Name: Solutions, part 1 Class #:______

1. ANATOMY OF A PURE POWER VVS

- ☐ Verbal ⇆ Nerdy language / Abuelita language
- ☐ visual
- ☐ symboic

1A. Consider the following equation:

$$b^n = a$$

Identify the names for each part of this equation and any special features of the notation that stick out in your mind.

Notation

$$b^n = a$$

Visual

(up top) superscript

$$b^n = a$$

Nerdy Language

verbal description:

- ☐ "b to the nth power equals a"
- ☐ "b to the n equals a"
- ☐ "b to the power of n"
- ☐

Abuelita Language

Brainstorm mode

- exponent
- product
- coefficient
- variable
- constant
- base
- value of power
- equals sign

power n in the superscipt (up top and to the right of base)

$$b^n = a$$

base b to the left and lower than than power n

output of a power operation (constant/product)

Question:

- [ ] why do we call n a power and not an exponent?

WTF does this notation mean:

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Look back on your work from problem 1A above. For each of the following power expressions, evaluate the expression. Then, specifically identify the value of base b and the value of power n

1B. 2^6

"two to the power of six"

Evaluate : $2^6 = \underbrace{2\cdot2\cdot2\cdot2\cdot2\cdot2}_{\text{two multiplied by itself six times}}$

$= (2\cdot2\cdot2)\cdot(2\cdot2\cdot2)$

$= 8 \cdot 8$

$= 64$

value of power is $\boxed{n=6}$ in superscript

$\Rightarrow \quad 2^6 = 64$ ← output of the power evaluation is $\boxed{a=64}$

base $\boxed{b=2}$ is below and to the left of power $n=6$

1C. 3^4

Evaluate : $3^4 = \underbrace{3\cdot3\cdot3\cdot3}$

- "three multiplied by itself four time"
- "take the base 3, write that base four times, and then multiply them."

$= (3\cdot3)\cdot(3\cdot3)$

$= 9\cdot9$

$= \boxed{81}$

$$\boxed{\textbf{Evaluate :}} \quad 2^6 = \underbrace{2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2}_{\text{two multiplied by itself six times}}$$

$$= \underbrace{(2\cdot 2\cdot 2)}\cdot\underbrace{(2\cdot 2\cdot 2)}$$

$$= 8 \cdot 8$$

$$= 64$$

value of power is $\boxed{n=6}$ in superscript

$$\Rightarrow \quad 2^6 = 64$$

output of the power evaluation is $\boxed{a = 64}$

base $\boxed{b = 2}$ is below and to the left of power $n=6$

value of power in superscript

$$b^n = a$$

base b is "below and to the left" of power n

Output of the power evaluation

Specific

power
$n = 4$

$$3^4 = 81$$

base
$b = 3$

"three to the fourth equals eighty one"

General

$$b^n = a$$

"base b raised to the nth power results in an output of a"

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1D. $\underbrace{4^1}$

"four to the power one"

or "four to the one"

$\boxed{\text{Evaluate :}}$ $4^1 = \underbrace{4}$

"four multplied by itself one time"

OR

"take base 4 and write that base one time... no need for any multiplication"

$\Rightarrow \boxed{4^1 = 4}$

$\boxed{\text{conjecture: } 5^0 = 1}$

1E. $\underbrace{5^0}$

"five to the power of zero"

"take base five and then you write that base zero times?"

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2. EXPLORE BEHAVIOR OF POWERS

2A. Please fill out the table below. Specifically identify patterns in your work. Work slowly and methodically on this problem. We're going to use this work to create power rules and the work you do is going to be with us for the rest of this quarter (and many more years if you go on in your math career).

n	2^n
-4	$\frac{1}{16} = \frac{1}{2^4}$
-3	$\frac{1}{8} = \frac{1}{2^3}$
-2	$\frac{1}{4} = \frac{1}{2^2}$
-1	$\frac{1}{2} = \frac{1}{2^1}$
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512
10	1024

Power $n = 2$

$$2^n = 2^2$$

$$= \underbrace{2 \cdot 2}_{\text{two multiplied by itself twice}}$$

$$= 4$$

Power $n = 3$

$$2^n = 2^3$$

$$= 2 \cdot 2 \cdot 2$$

$$= (2 \cdot 2) \cdot 2$$

$$= 4 \cdot 2$$

$$= 8$$

Conjecture: to go from one row n down to the row immediately beneath $n+1$, we take the last entry and times it by 2 ...

$$\text{eg: } 2^3 = 2^2 \cdot 2 = 4 \cdot 2$$

$$\text{eg } 2^4 = 2^3 \cdot 2 = 8 \cdot 2$$

Conjecture: to go from row n back upwards to row $n-1$ above, we take the entry in row n and divide by 2

$$\text{eg 1: } 2^3 = \frac{2^4}{2} = \frac{16}{2} = 8$$

$$\text{eg2: } 2^2 = \frac{2^3}{2} = \frac{8}{2} = 4$$

eg 3: $2^1 = \frac{2^2}{2} = \frac{4}{2} = 2$

eg 4: $2^0 = \frac{2^1}{2} = \frac{2}{2} = 1$

eg 5: $2^{-1} = \frac{2^0}{2} = \frac{1}{2} = 0.5$

eg 6: $2^{-2} = \frac{2^{-1}}{2}$

$= 2^{-1} \div 2$

$= \frac{1}{2} \div \frac{2}{1}$

$= \frac{1}{2} \cdot \frac{1}{2}$

Math memories make money

symbolic $\frac{A}{B} \div \frac{C}{D} = \frac{A}{B} \cdot \frac{D}{C}$

Verbal "when I divide the left fraction A over B by the right fraction C over D, I take the reciprocal of the right fraction (flip it / opposite) and turn division into multiplication left fraction | right fraction

$$\Rightarrow 2^{-2} = \boxed{\frac{1}{2}} \cdot \boxed{\frac{1}{2}}$$

$$= \frac{1 \cdot 1}{2 \cdot 2}$$

$$= \frac{1}{4}$$

$$2^{-3} = \frac{2^{-2}}{2}$$

$$= 2^{-2} \div 2$$

$$= \frac{1}{4} \div \boxed{\frac{2}{1}}$$

$$= \frac{1}{4} \cdot \frac{1}{2}$$

$$= \frac{1 \cdot 1}{4 \cdot 2} = \boxed{\frac{1}{8}}$$

Math Memories make Money

symbolic $\left\{ \frac{A}{B} \cdot \frac{C}{D} = \frac{A \cdot C}{B \cdot D} \right.$ ← numerator ← denomin

verbal description

"when I multiply the left fraction A over B by the right fraction C over D, I take the numerator of the fraction A multiply it by the top of the right fraction C and put the product A·C in the numerator

And I take the bottom of the first fraction and multiply by the bottom of second fraction and turn that into denominator

VVS: verbal, visual, symbolic

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2B. Using the table above, come up with a conjecture (a mathematical guess) about the following power expressions.

One as an exponent: b^1 $= b$

Symbolic: $\boxed{b^1 = b}$

verbal: when we take any base b to the first power, we get a single copy of that base back again.

Remember

$$b^0 = \frac{b^1}{b} = \frac{b}{b}$$

Zero as an exponent: b^0

Symbolic $\boxed{b^0 = 1}$

verbal: any base b raised to the zeroth power is equal to the number one

Negative exponent: b^{-n}

$$\boxed{b^{-n} = \frac{1}{b^n}}$$

when we take base b to a negative power we can get rid of the negative by taking base b to the positive power in denominator.

Hidden in the table from problem 2 above are other "laws of powers." These are algebraic properties for taking powers of any base. In problem 3, we explore those laws in more detail.