Name: Solution Class #:______

Math 48B, Lesson 11: Exponential Functions

In Math 48B Lessons 11, 12, and 13, we study exponential functions:

$$y = b^x$$

To begin our exploration, we investigate exponential notation and how this differs from power notation.

1. WHAT IS POWER NOTATION?

"y equals x to the nth power"

Explain power notation:

$$y = \boxed{x^n}$$

Power Notation

Where is input? Where is output? What do you notice about value in the superscript? What type of function(s) use power notation?

In power notation, we see

Input is x ← unknown variable is in base

power n is in superscript

(when superscript is constant, we will call this a power)

In power functons,

Power
superscript
Value is constant

$$f(x) = x^n = y$$

output

input variable
is in base
(base is variable)
(input is at bottom)

Remember in function notation

name of
function

$$y = f(x)$$

output
value

input
variable

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2. WHAT IS EXPONENT NOTATION?

Explain exponent notation:

$$y = \boxed{b^x}$$

Exponent Notation

Where is input? Where is output? What do you notice about value in the superscript? What type of function(s) use power notation?

In an exponential function.

Input up top

input shows up in superscript

$$f(x) = b^{\boxed{x}} = y$$

input variable

base b is a given constant (this is a known value)

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3. COMPARE AND CONTRAST POWER AND EXPONENT NOTATION?

What are the similarities and differences between power and exponent notations:

$$y = \boxed{x^n} \qquad \qquad y = \boxed{b^x}$$

study of anatomy of function

(we have many options for x)

Power Notation

Exponent Notation

How do you know if you're looking at a power function or an exponential function?

In power notation: □ the superscript is constant n and base is variable

In exponentnotation: □ the base b is constant and superscript is variable

Cheat Sheet: At the end of each lecture, Hw, study session write a 1 page "cheat sheet" to capture major ideas as rem

The base in power notation is variable

In exponent notation, the base is constant

Rules: □ superscript constant = power notation

□ if power isn't constant = exponent problem

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3. WHAT IS THE PRODUCT RULE OF POWERS/EXPONENTS?

Let's explore the product rule of powers and exponents. Remember that superscript notation is used to count the number of times we multiply the base by itself. Starting with this definition, solve the problems below.

✓ 3A. $4^7 \cdot 4^5$ 3B. $2^3 \cdot 2^8$ 3C. $b^6 \cdot b^4$

study of numbers/arithmetic

Let's look at 3A ...

$$4^7 \cdot 4^5 = \underbrace{(4\cdot4\cdot4\cdot4\cdot4\cdot4\cdot4)}_{\text{base 4 multiplied by itself 7-times}} \cdot \underbrace{(4\cdot4\cdot4\cdot4\cdot4)}_{\text{base 4 multiplied by itself 5-times}}$$

Note: How superscipts relate to count

□ b^n ← this notation "counts" the number of times b is multiplied by itself

$$= \underbrace{4\cdot4\cdot4\cdot4\cdot4\cdot4\cdot4}_{7}\cdot\underbrace{4\cdot4\cdot4\cdot4\cdot4}_{5}$$

$$= 4^{12}$$

$$= 4^{7+5}$$

the addition in super-script counts the total number of tmes we multpiy 4 by itself

5

$\boxed{3C}$ $b^6 \cdot b^4$ ← power notation
(base value is variable
superscript is Known)

Note: superscripts count the number of times we multiply the base by itself

$$b^6 \cdot b^4 = \underbrace{b \cdot b \cdot b \cdot b \cdot b \cdot b}_{\text{base } b \text{ multiplied by itself 6 times}} \cdot \underbrace{b \cdot b \cdot b \cdot b}_{\text{base } b \text{ multiplied by itself 4 - time}}$$

$$= b^{10} = b^{\underbrace{6+4}}$$

values in superscript we add these together

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3D. Using what you learned in problems 3A – 3C, come up with a rule for the expression below. Describe your rule in VANVS (use language and notation).

$$b^n \cdot b^m$$

General Rule:

$$\boxed{b^n \cdot b^m = b^{n+m}}$$

this is a counting problem that translates multiplication into addition

multiplication (not addition)

☐ On the left-hand side, the problem $b^n \cdot b^m$ asks us to find the total amount that b multiplied ... thus, we must connect the two chains of multiplication using the same operation:

this count total number of units

$$\boxed{2^4} + 2^6 \overset{?}{\neq} 2^{10} = 1024$$

counts how many times we multiply base

$= 2 \cdot 2 \cdot 2 \cdot 2 + 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

$= 16 + 64$

$= 80$

$$b^n + b^m \neq b^{n+m}$$

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4D. Using what you learned in problems 4A – 4C, come up with a rule for the expression below. Describe your rule in VANVS (use language and notation).

$$\frac{b^n}{b^m}$$

Quotient Rule for superscripts

$$\boxed{\frac{b^n}{b^m} = b^{n-m}}$$

bases are identical

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5.	WHAT IS THE RULE FOR NEGATIVE POWERS/EXPONENTS?

5A. Using the quotient rule for exponents/powers, what do you notice about the following statement:

$$1 = \frac{b}{b} = \frac{b^1}{b^1}$$

power on top

$$\frac{\boxed{b^1}}{\boxed{b^1}} = b^{1-1} = b^0 = \frac{b}{b} = 1$$

power on bottom

$$\Rightarrow \boxed{b^0 = 1}$$

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5B. Using the quotient rule for exponents/powers, what do you notice about the following statement:

$$\frac{1}{b^n}$$

$$\frac{1}{b^n} = \frac{b^0}{b^n}$$

$$= b^{0-n}$$

$$= b^{-n}$$

$$\Rightarrow \boxed{b^{-n} = \frac{1}{b^n}}$$

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6. WHAT IS THE POWER RULE OF POWERS/EXPONENTS?

Let's explore the power rule of powers and exponents. Remember that superscript notation is used to count the number of times we multiply the base by itself. Starting with this definition, solve the problems below.

6A. $(5^3)^4$ 6B. $\left(\frac{1}{3^2}\right)^6$ 6C. $(b^3)^2$

superscript counts the number of times we multiply the base by itself

$$\left(\frac{1}{3^2}\right)^{\boxed{6}} = \underbrace{\frac{1}{3^2}\cdot\frac{1}{3^2}\cdot\frac{1}{3^2}\cdot\frac{1}{3^2}\cdot\frac{1}{3^2}\cdot\frac{1}{3^2}}_{6\text{-copies}}$$

$$= \frac{1}{3\cdot 3}\cdot\frac{1}{3\cdot 3}\cdot\frac{1}{3\cdot 3}\cdot\frac{1}{3\cdot 3}\cdot\frac{1}{3\cdot 3}\cdot\frac{1}{3\cdot 3}$$

$\frac{A}{B}\cdot\frac{C}{D} = \frac{AC}{BD}$

each copy has 2 threes

$$= \frac{1}{3\cdot 3\cdot 3\cdot 3\cdot 3\cdot 3\cdot 3\cdot 3\cdot 3\cdot 3\cdot 3\cdot 3}$$

$$= \frac{1}{3^{12}}$$

$$= \left(\frac{1}{3}\right)^{12} \Rightarrow \left(\left[\frac{1}{3}\right]^2\right)^6 = \left(\frac{1}{3}\right)^{2\cdot 6} = \left(\frac{1}{3}\right)^{12}$$

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6D. Using what you learned in problems 6A – 6C, come up with a rule for the expression below. Describe your rule in VANVS (use language and notation).

$$(b^n)^p$$

$$\boxed{(b^n)^p = b^{n \cdot p}}$$

Review of these formula

superscript is constant

Power notation: $x^n = y$

base is variable

superscript is variable

Exponent notation $b^x = y$

base is constant

) Product Rule: $b^n \cdot b^m = b^{n+m}$

) Quotient Rule: $\dfrac{b^n}{b^m} = b^{n-m}$

Zero power: $1 = \dfrac{b}{b} = \dfrac{b^1}{b^1} = b^{1-1} = b^0 \Rightarrow \boxed{b^0 = 1}$

negative power : $\frac{1}{b^n} = \frac{b^0}{b^n} = b^{0-n} = b^{-n}$

Power to a power : $(b^n)^p = b^{n \cdot p}$

Note: $f(x) = 2^{x}$ ← variable in superscript

↑ base is constant

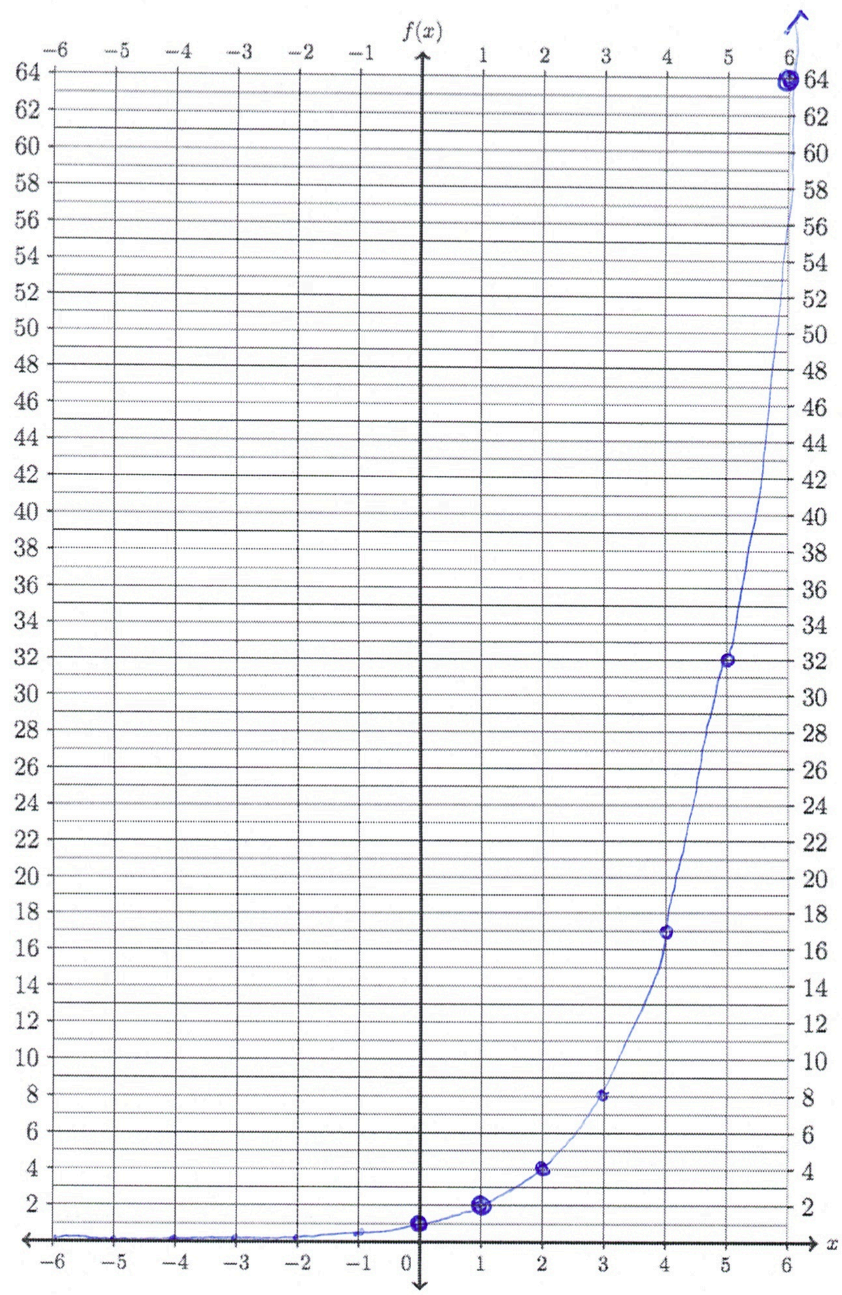
← This is an exponential function (base is constant and variable is up top)

7. WHAT IS EXPONENTIAL GROWTH?

7A. Exponential growth curve: Fill in the tables below and graph this function.

y-intercept (point where x=0)

x	$f(x) = 2^x$
−4	1/16
−3	1/8
−2	1/4
−1	$\frac{1}{2}$
0	$1 = 2^0$
1	$2 = 2^1$
2	4
3	8
x = 4	16
5	32
6	64
7	128
8	256
9	512
10	1024



☐ to move down from one row to the next, we multiply previous output by 2 (this is because base is 2)

☐ to move upward in table

☐ notice that we have a horizontal asymptote as $x \to -\infty$ with $\boxed{\lim_{x \to -\infty} f(x) = 0^{+}}$

$x = 4 \Rightarrow 2^x = 2^4 = 2 \cdot 2 \cdot 2 \cdot 2 = \boxed{16}$

8

$x = 3 \Rightarrow 2^x = 2^3 = 2 \cdot 2 \cdot 2 = \boxed{8} =$ 16

4

$x = 2 \Rightarrow 2^x = 2^2 = 2 \cdot 2 = \boxed{4} =$

2

$x = 1 \Rightarrow 2^x = 2^1 = \boxed{2}$

$2 \times 128 = 2(100 + 20 + 8)$

$= 200 + 40 + 16$

$= 256$

$2 \times 256 = 5$

$1 \div 2 = \frac{1}{2} =$

$b^{-n} = \frac{1}{b}$

$2^{-2} = \frac{1}{2^2} = \frac{1}{4}$

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7B. As the x-values get larger, do the output values of $f(x)$ grow or decay? Why does this occur?

☐ As x gets bigger, we notice $y = 2^x$ also gets bigger since base $b = 2$.

CONJECTURE

☐ In fact, for any $b > 1$ (positive value for b bigger than 1

$f(x) = b^x$ ← this will grow.

CONJECTURE

☐ In fact, for any base $0 < b < 1$ (positive base b less than 1)

$f(x) = b^x$ ← this will decay

Note: for $f(x) = \left(\frac{1}{2}\right)^x$ ← superscript variable

base is constant

☐ This is an exponential function!

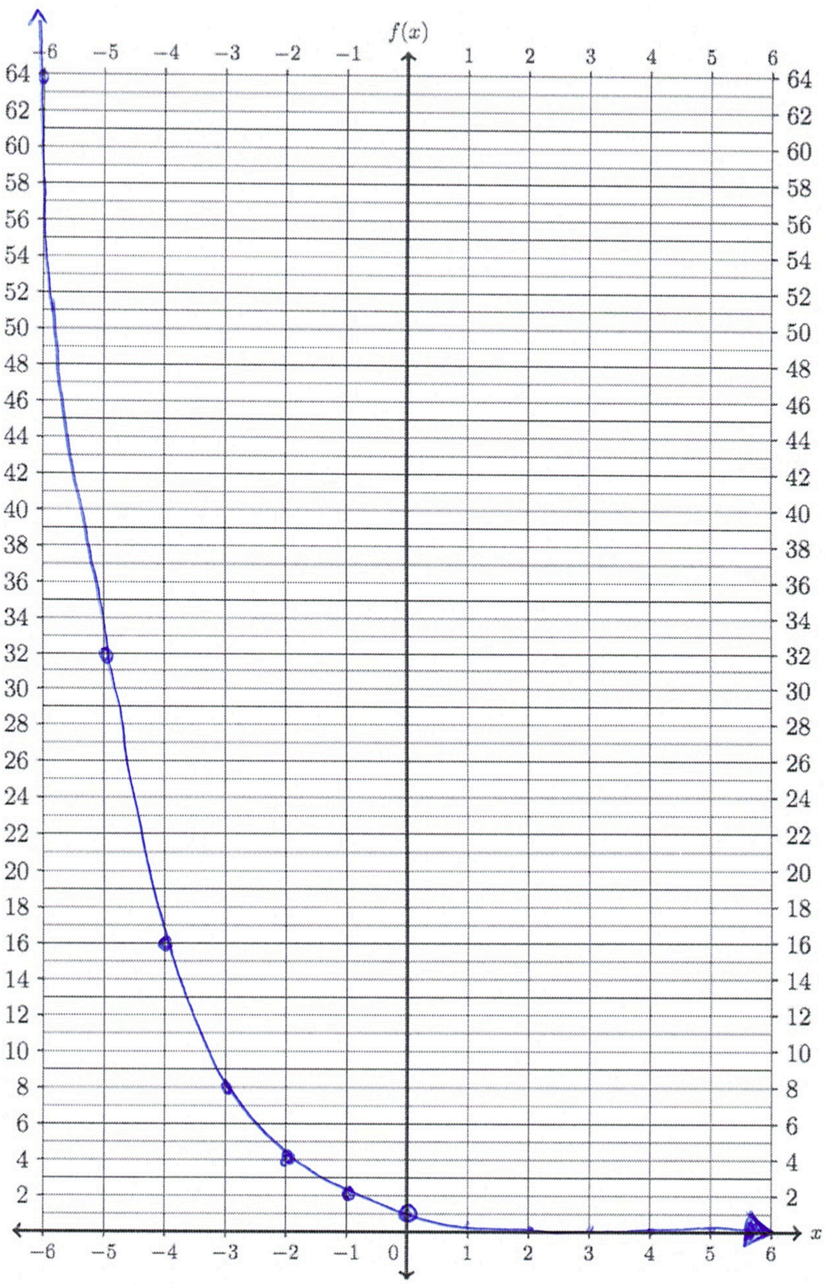
base $b = \frac{1}{2} > 0$ positive

8. WHAT IS EXPONENTIAL DECAY?

8A. Exponential decay curve: Fill in the tables below and graph this function.

y-intercept (were x=0)

x	$f(x) = \left(\frac{1}{2}\right)^x$
−8	256
−7	128
−6	64
−5	32
−4	16
−3	8
−2	4
−1	2
0	$1 = \left(\frac{1}{2}\right)^0$
1	1/2
2	1/4
3	1/8
4	1/16



☐ Horizontal asymptote

$$\lim_{x \to +\infty} f(x) = 0^+$$