we seperate N into groups, each group has D units, we find we have q number of groups
- division is really special because the denominator is guaranteed amount: the number of units in each group
- Division forces equal sizing

Name: Solutions ______ -

V:	Visual
V:	Verbal
S:	Symbolic

Class #:______

4. HOW TO USE REMAINDER NOTATION?

Take a look at the following notation:

$$\frac{N}{D} = \boxed{q} + \frac{\boxed{r}}{D} \longleftarrow \text{Addition notation}$$

numerator, denominator, quotient, remainder

$$= \boxed{q}\frac{\boxed{r}}{D} \longleftarrow \text{Mixed-number notation}$$

quotient, remainder

$$= \boxed{q}\ \mathrm{R}\ \boxed{r} \longleftarrow \text{Remainder notation}$$

quotient, remainder

This is called remainder notation.

Division Problem

Multiplication Problem

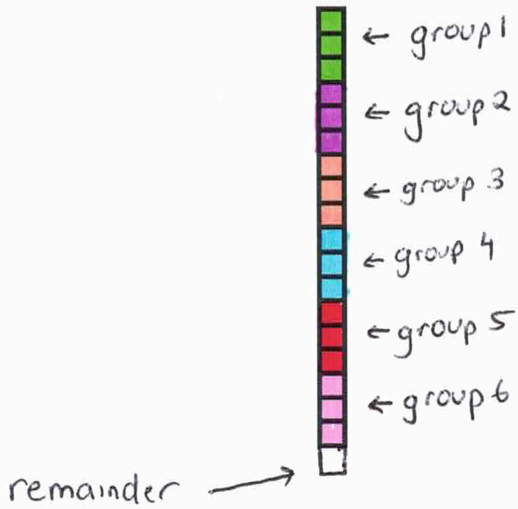
$$\frac{N}{D} = q\ R\ r \quad \Leftrightarrow \quad N = D \cdot q + r$$

Name:________________________ Class #:______

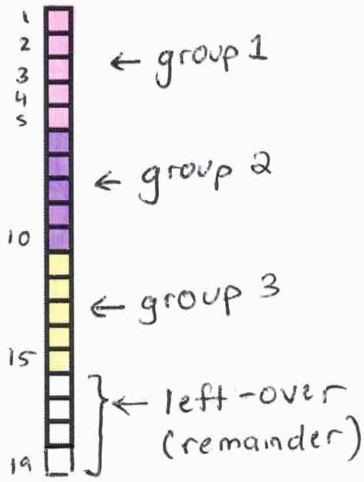
4A. Use using this notation, develop a verbal, visual, and symbolic description for the solutions of each of the following division problems:

break the number 19 into groups of size 5

$\frac{19}{3}$



$\frac{19}{5}$



numerator

$$\frac{N}{D} = \frac{19}{3}$$

denominator

☐ break 19 into groups of size 3

☐ in the case of 19, there are uneven groups: we're left over with an extra one

☐ we can't break 19 into equal-sized groups of 3 without having something left over (remainder)

$$\Rightarrow \frac{19}{3} = 6 \text{ remainder } 1 = 6R1 = 6\frac{1}{3}$$
$$= 6 + \frac{1}{3}$$

☑ Question: Does it matter which way we write our factors? In other words, can we write left & right factors in reverse order?

(Thurs 4/22/21 @ 10:49am)

Answer:
$$\boxed{19} = \underbrace{\boxed{5}}_{\text{left factor}} \cdot \underbrace{\boxed{3}}_{\text{right factor}} + \underbrace{\boxed{4}}_{\text{remainder}} \qquad \frac{19}{5} = 3\,R\,4$$
$$= \boxed{3} \cdot \boxed{5} + 4$$

Note:
$$\underbrace{\boxed{5 \cdot 3}}_{LHS_1} = \underbrace{\boxed{15}}_{RHS_1} \qquad \text{equation 1}$$

$$\underbrace{\boxed{3 \cdot 5}}_{LHS_2} = \underbrace{\boxed{15}}_{RHS_2} \qquad \text{equation 2}$$

$$\Rightarrow \underbrace{\boxed{5 \cdot 3}}_{LHS_1} = \underbrace{\boxed{15}}_{RHS_1} = \underbrace{\boxed{15}}_{RHS_2} = \underbrace{\boxed{3 \cdot 5}}_{LHS_2}$$

$$\Rightarrow 5 \cdot 3 = 3 \cdot 5$$ ← transitivity of equals sign

$\boxed{\text{Transivity of equals sign}}$ if $a = b$ and $b = c$, then $a = b = c$ and $a = c$

see reverse

quotient

addition

remainder

$$\frac{19}{5} = 3 + \frac{4}{5}$$

addition notation

fraction remainder

$$= \boxed{3}\boxed{\tfrac{4}{5}}$$

integer

mixed-number notation

we say this is mixed because there is a combined integer & fraction

CAREFUL:
this is not multiplication
this is a lazy person's way of writing remainders without addition symbol

$$\frac{19}{5} = 3 \text{ R } 4$$

remainder notation

"nineteen divided by five" is "three remainder four"

$$\frac{19}{5} = 3 \text{ remainder } 4$$

$$= 3 \text{ with } 4 \text{ left over}$$

$$= 3R4$$

$$= 3\tfrac{4}{5}$$

$$= 3 + \frac{4}{5}$$

Note:

- ☐ product is an output of a "pure" multiplication problem (multiplying a chain of factors
- ☐

Division Problem

numerator (product)

quotient (right factor)

$$\frac{\boxed{19}}{\boxed{5}} = \boxed{3}R\boxed{4}$$

remainder (left over)

denominator (left factor)

Multiplication Problem

$$\Longleftrightarrow \boxed{19} = \boxed{5} \cdot \boxed{3} + \boxed{4}$$

"product" left factor right factor remainder

"nineteen divided by five is three remainder four"

$$\Longleftrightarrow 19 = 15 + 4$$

Long Division: Math Memories Make Money

Side note: mental math

$$81 \cdot 7 = (80+1)\cdot 7$$
$$= (10\cdot 8 + 1)\cdot 7$$
$$= 10\cdot 8\cdot 7 + 1\cdot 7$$
$$= 10\cdot 56 + 7$$
$$= 560 + 7$$
$$= 567$$

$$\begin{array}{r} 081 \\ 7\overline{)572} \\ -56 \\ \hline 12 \\ -7 \\ \hline 5 \end{array}$$

$$\Rightarrow \frac{572}{7} = 81 \text{ R } 5$$ ← remainder notation

$$= 81\tfrac{5}{7}$$ ← mixed-number notation

$$= 81 + \frac{5}{7}$$ ← addition notation

Division Problem

numerator

$$\frac{\boxed{572}}{\boxed{7}} = \underbrace{\boxed{81}}_{\text{quotient}} \text{ R } \underbrace{\boxed{5}}_{\text{remainder}}$$

denominator

$\Longleftrightarrow$

Multiplication Problem

$$\underbrace{\boxed{572}}_{\text{numerator}} = \underbrace{\boxed{81}}_{\text{quotient}} \cdot \overbrace{\boxed{7}}^{\text{denom}} + \underbrace{\boxed{5}}_{\text{remainder}}$$
$$= 567 + 5$$