Math 48B, Lesson 12: Exponential Functions

In Math 48B Lessons 11, 12, and 13, we study exponential functions:

$$y=b^{x}$$

To begin our exploration, let’s recall the rules of powers/exponents.

1. WHAT ARE RULES OF POWERS/EXPONENTS?

Powers vs exponents: $y=x^{n}$ $y=b^{x}$

Product Rule: $b^{n}∙b^{m}$

Quotient Rule: $\frac{ b^{n} }{b^{m}}$

Zero Power: $1=\frac{ b }{b}=\frac{ b^{1} }{b^{1}}$

Negative Powers: $\frac{ 1 }{b^{n}}$

Power to a Power: $\left(b^{n}\right)^{p}$

2. WHAT IS EXPONENTIAL GROWTH?

2A. Fill in the table below. To the best of your ability, fill this table out by hand.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| $$x$$ | $$f(x)=2^{x}$$ | $$g(x)=4^{x}$$ | $$h(x)=5^{x}$$ | $$j(x)=10^{x}$$ |
| $$-4$$ |  |  |  |  |
| $$-3$$ |  |  |  |  |
| $$-2$$ |  |  |  |  |
| $$-1$$ |  |  |  |  |
| $$0$$ |  |  |  |  |
| $$1$$ |  |  |  |  |
| $$2$$ |  |  |  |  |
| $$3$$ |  |  |  |  |
| $$4$$ |  |  |  |  |

2B. Graph the functions $f(x)$, $g\left(x\right),$ $h(x)$, and $j(x)$ from problem 2A above.



2C. Identify patterns in the graphs of the functions $f(x)$, $g\left(x\right),$ $h(x)$, and $j(x)$ from problems 2AB above. Make a conjecture about the general behavior of the graph of the function

$y=b^{x}$ for $1<b$

In your conjecture, identify the domain, range, y-intercept, and the end behavior as $x\rightarrow -\infty $ as well as $x\rightarrow +\infty $.

3. WHAT IS EXPONENTIAL DECAY?

3A. Fill in the table below. To the best of your ability, fill this table out by hand.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| $$x$$ | $$F(x)=\left(\frac{1}{ 2 }\right)^{x}$$ | $$G(x)=\left(\frac{1}{ 4 }\right)^{x}$$ | $$H(x)=\left(\frac{1}{ 5 }\right)^{x}$$ | $$J(x)=\left(\frac{1}{ 10 }\right)^{x}$$ |
| $$-4$$ |  |  |  |  |
| $$-3$$ |  |  |  |  |
| $$-2$$ |  |  |  |  |
| $$-1$$ |  |  |  |  |
| $$0$$ |  |  |  |  |
| $$1$$ |  |  |  |  |
| $$2$$ |  |  |  |  |
| $$3$$ |  |  |  |  |
| $$4$$ |  |  |  |  |

3B. Graph the functions $F(x)$, $G\left(x\right),$ $H(x)$, and $J(x)$ from problem 3A above.



3C. Identify patterns in the graphs of the functions $F(x)$, $G\left(x\right),$ $H(x)$, and $J(x)$ from problems 3AB above. Make a conjecture about the general behavior of the graph of the function

$y=b^{x}$ for $0<b<1$

In your conjecture, identify the domain, range, y-intercept, and the end behavior as $x\rightarrow -\infty $ as well as $x\rightarrow +\infty $.

4. TRANSFORMATIONS OF EXPONENTIAL FUNCTIONS?

4A. For exponential function $y=a∙b^{x-h}+k$, what do parameters $a$ *,*$h$*,* and $k$ do to the graph of $y= b^{x}$?

4B. Test your hypothesis from Problem 4A above by graphing the function below.

|  |  |  |
| --- | --- | --- |
| *x* | $$f(x)= 2^{x}$$ | $$g(x)= 2^{x+2}$$ |
| -4 |  |  |
| -3 |  |  |
| -2 |  |  |
| -1 |  |  |
| 0 |  |  |
| 1 |  |  |
| 2 |  |  |
| 3 |  |  |
| 4 |  |  |
| 5 |  |  |
| 6 |  |  |
| 7 |  |  |



4C. Test your hypothesis from Problem 4A above by graphing the function below.

|  |  |  |
| --- | --- | --- |
| *x* | $$f(x)= 2^{x}$$ | $$g\left(x\right)=2^{x}+6$$ |
| -4 |  |  |
| -3 |  |  |
| -2 |  |  |
| -1 |  |  |
| 0 |  |  |
| 1 |  |  |
| 2 |  |  |
| 3 |  |  |
| 4 |  |  |
| 5 |  |  |
| 6 |  |  |
| 7 |  |  |



4D. Test your hypothesis from Problem 4A above by graphing the function below.

|  |  |  |
| --- | --- | --- |
| *x* | $$f(x)= 2^{x}$$ | $$g\left(x\right)=-2^{x}$$ |
| -4 |  |  |
| -3 |  |  |
| -2 |  |  |
| -1 |  |  |
| 0 |  |  |
| 1 |  |  |
| 2 |  |  |
| 3 |  |  |
| 4 |  |  |
| 5 |  |  |
| 6 |  |  |
| 7 |  |  |

4E. How is your work on problems 4ABC related to the general transformations:

$$g\left(x\right)=a f\left(x-h\right)+k$$

5. QUADRATIC VERSUS EXPONENTIAL GROWTH?

5A. Fill out the table below

|  |  |  |
| --- | --- | --- |
| *x* | $$f(x)= x^{2}$$ | $$f\left(x+1\right)-f(x)$$ |
| 0 |  |  |
| 1 |  |  |
| 2 |  |  |
| 3 |  |  |
| 4 |  |  |
| 5 |  |  |
| 6 |  |  |

5B. Fill out the table below

|  |  |  |
| --- | --- | --- |
| *x* | $$g(x)= 2^{x}$$ | $$g\left(x+1\right)-g(x)$$ |
| 0 |  |  |
| 1 |  |  |
| 2 |  |  |
| 3 |  |  |
| 4 |  |  |
| 5 |  |  |
| 6 |  |  |

5C. Graph $f(x)= x^{2}$ and $g(x)= 2^{x}$ below. What behavior do you notice? Which one is growing more quickly well as $x\rightarrow +\infty $.

