Name: In-class notes : Lesson 5 Class #:

1. SOLVE QUADRATIC EQUATIONS USING GRAPHS

1A. Look back on your work on problems 6A – 6C on Lesson 4 handout. Recall that we tried to solve the quadratic equation:

$$\boxed{2x^2 - 8x - 1} = \boxed{3 - x}$$

← RHS: right-hand side of equals sign

LHS: left-hand side of equals sign

Create a table of values and graph the resulting curves on this axes below. Using that work, solve each of the following problems. For each problem, graph the solution interval on the axis provided. Make explicit connections between your solution and the graphs that you draw.

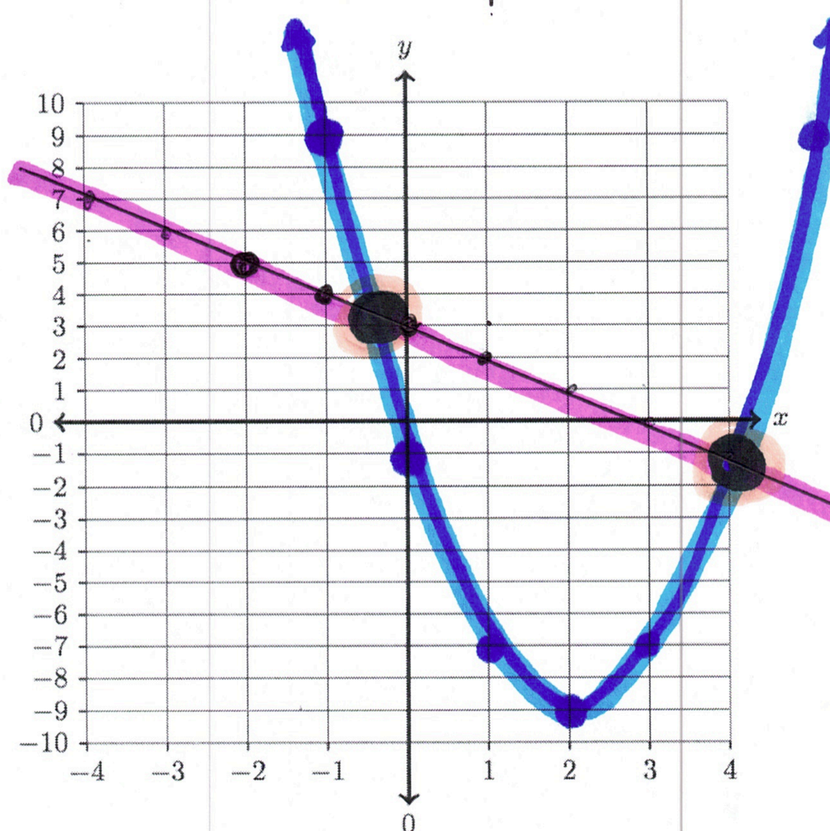
Notice : $3 - x = -x + 3$

$= \frac{-1}{1} \cdot x + 3$

Input x-value

output y-value

x	Left-hand side: $2x^2-8x-1$	Right-hand side: $3-x$
−2	23	5
−1	9	4
−0.5	3.5	3.5
0	−1	3
1	−7	2
2	−9	1
3	−7	0
4	−1	−1
5	9	−2



Points of intersection are at $(-0.5, 3.5)$ or $(4, -1)$

(solutions to equations happen when the graph cross)

⇒ to solve this equation, we say

$\boxed{x = -0.5 = -\frac{1}{2}}$ OR $\boxed{x = 4}$

①

$$\boxed{3 - x} = 3 + -x$$

$$= -x + 3$$

$$= -1 \cdot x + 3$$

$$= mx + b$$

$$= \boxed{\frac{-1}{1} \cdot x + 3}$$

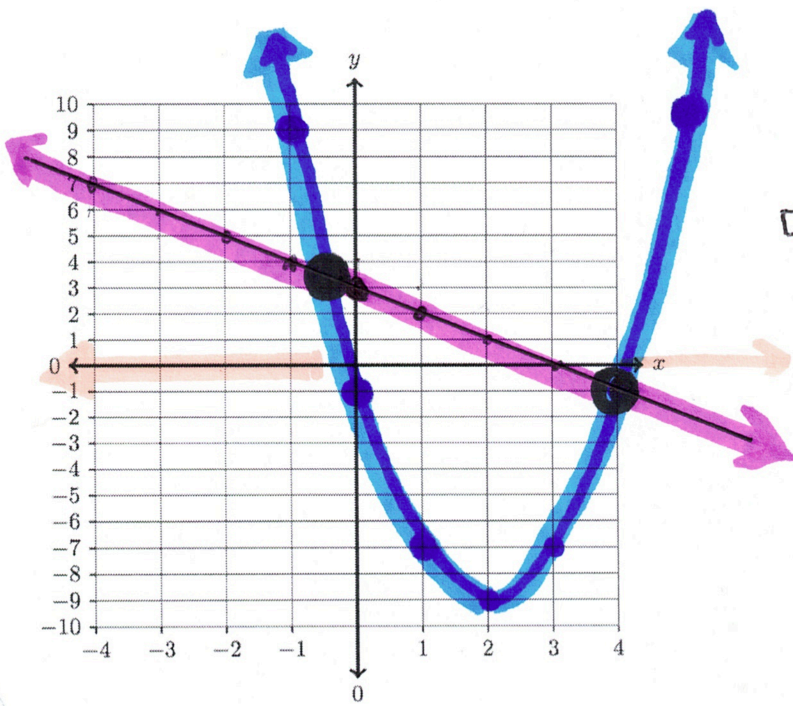
slope

y-intercept (0,3)

$$m = \frac{rise}{run}$$

a strict inequality is a greater than or less than sign w/ no line underneath

1B. Find all x-values such that: $\boxed{2x^2 - 8x - 1} > \boxed{3 - x}$



- "when is the left-hand side greater than the right-hand sides"
- "when is the output y-values on the blue parabola above the output y-values on the purple line"

We see this happens

- "before" the two curves touch at the point $(\boxed{-0.5}, 3.5)$
- "after" the two curves touch at the point $(4, -1)$

In symbols, we write the following intervals

$(-\infty, -0.5)$ OR $(4, +\infty)$

open parenthesis (Never touches at negative infinity)

open parenthesis (we have a strict inequality with no line underneith)

open parenthesis

open parenthesis

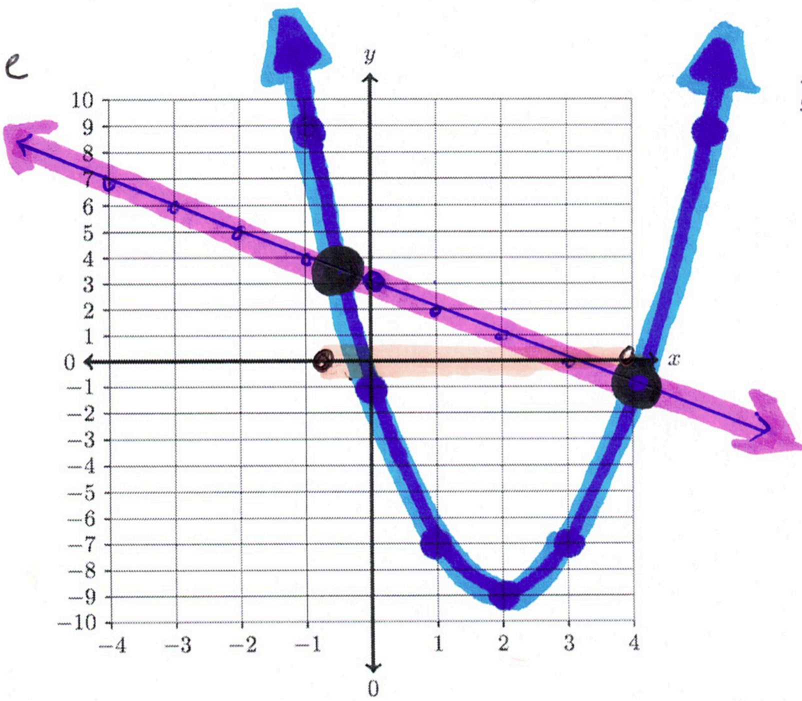
3

inequality: less than or equal to sign" has the line underneath the alligator mouth

1C. Find all x-values such that: $\boxed{2x^2 - 8x - 1} \leq \boxed{3 - x}$

parabola

line



☐ "when is the left-hand side less-than-or-equal to the right-hand side"

☐ "when are the output y-values on blue parabola below or touching the output y-values on the purple line."

We see this happens when:

☐ the blue parabola is under or touching between two points which are

left point: $(\boxed{-0.5}, 3.5)$

right point: $(\boxed{4}, -1)$

☐ in symbol we write that the inequality in this problem is true when x is in

closed bracket (we touch this point because we start with less than OR Equal)

$[-0.5, 4]$

closed bracket

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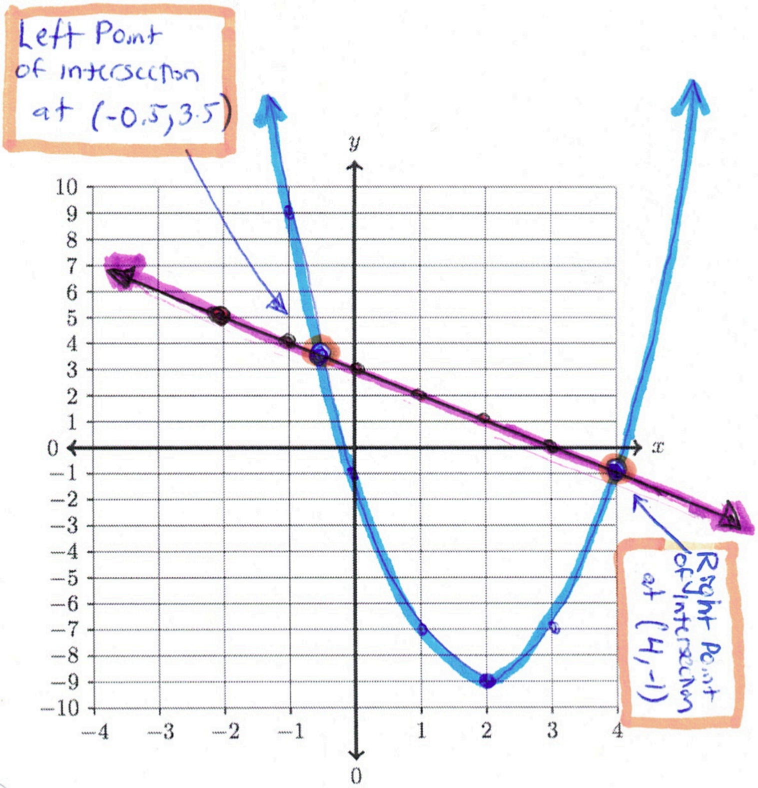
Home work

1D. Find all x-values such that: $\boxed{2x^2 - 8x - 1} < \boxed{3 - x}$



- This is asking us the question:

  when is the purple curve strictly above the blue curve

Note: because we have a strict inequality $<$, we are NOT interested in where the two curves touch.

- We notice that the purple curve ($3-x$ on right-hand side) is above the blue curve ($2x^2 - 8x - 1$ on left-hand side) in between the two points of intersection, not including the exact points where the two curves touch.

- We see that this corresponds to the interval where

$$-0.5 < x < 4$$

$$\Rightarrow \quad x \in (-0.5, 4)$$

interval notation

open parenthesis (we have a strict inequality with no line underneath)

open parentheses

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Homework

2. SOLVE ABSOLUTE VALUE EQUATION USING GRAPHS

Graphical Technique to solve an algebraic equation

To find the solution to algebraic equations using a graphical technique, we use the following five step program for salvation:

Step 1: Graph the function y_1 on the left-hand side of the equals sign.
Step 2: Graph the function y_2 on the right-hand side of the equals sign.
Step 3: Find the point(s) of intersection between the graphs of the two functions.
Step 4: Write each point of intersection as an ordered pair in the form: (x, y)
Step 5: Set the variable from the original algebraic equation equal to the 1st coordinate of each point of intersection. These "x"-values are the solution(s) to the algebraic equation.

2A. Consider the following equation : $2 \cdot |x - 2| - 4 = 1 - x.$

A. Identify and graph the function on the left-hand side of the equals sign: $2 \cdot |x-2| - 4$

B. Identify and graph the function on the right-hand side of the equals sign: $1 - x$

C. Find and label the points of intersection on the graph below. Make sure to write each point of intersection as an ordered pair in the form (x, y).

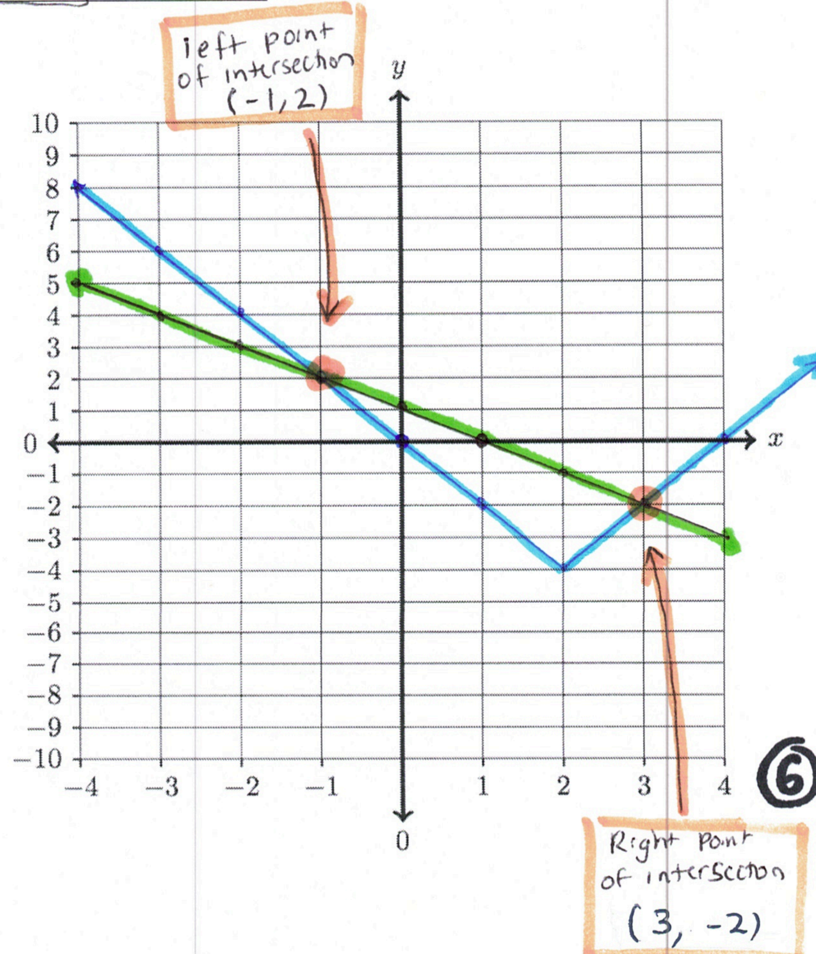
D. Identify the x – value for each point of intersection.

E. Identify the solution(s) to this equation: $\boxed{x=-1}$ or $\boxed{x=3}$

x	Left-hand side:	Right-hand side: $1-x$
−4	8	5
−3	6	4
−2	4	3
−1	2	2
0	0	1
1	-2	0
2	-4	-1
3	-2	-2
4	0	-3

left point of intersection

right point of intersection



with $x = 3$

⑥

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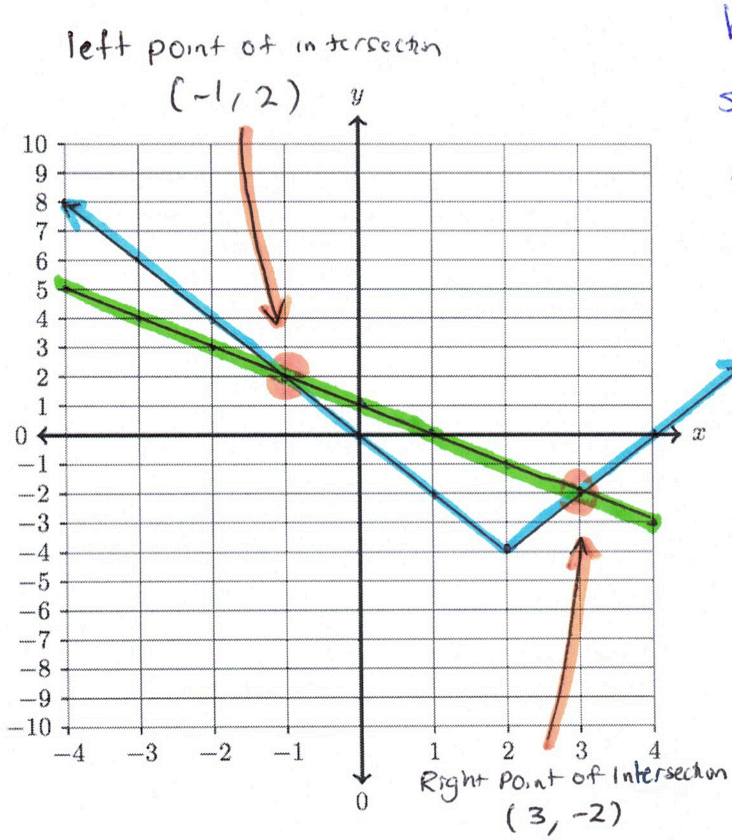
Homework

2B. Use the table you drew in Problem 2A and re-draw your graph below. Then using this graph to for the x-values that satisfy the inequality:

$$\boxed{2 \cdot |x - 2| - 4} > \boxed{1 - x}$$

• We want to know where the blue left-hand side curve is strictly above the green right-hand side curve (not including the points where the two curves touch since we have a strict inequality >).



Looking at our graph, we see the blue curve $f(x) = 2|x-2| - 4$ is above the green curve $g(x) = 1 - x$ everywhere when

$x < -1$ or $x > 3$

$\Rightarrow$ $(-\infty, -1)$ or $(3, \infty)$

open parenthesis since $x < -1$ and $x \neq -1$

open parenthesis correspond to strict inequalities

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3. ANALYZE THE GRAPH OF A CUBIC POLYNOMIAL

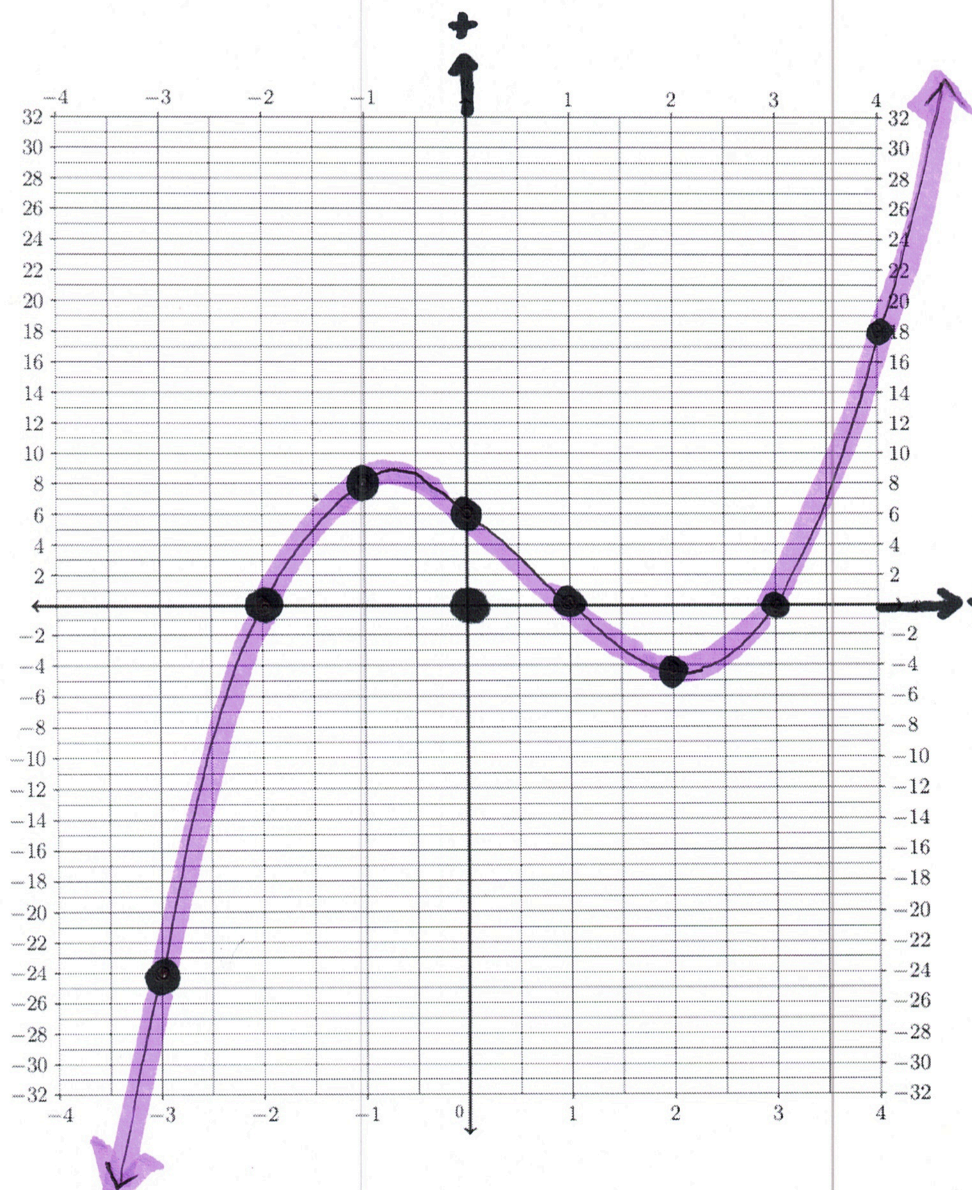
3A. Use any method you'd like to fill out the table of value and graph the cubic polynomial function:

$f(x) = x^3 - 2x^2 - 5x + 6$ ← cubic polynomial (power function)

input x-values

output y-value

x	$f(x) = x^3 - 2x^2 - 5x + 6$
−5	−144
−4	−70
−3	−24
−2	0
−1	8
0	6
1	0
2	−4
3	0
4	18
5	56



Question :

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3B. Write your first draft of a definition for what it means for a function to be **increasing**.

Make sure to include:

First priority: A definition in your own language using street knowledge

Use abuelita language to describe the idea. In other words, use language that even your abuelita can understand.

Second priority: Write this out using nerdy language. See if you can include formal mathematical symbols. This is the formal concept definition found in your textbook.

Abuelita language (street knowledge):

- ☐ a graph is "increasing" when it rises
- ☐ we say a function is "increasing" if as we increase x-value, then y-values also increase
- ☐ we say function is increasing when the "slope" positive (CAREFUL: not all graphs are lines and we won't study tangent lines until calculus)
- ☐ as inputs increase so to do output increase

Jasiri conjecture: ☐ when the curve is above the x-axis the

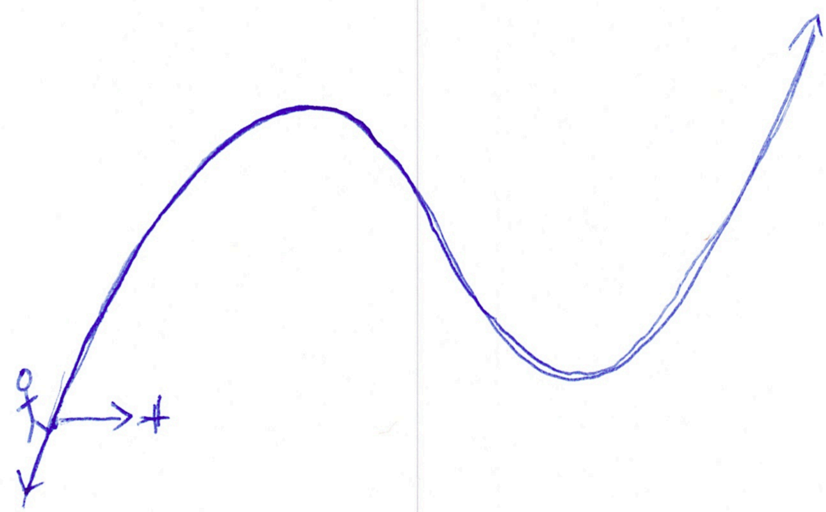
FALSE ☐ we see the curve increasing when the curve is above the x-axis

Vincent conjecture ☐ the curve is increasing when it is positive FALSE

☐

☐ As we go right, we go up

☐ Imagine we have a little person on the graph



as time goes he alway tries to walk to the right (in positive input directions). If, when he does so, he has to climb upward, we say the graph is increasing

☐ as the input increase the output increases

Formal math language: If $x_1 < x_2$

the $f(x_1) < f(x_2)$

A function $f(x)$ is increasing if and only if

when $\boxed{x_1 < x_2}$, then $f(x_1) < f(x_2)$

(if $\boxed{\text{the input increases}}$) (then output gets bigger)

$x_1 < x_2$: means that we are moving to the right on the x-axis

$f(x_1) < f(x_2)$ means that we are moving upward on the y-axis (increasing output values)

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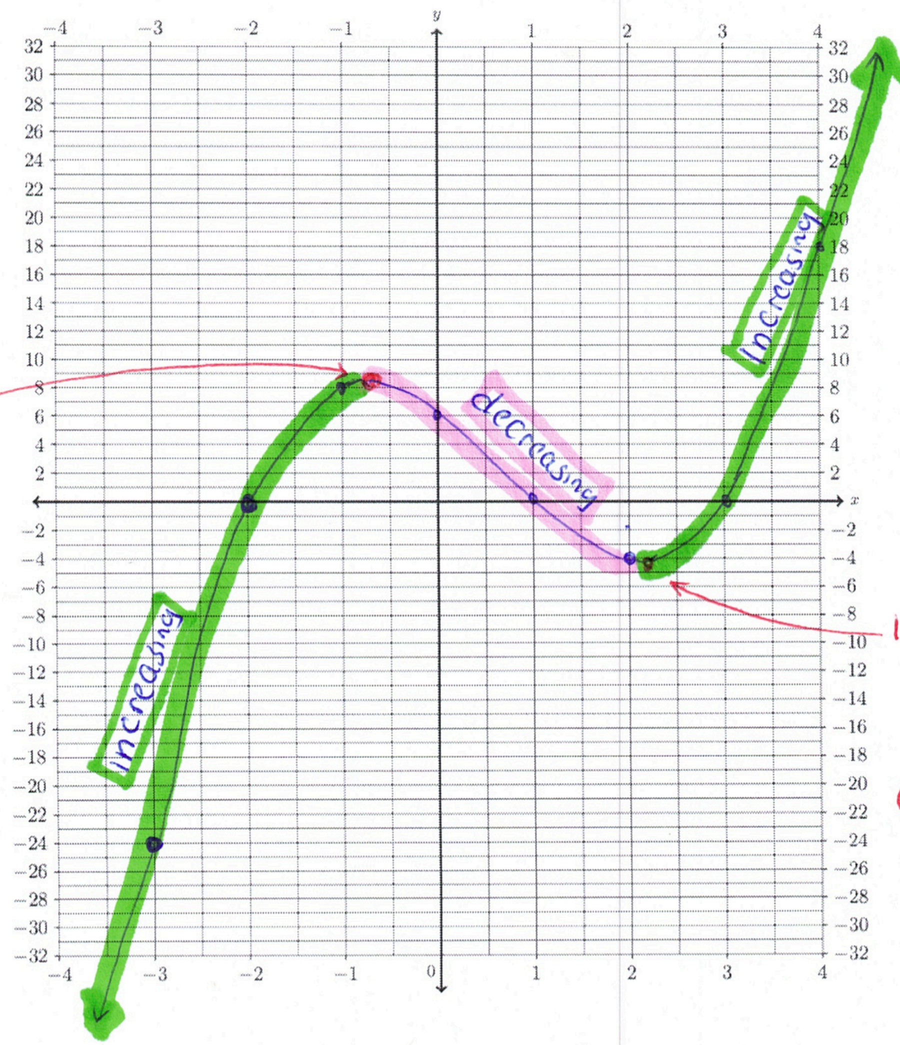
3C. Where is the function $f(x) = x^3 - 2x^2 - 5x + 6$ increasing? Why? Please specifically identify the location(s) on the graph below.

local maximum point at

(-0.786, 8.209)

(approximation using desmos.com)



local minimum point at

(2.12, -4.061)

(use desmos.com)

- This graph is increasing when $-\infty < x < -0.786$ and again when $2.12 < x < \infty$

- These intervals correspond to the parts of the graph that are colored in green above.