Name: Solutions, part 1 ... Class #:______

Math 48B, Lesson 2: Graphs of Polynomial Functions

In this lesson, we make connections between the general form of a polynomial function of degree n, given by

$$f(x) = a_n\, x^n + a_{n-1}\, x^{n-1} + \cdots + a_1\, x^1 + a_0\, x^0$$

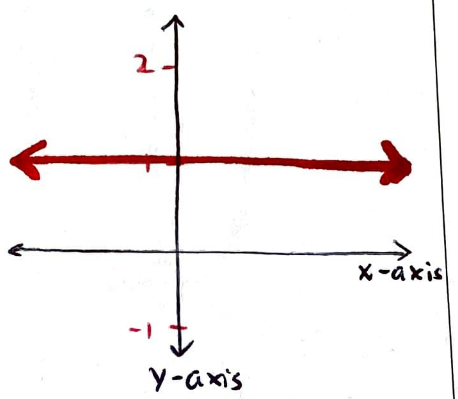
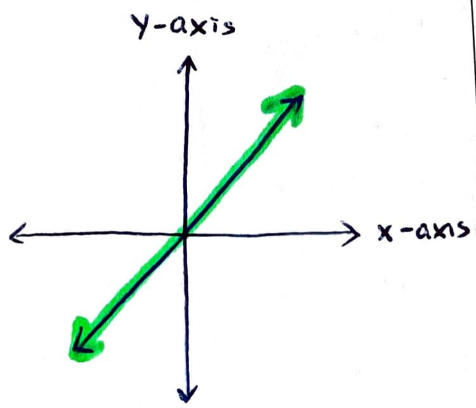
and the graphs of such functions.

1. GRAPHING BASIC MONOMIALS

The simplest polynomial functions are monomials in the form

$$f(x) = x^n \quad \begin{cases} \text{mono} = \text{one} \\ \text{nomial} = \text{term} \end{cases}$$

Let's use the graphing calculator on Desmos.com to complete the table below.

<table>
<tr><th>Polynomial</th><th>Degree</th><th>Shape of Graph</th><th>Notes</th></tr>
<tr><td>$P(x) = x^0 = 1$</td><td>0</td><td>
</td><td>Remember: $x^0 = 1$</td></tr>
<tr><td>$P(x) = x = x^1$</td><td>1</td><td>
</td><td>• The graph of this monomial is a straig
• mono = one
• poly = many
• monomial = one term
• polynomial = many te</td></tr>
</table>

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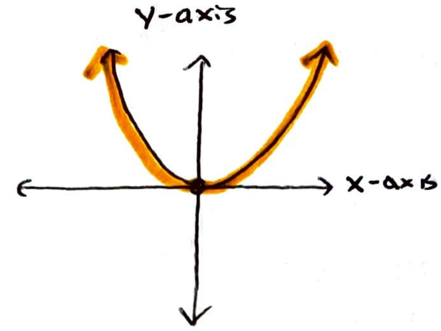
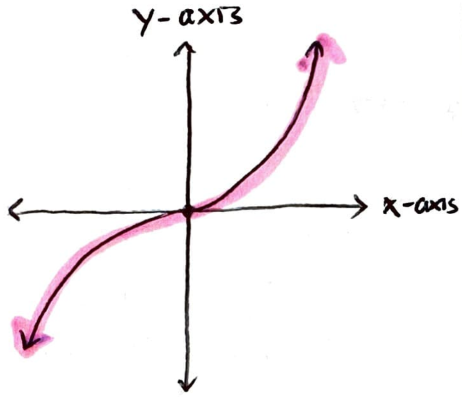
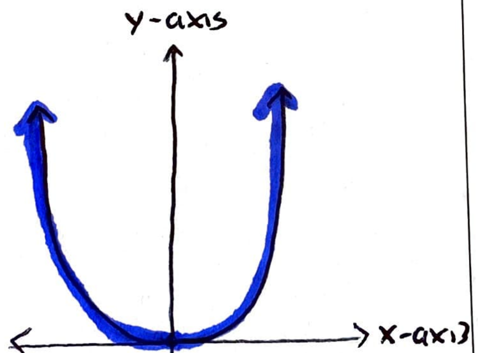
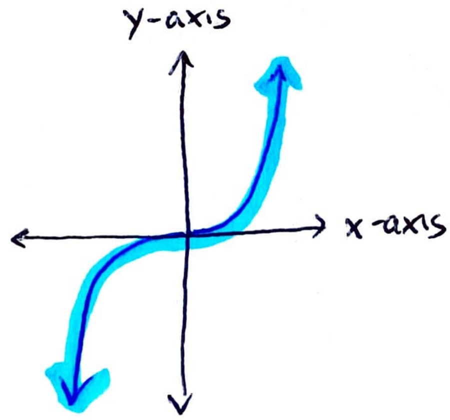
Polynomial	Degree	Shape of Graph	Notes
$P(x) = x^2$	2		• The shape of x^2 is called a parabola • x^2 is a second degree monomial
$P(x) = x^3$	3		• x^3 is degree three monomial • this graph, that looks like a snake, is called a cubic curve
$P(x) = x^4$	4		• x^4 is a degree four monomial: there is only term $1 \cdot x^4 = x^4$ coefficient • past one it's much vertical (slope is more vertical)
$P(x) = x^5$	5		• x^5 is a degree five monomial:

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2. TRANSFORMATION OF MONOMIALS

Use Desmos.com to graph each of the following functions.

✓ 2A. $P(x) = \underbrace{-x^3}_{\text{monomial}}$ ✓ 2B. $P(x) = \underbrace{(x-4)^2}_{\text{perfect square}}$ 2C. $P(x) = \underbrace{-\frac{1}{2}x^4 + 4}_{\text{binomial}}$

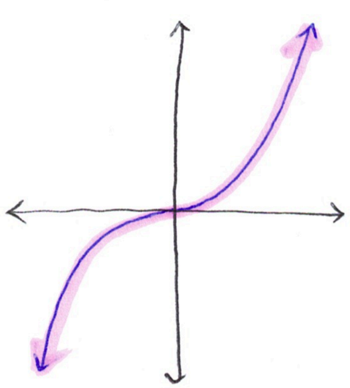
Compare these graphs to the graphs you found in Problem 1 above. For each function, describe why what you are seeing makes sense. In other words, for the transformations you see in each graph, explain how these transformations relate to the definition of each function.

Let's graph our transformations. We start

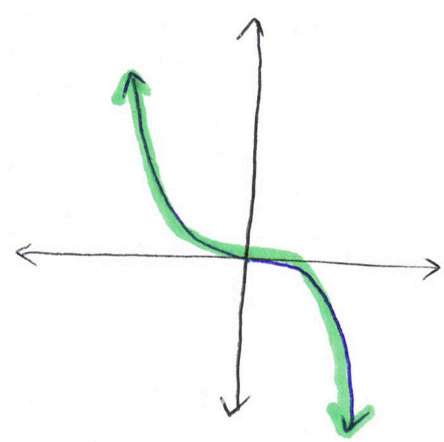
Problem 2A : $P(x) = -(x^3) = \underbrace{-1} \cdot x^3$

this scalar takes the original graph of x^3 and flips it (or mirror it) about the x-axis

Start $\xrightarrow[\text{(multiply by } -1)]{\text{transform}}$ End



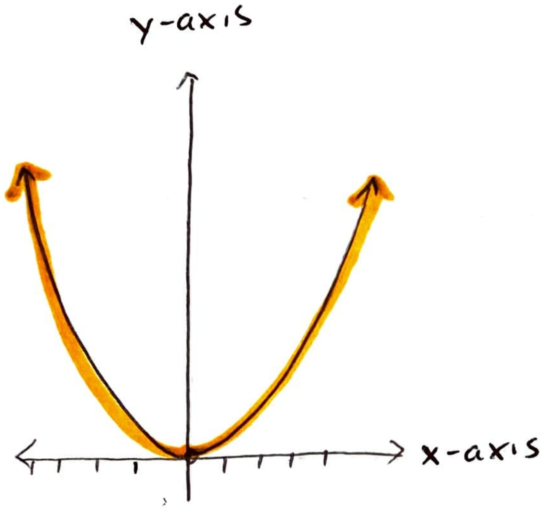
Reflection about the x-axis



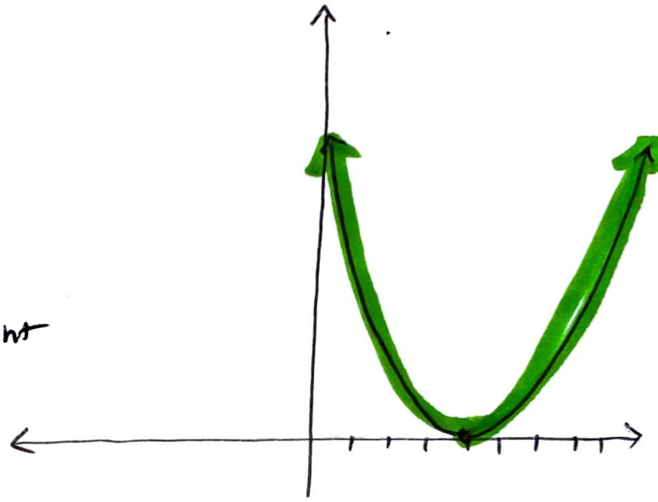
Problem 2B

Let's graph $P(x) = (x-4)^2$

Start: x^2 $\xrightarrow[\text{(subtract 4 from } x)]{\text{transform}}$ end: $(x-4)^2$

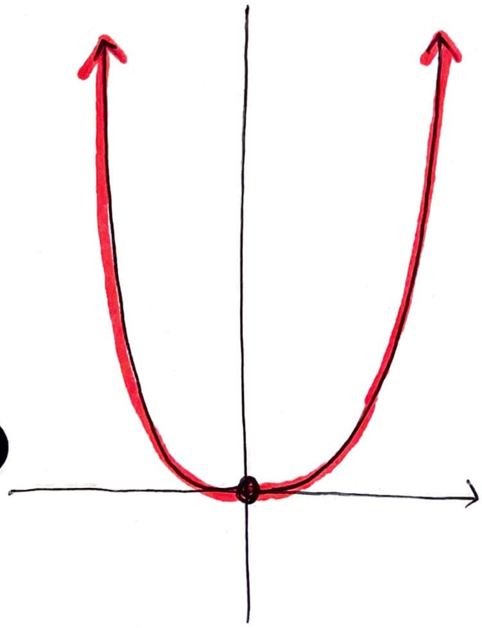


horizontal shift by four units to the right



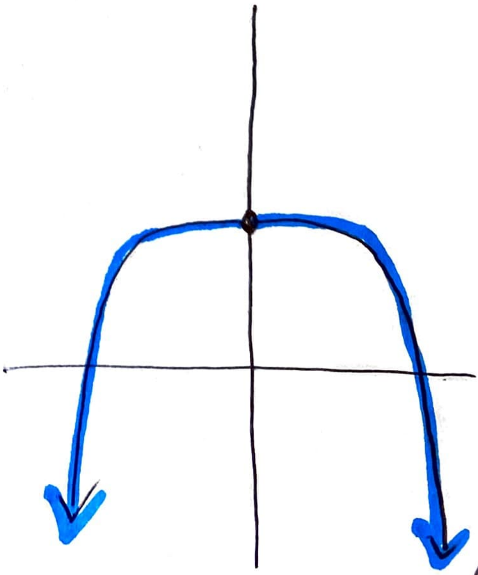
Problem 2C

Start: x^4 $\xrightarrow{\text{transformation}}$ end: $\boxed{-\frac{1}{2}}x^4 + 4$



$-\frac{1}{2}$: stretch and reflects/flips around x-axis

$+4$: vertical shift upwards by four units

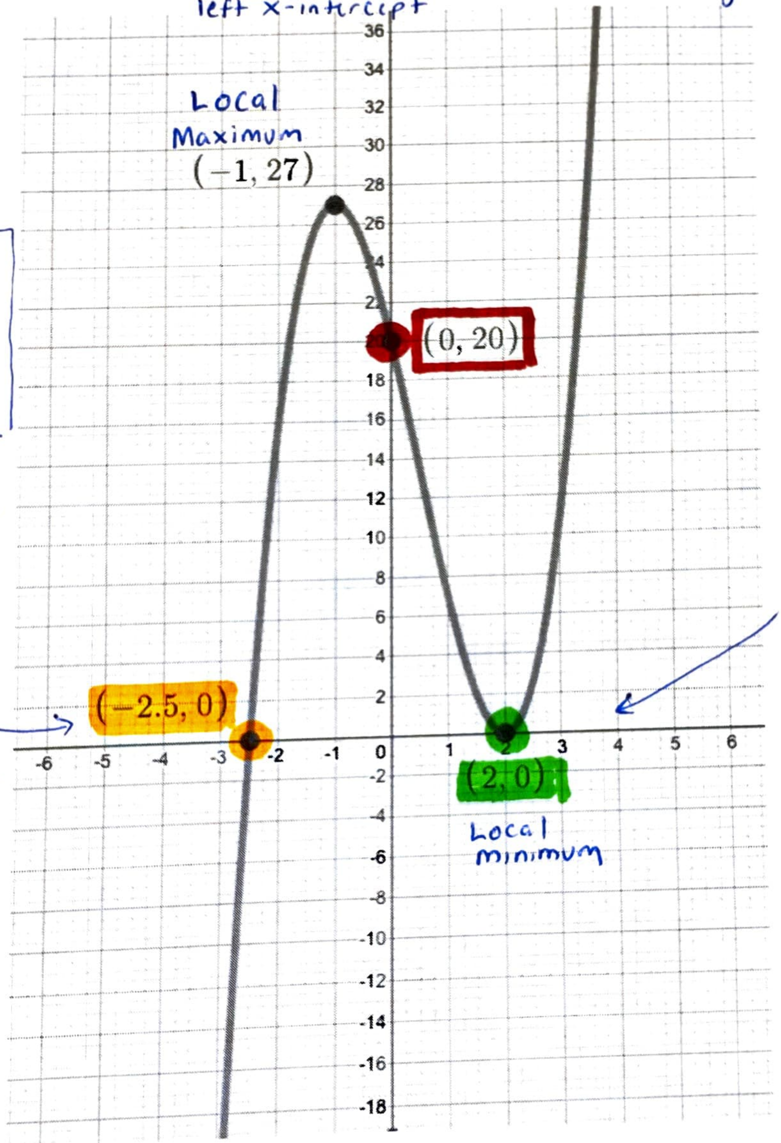


Problem 3A) Graph $P(x) = (x-2)^2 \cdot (2x+5)$

We see by the graph below,

- there are two x-intercepts at points $\boxed{(-2.5, 0)}$ and $\boxed{(2, 0)}$
  left x-intercept — right x-intercept

- Recall: x-intercept happen when graph touches x-axis (when output value is zero)



Local maximum occur at point $(-1, 27)$

- local maximum value is 27
- this maximum value happens at $x = -1$

Note: any time a mathematician uses the word "value" to describe a function, we are refering to the output of the function.

- When we say "local minimum value" we mean y-value at the minimum point(s) on the graph

Local minimum is at point $(2, 0)$

- local minimum value is 0
- local minimum is achieved at $x = 2$

- We see one y-intercept at point $\boxed{(0, 20}$
- We notice the domain of $P(x)$ is $(-\infty, \infty) = \mathbb{R} = \{\text{all real numbers}\}$
- In other words, we can use all values of x that are part of the function (all real numbers that can be a possible input)

☐ The symbols $x \to -\infty$ are read outloud

"x heads toward negative infinity"

☐ End behavior of output $P(x)$ ← nerdy language

Ryan's Language:

Calan's Language: ☐ End behavior can also be called long run behavior

☐ Its the behavior of the graph as we approach infinity in either direction

e.

The end behavior of $P(x) = (x-2)^2 (2x+5)$

as $x \to -\infty$ we see $y = P(x)$ goes to $-\infty$

(the graph keeps going down forever):

$$\lim_{x \to -\infty} P(x) = -\infty$$

"the limit as input x goes to negative infinity of output P(x) is negative infinity."

The end behavior on the right as $x \to +\infty$

we see the output $y = P(x)$ goes to $+\infty$

(the graph keeps on going up as we move right)

$$\lim_{x \to +\infty} P(x) = +\infty$$

"the limit as input x positive infinity

of output P(x) is positive infinity"

4. USE THE ZEROS TO GRAPH A POLYNOMIAL FUNCTION

Consider the following polynomial function:

$$P(x) = (x + 2)(x - 1)(x - 1)$$

Find the zeros of this function. Then, Desmos.com to graph this polynomial and specifically identify the location of each zero that you find on the graph.

Remember : ☐ the zeros of a function happen at the x-intercept (the location where the graph intersects with x-axis)

☐ The output value at any x-intercept is alway $y = 0$ (the "height" of an x-intercept is zero : hence we call these zeros).

☐ To find the x-intercepts, we set output $P(x) = 0$ and try to find the inputs that produce that output:

Let's find the zeros

$$P(x) = (x+2)\cdot(x-1)\cdot(x-1) = 0$$

$$\Rightarrow \quad x+2 = 0 \quad \text{or} \quad x-1 = 0$$

$$\quad -2 \quad -2 \qquad\qquad +1 \quad +1$$

$$\Rightarrow \quad \boxed{x = -2} \quad \text{or} \quad \boxed{x = 1}$$

The zeros of $P(x)$ are at points $(-2, 0)$ and $(1, 0)$

Note: zero-product property

INTUITIVE VERBAL

- If I multiply two numbers together and get zero, one of the two numbers must be zero

FORMAL SYMBOLIC

- If $a \cdot b = 0$, then either $a = 0$ or $b = 0$.

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5. MORE ABOUT ZEROS OF A POLYNOMIAL FUNCTION

5A. Read the following statement about the real zeros of a polynomial

FORMAL: NERDY

REAL ZEROS OF POLYNOMIALS

If P is a polynomial and c is a real number, then the following are equivalent:

1. c is a zero of P.
2. $x = c$ is a solution of the equation $P(x) = 0$.
3. $x - c$ is a factor of $P(x)$.
4. c is an x-intercept of the graph of P.

Translate this statement into abuelita language (simple words that you could use to explain the idea to your grandmother). Make sure to capture each of the ideas in full.

If $P(x) = \boxed{a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x^1 + a_0 x^0}$ *

(Standard form of a nth degree polynomial)

1. c is a zero of P

if we take $x = c$ as the input, then output

$$P(x)\Big|_{x=c} = P(c) = 0$$

(the output of P at input c is equal to zero)

2. $x = c$ is a solution to equation $P(x) = 0$

Notice $P(x) = 0$, we can write

$$0 = \underbrace{a_n x^n + a_{n-1} \cdot x^{n-1} + \cdots + a_1 x^1 + a_0 x^0}$$

Known/desired output

Here we have an nth degree polynomial with unknown value for input x

$\Rightarrow$ WTF the value of input x that forces output $P(x)$ to equal zero

Here the value $x = c$ forces $P(x) = P(c) =$

OR

That $x = c$ maks $P(x) = 0$

3. $x - c$ is a "factor" of $P(x)$

Recall: The standard form of a polynomial

$$P(x) = \underbrace{a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x^1 + a_0 x^0}_{\text{standard form}}$$

$$= \underbrace{a(x - c_1)\cdot(x - c_2)\cdots(x - c_n)}_{\text{complete factorization of } P(x)}$$

linear binomial factors

When we write $P(x)$ in factored form, we get a copy of $x - c$ in the factorization

4. c is x-intercept of the graph of P

The point $(c, 0)$ is a point where the graph of $P(x)$ touches the x-axis.

Let's look back at Problem 3 A

$$P(x) = (x-2)^2 \cdot (2x+5)$$

$$= \underbrace{(x-2)\cdot(x-2)\cdot(2x+5)}_{\text{complete factorization of } P(X)}$$

linear binomial factor

we saw the zeros were at

$(-2.5, 0)$ and $(2, 0)$

$\Rightarrow$ $x = -2.5$ and $x = 2$

are both zeros OR solution

to $P(x) = 0$