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Math 48B, Lesson 14: Exponential Functions

In Math 48B Lessons 14, 15, 16, 17, and 18, we study logarithmic functions:

Logarithmic Form	Exponential Form
$y = \log_b(x)$	$x = b^y$

To begin our exploration, let's recall the rules of powers/exponents.

1. WHAT IS AN INVERSE FUNCTION?

1A. What does it mean for a function to be one-to-one?
1B. What is the horizontal line test?
1C. What is an inverse function?
1D. When do inverse functions exist?
1E. Assume we start with function $y = f(x)$ where every point on the graph of this function is given as (x, y). How do we create the graph of the inverse function?
1F. Explain inverse function notation $f^{-1}(x) = y$.

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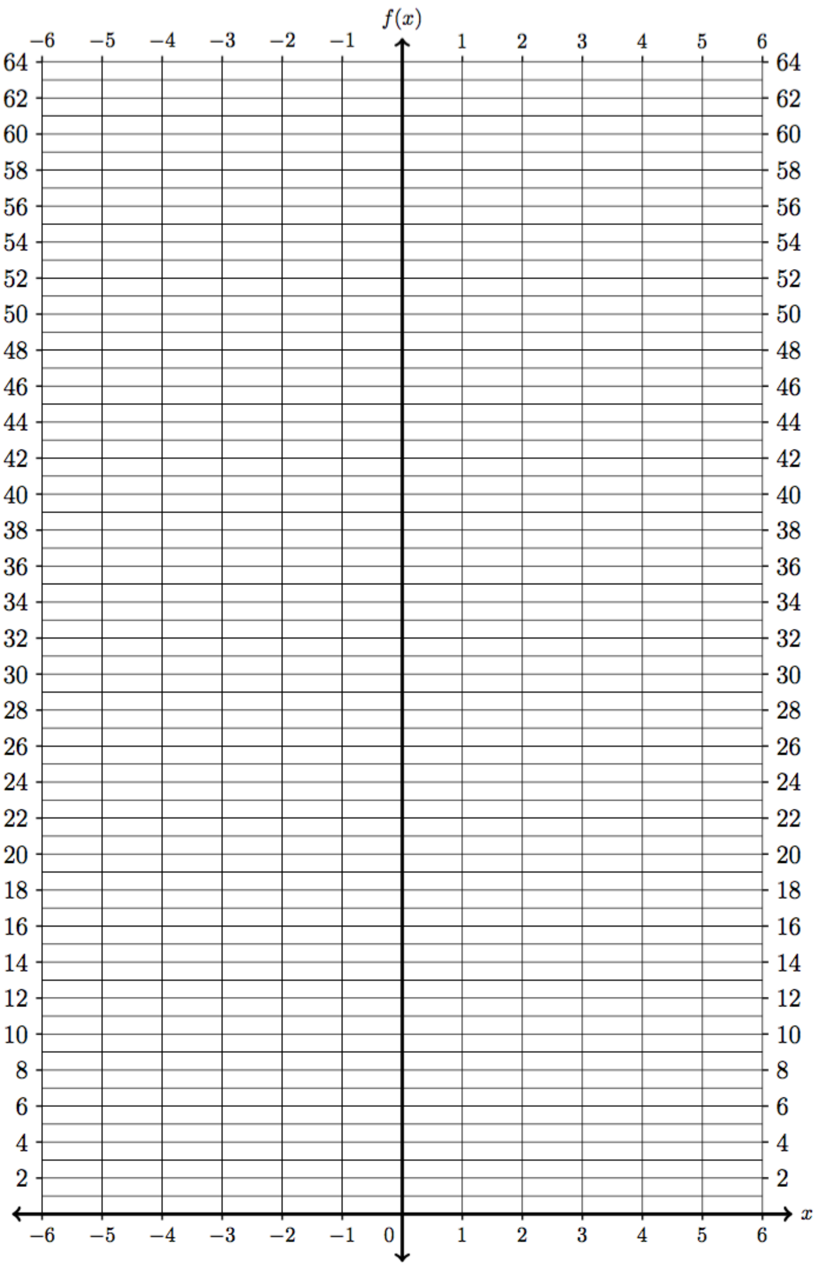
2. WHAT IS THE INVERSE OF AN EXPONENTIAL FUNCTION?

2A. Graph of the exponential function $y = 2^x$ below and then apply the horizontal line test to this function. What can you say about the existence of an inverse?

x	y	(x, y)
-4		
-3		
-2		
-1		
0		
1		
2		
3		
4		
5		
6		



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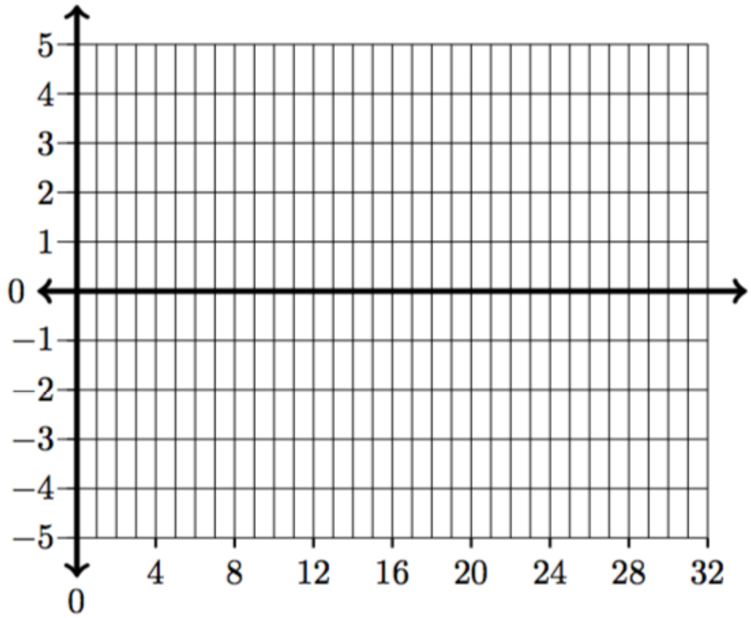
2B. Graph the inverse of the exponential function $y = 2^x$ below and then apply the horizontal line test to this function. What can you say about the existence of an inverse?

y	x	(y, x)



2C. Write the input-output relation for the inverse function from Problem 2B above. Then, translate this relation into logarithmic form. Finally, develop a verbal description for logarithmic functions.

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3. HOW TO MOVE BACK AND FORTH FROM LOGS INTO EXPONENTS?

Consider the two equivalent forms for logarithmic functions:

Logarithmic Form	Exponential Form
$y = \log_b(x)$	$x = b^y$

Write each of the following problems in both forms to find the desired output value.

3A. $y = \log_{10}(1000)$

3B. $y = \log_3(81)$

3C. $y = \log_2\left(\frac{1}{32}\right)$

3D. $y = \log_5\left(\frac{1}{125}\right)$

3E. $y = \log_{16}(64)$

3F. $y = \log_{27}(9)$

3G. $y = \log_9(9^5)$

3H. $y = 5^{\log_5(y)}$

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4. WHAT ARE PROPERTIES OF LOGARITHMS?

Consider the two equivalent forms for logarithmic functions:

Logarithmic Form	Exponential Form
$y = \log_b(x)$	$x = b^y$

Using these two equivalent forms, come up with formula to describe the properties of logarithmic functions in each of the following cases.

4A. $\log_b(1)$

4B. $\log_b(b)$

4C. $\log_b(b^x)$

4D. $b^{\log_b(x)}$

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5. WHAT DOES THE GRAPH OF A LOGARITHM LOOK LIKE?

Fill out the table for the logarithmic function $y = \log_3(x)$ below. The, use Desmos.com to create a graph and describe the relevant features of that graph including the domain, range, x-intercept, and the end behavior as $x \to +\infty$.

x	y
$\frac{1}{81}$	
$\frac{1}{27}$	
$\frac{1}{9}$	
$\frac{1}{3}$	
1	
3	
9	
27	
81	
243	
729	

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6. HOW TO EVALUATE LOGARITHMS?

Consider the two equivalent forms for logarithmic functions:

Logarithmic Form	Exponential Form
$y = \log_b(x)$	$x = b^y$

Use these two equivalent forms to evaluate each of the following logarithms.

6A. $y = \log_{10}(100{,}000)$

6B. $y = \log_4(32)$

6C. $y = \log_5(1)$

6D. $y = \log_{64}(8)$

6E. $y = \log_{64}(8)$

6F. $y = \log_7(7)$