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Math 48B, Lesson 16: Properties of Logarithms

In Math 48B Lessons 14, 15, 16, 17, and 18, we study logarithmic functions:

Logarithmic Form	Exponential Form
$y = \log_b(x)$	$x = b^y$

To begin our exploration, let's recall the rules of powers/exponents.

1. WHAT ARE RULES OF POWERS/EXPONENTS?

Exponent Notation: $N = b^n$

Product Rule: $b^n \cdot b^m$

Quotient Rule: $\frac{b^n}{b^m}$

Power to a Power: $(b^n)^p$

Zero Power: $1 = \frac{b}{b} = \frac{b^1}{b^1}$

Negative Powers: $\frac{1}{b^n}$

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2. WHAT ARE RULES OF LOGARITHMS?

Logarithmic Notation: $n = \log_b(N)$ and $m = \log_b(M)$

Product Rule: $\log_b(M \cdot N) = \log_b(M) + \log_b(N)$

Quotient Rule: $\log_b\left(\frac{M}{N}\right) = \log_b(M) - \log_b(N)$

Power to a Power: $\log_b(N^p) = p \cdot \log_b(N)$

Inverse Exponential: $\log_b(b^n) = n$

Inverse Log: $b^{\log_b(N)} = N$

Change of Base: $\log_b(N) = \frac{\log_a(N)}{\log_a(b)}$

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3. HOW TO USE LOG RULES?

Use the properties of logs we explored in problem 2 above to evaluate the logarithm in each problem:

3A. $\log_{10}\left(\frac{10000}{100}\right)$

3B. $\log_{9}(27 \cdot 81)$

3C. $\log_{2}(16^{2})$

3D. $\log_{5}\left(\frac{125^{2}}{\sqrt{625}}\right)$

3E. $\log_{2}(16 \cdot 32)$

3F. $\log_{5}(125) - \log_{5}(25)$

3G. $\log_{6}(6^{7}) - \log_{6}(6^{10})$

3H. $\log_{2}\left(\frac{2048}{\sqrt[10]{32}}\right)$

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4. HOW TO USE THE CHANGE OF BASE FORMULA?

Use the change of base formula to calculate each of the following values. Change the base to both the common log and the natural log. Check your work using a calculator.

4A. $\log_2\left(\frac{2}{16}\right)$

4B. $\log_9\left(\frac{9}{243}\right)$

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5. HOW TO USE LOG RULES WITH VARIABLES?

Use the properties of logs we explored in problem 2 above to expand each of the following expressions:

5A. $\log_2\left(\frac{36x^2(4x-1)}{(2x+1)}\right)$

5B. $\ln\left(\frac{x^4 \cdot y^3}{z^5}\right)$

5C. $\log(\sqrt[3]{x^2})$

5D. $\ln\left(\frac{\sqrt{(x+1)(2x-1)^2}}{x^2-4}\right)$

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6. HOW TO USE LOG RULES WITH VARIABLES?

Use the properties of logs we explored in problem 2 above to condense each of the following expressions into a single logarithm:

6A. $\log(5) + \log(8) - \log(4)$

6B. $2\ln(x) - 4\ln(x+2) + \frac{1}{3}\ln(2x-5)$