Math 48B, Lesson 14: Exponential Functions

In Math 48B Lessons 14, 15, 16, 17, and 18, we study logarithmic functions:

|  |  |
| --- | --- |
| Logarithmic Form | Exponential Form |
| $$y=log\_{b}(x)$$ | $$x=b^{y}$$ |

To begin our exploration, let’s recall the rules of powers/exponents.

1. WHAT IS AN INVERSE FUNCTION?

1A. What does it mean for a function to be one-to-one?

1B. What is the horizontal line test?

1C. What is an inverse function?

1D. When do inverse functions exist?

1E. Assume we start with function $y=f(x)$ where every point on the graph of this function is given as $(x,y)$. How do we create the graph of the inverse function?

1F. Explain inverse function notation $f^{-1}(x)=y$.

2. WHAT IS THE INVERSE OF AN EXPONENTIAL FUNCTION?

2A. Graph of the exponential function $y=2^{x}$ below and then apply the horizontal line test to this function. What can you say about the existence of an inverse?



|  |  |  |
| --- | --- | --- |
| $$x$$ | $$y$$ | $$(x,y)$$ |
| $$-4$$ |  |  |
| $$-3$$ |  |  |
| $$-2$$ |  |  |
| $$-1$$ |  |  |
| $$0$$ |  |  |
| $$1$$ |  |  |
| $$2$$ |  |  |
| $$3$$ |  |  |
| $$4$$ |  |  |
| $$5$$ |  |  |
| $$6$$ |  |  |

2B. Graph the inverse of the exponential function $y=2^{x}$ below and then apply the horizontal line test to this function. What can you say about the existence of an inverse?

|  |  |  |
| --- | --- | --- |
| $$y$$ | $$x$$ | $$(y,x)$$ |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
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2C. Write the input-output relation for the inverse function from Problem 2B above. Then, translate this relation into logarithmic form. Finally, develop a verbal description for logarithmic functions.

3. HOW TO MOVE BACK AND FORTH FROM LOGS INTO EXPONENTS?

Consider the two equivalent forms for logarithmic functions:

|  |  |
| --- | --- |
| Logarithmic Form | Exponential Form |
| $$y=log\_{b}(x)$$ | $$x=b^{y}$$ |

Write each of the following problems in both forms to find the desired output value.

3A. $y=log\_{10}(1000)$ 3E. $y=log\_{16}(64)$

3B. $y=log\_{3}(81)$ 3F. $y=log\_{27}(9)$

3C. $y=log\_{2}( \frac{1}{32} )$ 3G. $y=log\_{9}(9^{5})$

3D. $y=log\_{5}( \frac{1}{125} )$ 3H. $y=5^{log\_{5}(y)}$

4. WHAT ARE PROPERTIES OF LOGARITHMS?

Consider the two equivalent forms for logarithmic functions:

|  |  |
| --- | --- |
| Logarithmic Form | Exponential Form |
| $$y=log\_{b}(x)$$ | $$x=b^{y}$$ |

Using these two equivalent forms, come up with formula to describe the properties of logarithmic functions in each of the following cases.

4A. $log\_{b}(1)$

4B. $log\_{b}(b)$

4C. $log\_{b}(b^{x})$

4D. $b^{log\_{b}(x)}$

5. WHAT DOES THE GRAPH OF A LOGARITHM LOOK LIKE?

Fill out the table for the logarithmic function $y=log\_{3}(x)$ below. The, use Desmos.com to create a graph and describe the relevant features of that graph including the domain, range, x-intercept, and the end behavior as $x\rightarrow +\infty $.

|  |  |
| --- | --- |
| $$x$$ | $$y$$ |
| $$\frac{1}{81}$$ |  |
| $$\frac{1}{27}$$ |  |
| $$\frac{1}{9}$$ |  |
| $$\frac{1}{3}$$ |  |
| $$1$$ |  |
| $$3$$ |  |
| $$9$$ |  |
| $$27$$ |  |
| $$81$$ |  |
| $$243$$ |  |
| $$729$$ |  |

6. HOW TO EVALUATE LOGARITHMS?

Consider the two equivalent forms for logarithmic functions:

|  |  |
| --- | --- |
| Logarithmic Form | Exponential Form |
| $$y=log\_{b}(x)$$ | $$x=b^{y}$$ |

Use these two equivalent forms to evaluate each of the following logarithms.

6A. $y=log\_{10}(100,000)$ 6E. $y=log\_{64}(8)$

6B. $y=log\_{4}(32)$ 6F. $y=log\_{7}(7)$

6C. $y=log\_{5}(1)$

6D. $y=log\_{64}(8)$