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Math 48B, Lesson 10: Rational Functions, Part 2

In Math 48B Lessons 8, 9, and 10, we study rational functions in the form:

$$R(x) = \frac{a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x^1 + a_0 x^0}{b_m x^m + b_{m-1} x^{m-1} + \cdots + b_1 x^1 + a_0 x^0}$$

Numerator:
Standard form of an
nth degree polynomial
Denominator:
Standard form of an
mth degree polynomial

Rational Function
(RATIO of polynomials)

To begin our exploration, we explore some fundamental properties of division.

1. WHAT ARE RULES OF FRACTIONS?

Recall each of the following rules for fractions:

$$\frac{0}{K}$$

$$\frac{K}{0}$$

$$\frac{A}{A}$$

$$\frac{A}{1}$$

$$\frac{A}{B} \cdot \frac{C}{D}$$

$$\frac{A}{B} \div \frac{C}{D}$$

$$\frac{1}{\text{HUGE NUMBER}}$$

$$\frac{1}{\text{tiny number}}$$

2. HOW TO GRAPH A RATIONAL FUNCTION?

Consider the instructions below.

HOW TO**Given a rational function, sketch a graph.**

1. Evaluate the function at 0 to find the y -intercept.
2. Factor the numerator and denominator.
3. For factors in the numerator not common to the denominator, determine where each factor of the numerator is zero to find the x -intercepts.
4. Find the multiplicities of the x -intercepts to determine the behavior of the graph at those points.
5. For factors in the denominator, note the multiplicities of the zeros to determine the local behavior. For those factors not common to the numerator, find the vertical asymptotes by setting those factors equal to zero and then solve.
6. For factors in the denominator common to factors in the numerator, find the removable discontinuities by setting those factors equal to 0 and then solve.
7. Compare the degrees of the numerator and the denominator to determine the horizontal or slant asymptotes.
8. Sketch the graph.

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2A. Use these instructions to graph the following function:

$$R(x) = \frac{x^2 - x - 6}{(x - 2)(x + 1)^2}$$

Use Desmos.com as a tool to run your analysis.

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2B. Use these instructions to graph the following function:

$$R(x) = 4 + \frac{x + 3}{x^2 - 9}$$

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2C. Use these instructions to graph the following function:

$$R(x) = \frac{4x + 2}{x^2 + 4x - 5}$$

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