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Math 48B, Lesson 12: Exponential Functions

In Math 48B Lessons 11, 12, and 13, we study exponential functions:

 $y = b^x$

To begin our exploration, let's recall the rules of powers/exponents.

1. WHAT ARE RULES OF POWERS/EXPONENTS?

Powers vs exponents:
$$y = x^n$$
 $y = b^x$ Product Rule: $b^n \cdot b^m$ Quotient Rule: $\frac{b^n}{b^m}$ Zero Power: $1 = \frac{b}{b} = \frac{b^1}{b^1}$ Negative Powers: $\frac{1}{b^n}$ Power to a Power: $(b^n)^p$

2. WHAT IS EXPONENTIAL GROWTH?

2A. Fill in the table below. To the best of your ability, fill this table out by hand.

x	$f(x) = 2^x$	$g(x) = 4^x$	$h(x) = 5^x$	$j(x) = 10^x$
-4				
-3				
-2				
-1				
0				
1				
2				
3				
4				

2B. Graph the functions f(x), g(x), h(x), and j(x) from problem 2A above.

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  -3
                 -2
                               ^{-1}
                                                                            \mathbf{2}
                                                                                          3
                                                             1
                                                                                           +25
25 -
24
                                                                                             24
23 \cdot
                                                                                           - 23
22 \cdot
                                                                                           22
21
                                                                                            - 21
20 -
                                                                                           20
19
                                                                                           19
18 \cdot
                                                                                           18
17 -
                                                                                           17
16 -
                                                                                           - 16
15 \cdot
                                                                                           - 15
14 -
                                                                                           - 14
13 \cdot
                                                                                           13
12 \cdot
                                                                                           - 12
11 -
                                                                                           11
10 \cdot
                                                                                           10
 9 -
                                                                                            - 9
 8
                                                                                            - 8
 7 -
                                                                                            7
 6 -
                                                                                            6
 5 \cdot
                                                                                            5
 4 \cdot
                                                                                           4
 3 -
                                                                                            - 3
 2 -
                                                                                           - 2
 1 -
                                                                                            1
                                                                                              → x
                                                                                          3
```

-3

-2

-1

0

 $\dot{2}$

1

2C. Identify patterns in the graphs of the functions f(x), g(x), h(x), and j(x) from problems 2AB above. Make a conjecture about the general behavior of the graph of the function

$$y = b^x$$
 for $1 < b$

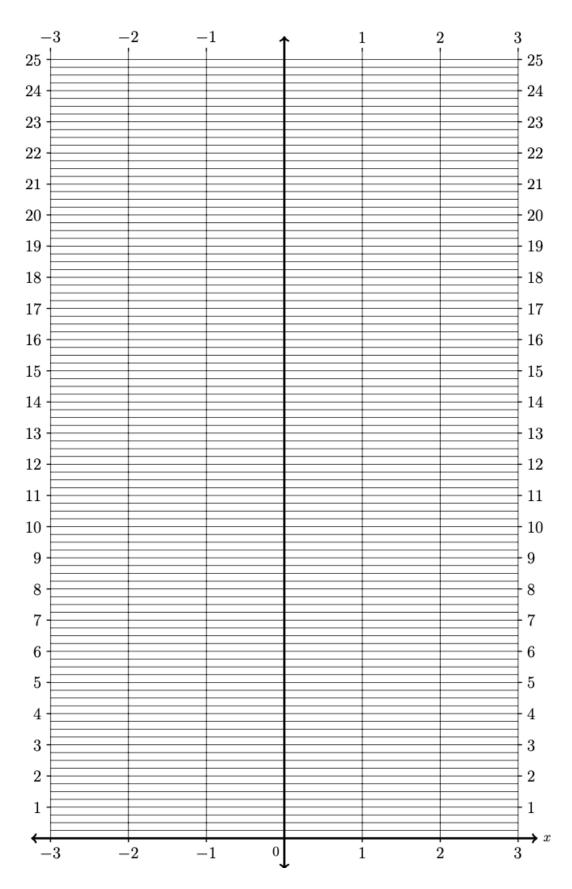
In your conjecture, identify the domain, range, y-intercept, and the end behavior as $x \to -\infty$ as well as $x \to +\infty$.

3. WHAT IS EXPONENTIAL DECAY?

3A. Fill in the table below. To the best of your ability, fill this table out by hand.

x	$F(x) = \left(\frac{1}{2}\right)^x$	$G(x) = \left(\frac{1}{4}\right)^x$	$H(x) = \left(\frac{1}{5}\right)^x$	$J(x) = \left(\frac{1}{10}\right)^x$
-4				
-3				
-2				
-1				
0				
1				
2				
3				
4				

3B. Graph the functions F(x), G(x), H(x), and J(x) from problem 3A above. <u>Math 48B</u> : Lesson 12 Handout Page 5 of 15



3C. Identify patterns in the graphs of the functions F(x), G(x), H(x), and J(x) from problems 3AB above. Make a conjecture about the general behavior of the graph of the function

$$y = b^x$$
 for $0 < b < 1$

In your conjecture, identify the domain, range, y-intercept, and the end behavior as $x \to -\infty$ as well as $x \to +\infty$.

4. TRANSFORMATIONS OF EXPONENTIAL FUNCTIONS?

4A. For exponential function $y = a \cdot b^{x-h} + k$, what do parameters a, h, and k do to the graph of $y = b^x$?

f(x) $g(x) = 2^{x+2}$ $f(x) = 2^x$ $^{6}_{+64}$ -3 -2 $^{-1}$ -6-5-4 $\mathbf{2}$ $\mathbf{5}$ х ↑ -4 -3 -2 -1 $\mathbf{24}$ $\leftarrow + -6$ $\rightarrow x$ -5-4-3 -2-1 $\dot{2}$

4B. Test your hypothesis from Problem 4A above by graphing the function below.

	$f(x) = 2^{\chi}$	a(m) = 2X + C	-6	$^{-5}$	_4	_3	-2	f(x = 1	2	3	4 5	(
x j	$f(x) = 2^x$	$g(x) = 2^x + 6$	64 +	-5	+	-0	1					4 0	(
			62				-					_	
-4			60 -									-	
			58 -										
			56 -									\rightarrow	
-3			54 -									\rightarrow	
			52 -										
•			50 -									_	
-2			48										
			46									+	
-1			44		-		-						
1			42										
			40										
0			38-										
			36										
1			34										
1													
			30										
2			28										
2			$\frac{20}{26}$										
3			22										
4			18										
4			16										
5			12										
5													
6			8										
7			$\frac{4}{2}$										
			\leftarrow										

4C. Test your hypothesis from Problem 4A above by graphing the function below.

4D. Test your hypothesis from Problem 4A above by graphing the function below.

			-6 -5 -4 -3 -2 -1 $\xrightarrow{y-axis}$ 1 2 3 4 5	6			
x	$f(x) = 2^x$	$g(x) = -2^x$		$\frac{6}{32}$			
			30	30			
4			28	28			
-4			26	26			
			24	-24			
-3				-22			
-3			20	20			
			18	18			
-2			16	-16			
2			14	-14			
			12	-12			
-1			10	-10			
•			8	8			
			6	-6			
0			4	-4			
			2	-2			
			y = 0	$\longrightarrow x$			
1				-2			
				-4			
				-6			
2				= -8			
			-10	-10			
			-12	-12			
3				-14			
			-16	-16			
4				-18			
4				= -20			
				-22			
5				-24			
5				-26			
				-20			
6				-20 -30			
			22	-30			
			$\begin{bmatrix} -32 \\ -6 \\ -5 \\ -4 \\ -3 \\ -2 \\ -1 \\ 0 \end{bmatrix}$ 1 2 3 4 5				
7			x = 0				

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4E. How is your work on problems 4ABC related to the general transformations:

$$g(x) = a f(x-h) + k$$

5. QUADRATIC VERSUS EXPONENTIAL GROWTH?

5A. Fill out the table below

x	$f(x) = x^2$	f(x+1) - f(x)
0		
1		
2		
3		
4		
5		
6		

x	$g(x) = 2^x$	g(x+1) - g(x)
0		
1		
2		
3		
4		
5		
6		

Name: 5B. Fill out the table below

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