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Math 48A, Lesson 12: Combining of Functions

1. COMPOSITION OF FUNCTIONS

COMPOSITION OF FUNCTIONS

Given two functions f and g, the **composite function** $f \circ g$ (also called the **composition** of f and g) is defined by

$$(f \circ g)(x) = f(g(x))$$

The domain of $f \circ g$ is the set of all x in the domain of g such that $g(x)$ is in the domain of f. In other words, $(f \circ g)(x)$ is defined whenever both $g(x)$ and $f(g(x))$ are defined. We can picture $f \circ g$ using an arrow diagram (Figure 4).

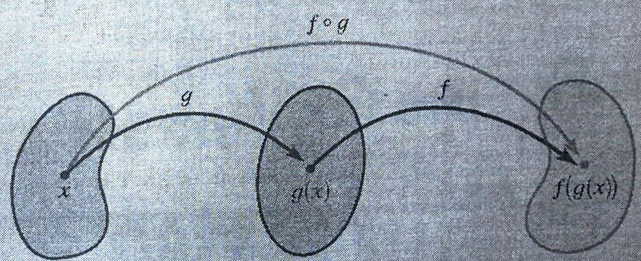

FIGURE 4 Arrow diagram for $f \circ g$

1A. Please translate this into abuelita language. In other words, translate this idea into language that your grandmother (abuelita) can understand.

Composition of functions: to mix or combine

$$f \circ g(x) = f(g(x))$$

small open circle

$$f \circ g(x) = f(g(x))$$

"f of g of x"

function composition

Notice: Recall from function notation that

$$y = \overbrace{g(x)}^{\text{output to } g(x)}$$

output

input (inside the parenthesis)

$$\Rightarrow \quad f(y) = f\Big(\underbrace{\overbrace{g(x)}^{\text{input to } f}}_{\text{output to function } g}\Big)$$

input to function f

$$\Rightarrow \quad f \circ g(x) = f(g(x))$$

functon composition is when the input of f is the output of g

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1B. Consider the following functions

$f(x) = \sqrt{x}$ and $g(x) = x^2 + 1$

input

We're going to study composition of functions in the form

$$f \circ g(x) = f(g(x))$$

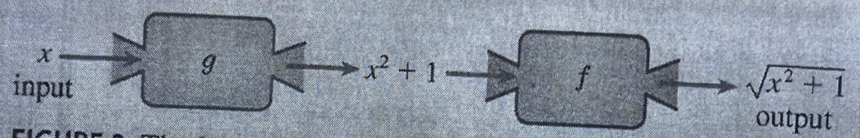

FIGURE 3 The h machine is composed of the g machine (first) and then the f machine.

$$f \circ g(x) = f(\underbrace{\boxed{g(x)}}_{})$$

let $y = g(x)$

$$= f(y)$$

$$= \sqrt{\boxed{y}}$$

$$= \sqrt{g(x)}$$

$$= \sqrt{x^2+1}$$

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2. COMPOSITIONS OF FUNCTIONS

Consider the following functions

$f(x) = x^2$ and $g(x) = x + 3$

Find each of the following functions:

2A. $f \circ g(x) = f(g(x))$ ✓

2B. $g \circ f(x) = g(f(x))$

f of g of x

outer function

$$f \circ g(x) = f(g(x))$$

open circle

inner function

the input of the outer function is the output of the inner function

$$= f(\boxed{x+3})$$

$$= \boxed{(x+3)}^2$$

$$= (x + 3) \cdot \boxed{(x+3)}$$

O F I L

$(A+B) \cdot \boxed{C}$

$= A \cdot C + B \cdot C$

$$= \boxed{x} \cdot (x+3) + \boxed{3} \cdot (x+3)$$

$A \cdot (B+C)$
$= A \cdot B + A \cdot C$

$$= x^2 + 3x + 3x + 9$$

F O I L

$$= \boxed{x^2 + 6x + 9} = (x+3)^2$$

"g of f of x"

outer function

$$g \circ f(x) = g(f(x))$$

inner function

$$= g(x^2)$$

$$= \boxed{x^2 + 3}$$ ✓

Function composition

- the input to g is the output of f
- the input to the outer function is the output to the inner function

Recall:

- $(2+x)\cdot(3-x)$

  parenthesis here represent multiplication

- $g(f(x))$

  do NOT represent multiplication

  (Abuse of notation: here we use parenthesis to identify inputs - the stuff inside)

Overloading notation

- the same notation represents two different ideas

In function notation; paranthesis tell us inputs

$$g(\boxed{x}) = \boxed{x} + 3$$

parenthesis helps us identify

$$g(\boxed{}) = \boxed{} + 3$$

$$g(\boxed{f(x)}) = \boxed{f(x)} + 3$$

$$= \boxed{x^2} + 3$$

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3. SUM AND DIFFERENCE OF FUNCTIONS

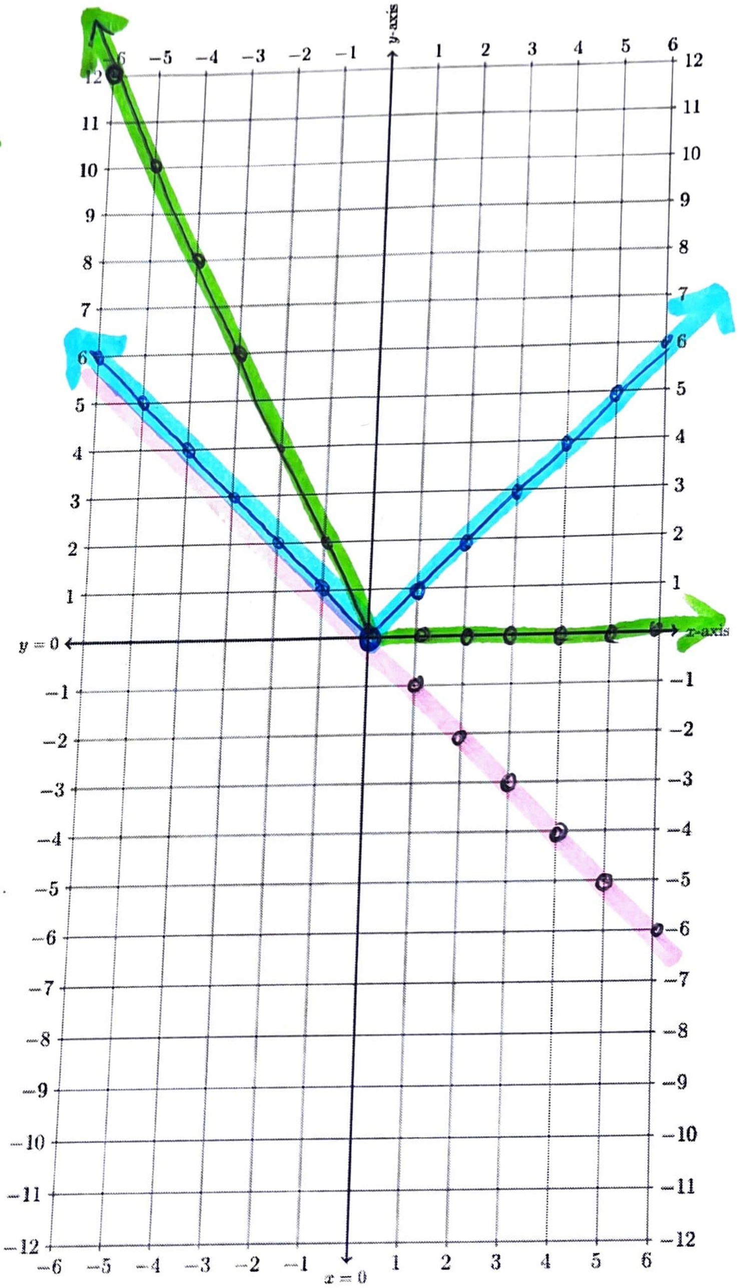
3A. Consider the following functions

sum of functions

$$f(x) = |x|, \qquad g(x) = -x, \qquad h(x) = (f+g)(x) = \underbrace{f(x) + g(x)}$$

Create a table of values and graph the resulting parabolas on these axes below.

Input	Output		
x	$f(x)$	$g(x)$	$h(x)$
−6	6	6	12
−5	5	5	10
−4	4	4	8
−3	3	3	6
−2	2	2	4
−1	1	1	2
0	0 vertex	0	0
1	1	−1	0
2	2	−2	0
3	3	−3	0
4	4	−4	0
5	5	−5	0
6	6	−6	0



3B. Consider the following functions