In Math 48B Lessons $14,15,16,17$, and 18 , we study logarithmic functions:

Logarithmic Form

$$
y=\log _{b}(x)
$$

Exponential Form

$$
x=b^{y}
$$

To begin our exploration, let's recall the rules of powers/exponents.

## 1. HOW TO EVALUATE LOGARITHMS?

Consider the two equivalent forms for logarithmic functions:

Logarithmic Form
Exponential Form

$$
y=\log _{b}(x)
$$

$$
x=b^{y}
$$

Use these two equivalent forms to evaluate the following logarithm problems.
1A. $4+\log _{10}(0.001)$
1B. $\log _{4}\left(\frac{1}{32}\right)$
1C. $\log _{e}\left(e^{2 / 5}\right)$

## 2. WHAT DOES THE GRAPH OF A LOGARITHM LOOK LIKE?

2A. Fill out the table for the logarithmic function $y=\log _{2}(x)$ below. The, use Desmos.com to create a graph and describe the relevant features of that graph including the domain, range, x-intercept, and the end behavior as $x \rightarrow+\infty$.

| $x$ | $y$ |
| :---: | :---: |
| $\frac{1}{32}$ |  |
| $\frac{1}{16}$ |  |
| $\frac{1}{8}$ |  |
| $\frac{1}{4}$ |  |
| $\frac{1}{2}$ |  |
| 1 |  |
| 2 |  |
| 4 |  |
| 8 |  |
| 16 |  |
| 32 |  |
| 64 |  |

$\qquad$
2B. Fill out the table for the common logarithmic function

$$
y=\log _{10}(x)=\log (x)
$$

The, use Desmos.com to create a graph and describe the relevant features of that graph including the domain, range, x -intercept, and the end behavior as $x \rightarrow+\infty$.

| $x$ | $y$ |
| ---: | ---: |
| 0.00001 |  |
| 0.0001 |  |
| 0.001 |  |
| 0.01 |  |
| 0.1 |  |
| 1 |  |
| 10 |  |
| 100 |  |
| 1000 |  |
| 10000 |  |
| 100000 |  |
| 1000000 |  |

2C. Suppose that $b>1$ and determine the characteristics of the function

$$
y=\log _{b}(x)
$$

Sketch a graph of this curve below and describe the relevant features of that graph including the domain, range, x-intercept, and the end behavior. Using Desmos.com, graph the log functions with $b=2, e$, and 10 on the same axes. Highlight the various features of each graph.

## 3. WHAT DOES THE GRAPH OF A LOGARITHM LOOK LIKE?

3A. Fill out the table for the logarithmic function $y=\log _{0.5}(x)$ below. The, use Desmos.com to create a graph and describe the relevant features of that graph including the domain, range, x -intercept, and the end behavior as $x \rightarrow+\infty$.

| $x$ | $y$ |
| :---: | :---: |
| $\frac{1}{32}$ |  |
| $\frac{1}{16}$ |  |
| $\frac{1}{8}$ |  |
| $\frac{1}{4}$ |  |
| $\frac{1}{2}$ |  |
| 1 |  |
| 2 |  |
| 4 |  |
| 8 |  |
| 16 |  |
| 32 |  |
| 64 |  |

$\qquad$
3B. Fill out the table for the common logarithmic function

$$
y=\log _{0.1}(x)=\log (x)
$$

The, use Desmos.com to create a graph and describe the relevant features of that graph including the domain, range, $x$-intercept, and the end behavior.

| $x$ | $y$ |
| ---: | ---: |
| 0.00001 |  |
| 0.0001 |  |
| 0.001 |  |
| 0.01 |  |
| 0.1 |  |
| 1 |  |
| 10 |  |
| 100 |  |
| 1000 |  |
| 10000 |  |
| 100000 |  |
| 1000000 |  |

3C. Suppose that $0<b<1$ and determine the characteristics of the function

$$
y=\log _{b}(x)
$$

Sketch a graph of this curve below and describe the relevant features of that graph including the domain, range, $x$-intercept, and the end behavior.
4. TRANSFORMATIONS OF EXPONENTIAL FUNCTIONS?

4A. For logarithmic function $y=a \cdot \log _{b}(x-h)+k$, what do parameters $a, h$, and $k$ do to the graph of $y=\log _{b}(x)$ ? Develop graphs on Desmos.com to highlight each parameter and demonstrate the effect on your graph. Capture

4B. Test your hypothesis from Problem 4A above by graphing the function

$$
f(x)=-2 \log _{3}(x-4)+5
$$

