Name: Solutions Class #:______

4. HOW TO SOLVE EXPONENTIAL EQUATIONS?

Solve the algebraic exponential equation: $4^{3x} = 16$

4A. Use the inverse operation known as "equating exponent" to solve algebraically
4B. Use the inverse operation known as logarithms to solve algebraically ✓
4C. Use a graphical technique to solve this algebraic equation

Problem 4A) Lets solve $4^{3x} = 16$ graphically

Let's identify LHS (left-hand side) and RHS (right-hand side)

$$\underbrace{4^{3x}}_{\text{left-hand side function}} = \underbrace{16}_{\text{right-hand side function}}$$

$f(x) = 4^{3x}$ $g(x) = 16$

Step 1: Graph $f(x) = 4^{3x}$ ✓

Step 2: Graph $g(x) = 16$ ✓

Step 3: Find point of intersection: $(\frac{2}{3}, 16)$

Step 4: $(x, y) = (\frac{2}{3}, 16) \Rightarrow x = \frac{2}{3}$

Step 5: Solution: $\boxed{x = \frac{2}{3}}$ ✓

Property of equal Exponents

If $b^x = b^w$, then $x = w$.

Problem 4A

$\Rightarrow \quad 4^{3x} = 16$

$\Rightarrow \quad 4^{3x} = 4^2$

$\Rightarrow \quad 3x = 2$

$\Rightarrow \quad \boxed{x = \frac{2}{3}}$ ✓

Inverse Property of exponents and logs

logs undo exponents : $\cancel{\log_b}(\cancel{b}^{x}) = x$

exponents undo logs : $\cancel{b}^{\cancel{\log_b}(x)} = x$

INVERSES

Foward	Backward
$+$	$-$
$\cdot$	$/$
x^n	$\sqrt[n]{x}$
b^x	$\log_b(x)$

Problem 4B

$$4^{\boxed{3x}} = 16$$

let's get rid base 4
(hit it with a log)

$$\Rightarrow \log_4\left(4^{3x}\right) = \log_4(16)$$

$$\Rightarrow 3x \cdot \underbrace{\log_4(4)} = \underbrace{\log_4(16)}$$

what exponent do I need on base 4 to get 16

$$\Rightarrow 3x \cdot 1 = 3x = 2$$

$$\Rightarrow \boxed{x = \frac{2}{3}} \checkmark$$

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5. HOW TO SOLVE EXPONENTIAL EQUATIONS?

Solve each of the algebraic equations below.

✓ 5A. $8^{x+3} = 64$

5B. $2^{x-4} = \sqrt[3]{2}$

5C. $\log_2(8 - 6x) = 5$

5D $\log(x) + \log(x - 3) = 1$

5E. $2^{x+3} = 3^x$

5F. $4\ln(2x) = 8$

✓ 5F. $\ln(x^2) = \ln(3x + 4)$

✓ Problem 5A) Let's solve $8^{x+3} = 64$

Let's use logarithms:

logs undo exponents: $\cancel{\log_b}(\cancel{b}^{x}) = x$

exponents undo logs: $\cancel{b}^{\cancel{\log_b}(x)} = x$

variable in superscript

$$\Rightarrow \quad 8^{\boxed{x+3}} = 64$$

exponent with base $b = 8$

$$\Rightarrow \quad \cancel{\log_8}(\cancel{8}^{x+3}) = \log_8(64)$$

$$\Rightarrow \quad x + 3 = \underbrace{\log_8(64)}$$

what exponent do we put on 8 to get 64 out

$8^2 = 8 \cdot 8 = 64$

Side note:

$$y = \log_8(64)$$

$$\Rightarrow \quad 8^y = 64$$

$$\Rightarrow \quad 8^y = 8^2$$

$$\Rightarrow \quad y = 2$$

$$\Rightarrow \quad \log_8(64) = 2$$

$$\Rightarrow \quad x + 3 = 2$$

$$-3 \qquad -3$$

$$\Rightarrow \quad \boxed{x = -1}$$ ✓

Let's solve $8^{x+3} = 64$ by equating exponents.

$$\boxed{\text{Principle of Equal Exponents}}$$

If $\underbrace{b^{\boxed{x}} = b^{\boxed{w}}}_{\substack{\text{two exponents} \\ \text{with the same} \\ \text{base that} \\ \text{are equal}}}$, then $\underbrace{x = w}_{\substack{\text{the superscipt} \\ \text{values must} \\ \text{be the same}}}$

$\Rightarrow \quad 8^{x+3} = 64$ ← write this as base $b = 8$

$\Rightarrow \quad 8^{\boxed{x+3}} = 8^{\boxed{2}}$

$\Rightarrow \quad x + 3 = 2$

$\quad\quad -3 \quad -3$

$\Rightarrow \quad x = -1$ ✓

Let's solve $\underbrace{8^{x+3}}_{\text{left hand side}} = 64$ graphically

Step 1: Let $\underbrace{f(x) = 8^{x+3}}_{\text{function on left-hand side}}$ and graph

Step 2: Let $\underbrace{g(x) = 64}_{\text{function on Right-hand side}}$ and graph

Step 3: Find point of intersection: $(-1, 64)$

Step 4: Note $(x, y) = (-1, 64)$

x-value

Step 5: Solution $\boxed{x = -1}$ ✓

Problem 5F) Lets solve $\ln(x^2) = \ln(3x+4)$

use inverse operations:

logs undo exponents: $\cancel{\log_b}(\cancel{b}^x) = x$

exponents undo logs: $\cancel{b}^{\cancel{\log_b}(x)} = x$

$$\Rightarrow \quad \ln(x^2) = \ln(3x+4)$$

$$\Rightarrow \quad \log_e(x^2) = \log_e(3x+4)$$

$$\Rightarrow \quad \cancel{e}^{\cancel{\ln}(x^2)} = \cancel{e}^{\cancel{\ln}(3x+4)}$$

$$\Rightarrow \quad x^2 = 3x+4$$

$$\Rightarrow \quad x^2 - 3x - 4 = 0$$

$$\Rightarrow \quad (x-4)\cdot(x+1) = 0$$

$$\Rightarrow \quad x-4=0 \quad \text{OR} \quad x+1=0 \quad \Rightarrow \boxed{x=4} \text{ or } \boxed{x=-1}$$

Problem 5F) let's solve $\ln(x^2) = \ln(3x+4)$

use inverse operations:

Step 1: Graph LHS $f(x) = \ln(x^2)$

Step 2: Graph RHS $g(x) = \ln(3x+4)$

Step 3: Find points of intersection

$(-1, 0)$ point 1

$(4, 2.773)$ point 2

Step 4: $(x, y) = (-1, 0)$ OR

$(x, y) = (4, 2.773...)$

Step 5: $\boxed{x = -1}$ ✓ or $\boxed{x = 4}$ ✓