

Math 48A, Lesson 4: Read Graphs to Solve Equations

1. EVALUATE LINEAR FUNCTIONS

Recall our discussion of function notation:

$$f(x) = -2x - 3$$

↑
output

input

1. Let $f(x) = -2x - 3$. Evaluate each of the following:

$f(-4)$

$f(-\frac{1}{2})$

$f(\frac{1}{2})$

and

$f(4)$

Input: $x = -\frac{1}{2} \Rightarrow \boxed{f(-\frac{1}{2}) = -2}$

Output: $f(-\frac{1}{2}) = (-2x - 3) \Big|_{x = -\frac{1}{2}}$

evaluation bar

$$= -2\left(-\frac{1}{2}\right) - 3$$

$$= \boxed{-2} \cdot \boxed{\frac{-1}{2}} - 3$$

multiplication

fraction

①

$$= \frac{-2}{1} \cdot \frac{-1}{2} - 3$$

fraction fraction

↑
multiplication

$$= \frac{-2 \cdot -1}{1 \cdot 2} - 3$$

$$= \frac{2}{2} - 3$$

$$= 1 - 3$$

$$= -2$$

$$\boxed{\frac{2}{1}} \cdot \boxed{\frac{1}{2}} = \frac{2 \cdot 1}{1 \cdot 2} = \frac{2}{2} = 1$$

fraction fraction
multiplication

Famous Algebra Tricks

$$\square \quad \boxed{\frac{A}{B}} \cdot \boxed{\frac{C}{D}} = \frac{A \cdot C}{B \cdot D} = \frac{AC}{BD}$$

fraction fraction
multiplication

they make a bond
(multiplication bond
on top and
on bottom)

When multiplying two fraction,

they come together

and divide

$$\square \quad \boxed{\frac{A}{B}} \div \boxed{\frac{C}{D}} = \boxed{\frac{A}{B}} \cdot \boxed{\frac{D}{C}} = \frac{A \cdot D}{B \cdot C}$$

fraction fraction
divide

$$\square \quad \text{Division by 1: } A = \frac{A}{1}$$

(division by 1 doesn't
change anything)

- When multiplying two numbers together, remember sign matters

Sign of one number

Sign of other number

	+	-
+	+	-
-	-	+

- $\frac{A}{A} = 1$ if $A \neq 0$

any nonzero number divided by itself is zero

Algebraic Technique to solve algebraic equations

To find the solution of an algebraic equation using an algebraic method, isolate the unknown variable by using inverse operations.

2. Solve the equation $-2x - 3 = \frac{1}{2}x + 2$ using an algebraic technique. Show your steps. If possible, please solve this equation in more than one way.

Linear equation: $-2x - 3 = \frac{1}{2}x + 2$

ouch... let's annihilate this first

anything I do to RHS, I gotta do to LHS

$$\Rightarrow -2x - 3 \neq \frac{2}{1} \cdot \left(\frac{1}{2}x + 2 \right)$$

multiply to every value (distributivity)

$$\Rightarrow -2x - 3 \neq 1 \cdot x + \frac{2}{1} \cdot 2$$

$$\Rightarrow -2x - 3 \neq x + 2 \cdot 2$$

⑥

$$\Rightarrow \begin{array}{l} -2x - 3 \\ +2x \end{array} \neq \begin{array}{l} x + 4 \\ +2x \end{array}$$

~~Cancel~~

$$\Rightarrow -3 \neq 3x$$

CORRECT MV MISTAKE:

$$2(-2x - 3) = \frac{2}{1} \left(\frac{1}{2}x + 2 \right)$$

distribute

$$\begin{array}{l} -4x - 6 \\ +4x \end{array} = \begin{array}{l} x + 4 \\ +4x \end{array}$$

~~Cancel~~

$$\begin{array}{l} -6 \\ -4 \end{array} = \begin{array}{l} 5x + 4 \\ -4 \end{array}$$

~~Cancel~~

$$-10 = 5x$$

$$\frac{-10}{5} = \frac{5 \cdot x}{5}$$

$$-2 = 1 \cdot x$$

$$\boxed{-2 = x}$$

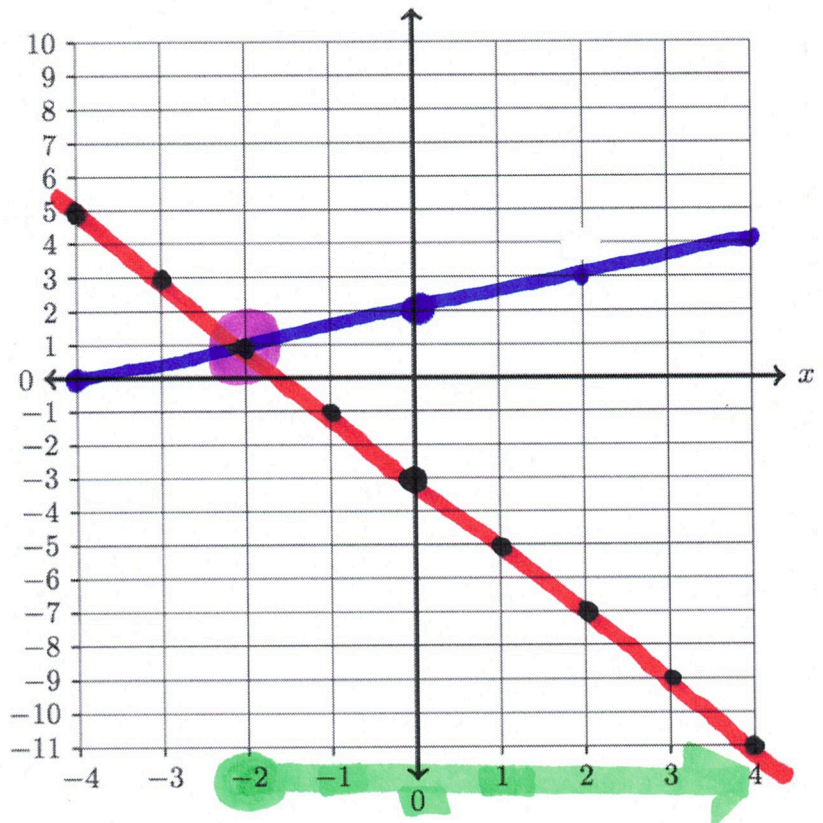
Graphical Technique to solve algebraic equations

To find the solution(s) to an algebraic equation using a graphical technique, complete the five steps:

- Step 1: Identify and graph the function on the left-hand side of the equals sign.
 Step 2: Identify the function on the right-hand side of the equals sign and graph this function on the same axis you used in step 1.
 Step 3: Find the point(s) of intersection between the graphs of the two functions.
 Step 4: Write each point of intersection as an ordered pair in the form (x, y) . Then, identify the 1st coordinate (the "x" value) of the points of intersection.
 Step 5: Set the variable from the equation equal to the 1st coordinate of each point of intersection. This represents the solution(s) to the algebraic equation.

3A. Complete the 5 steps outlined above to solve the equation $-2x - 3 = \frac{1}{2}x + 2$ using a graphical technique.

	Left-hand side:	Right-hand side:
x	$y_1 = -2x - 3$	$y_2 = \frac{1}{2}x + 2$
-4	5	0
-3	3	0.5
-2	1	1
-1	-1	1.5
0	-3	2
1	-5	2.5
2	-7	3
3	-9	3.5
4	-11	4



Point of
Intersection:
(Intersection)

$(-2, 1)$

$$x = -2$$

x-coordinate

$$y = 1$$

y-coordinate

3B. At what point do the two graphs in Problem 3A above intersect? Specifically identify this point on the graph and write it as an ordered pair below.

$(-2, 1)$

3C. What is the x -coordinate of the point of intersection you found in Problem 3B? In other words, what does x equal when the two lines cross in the graph you found in Problem 3A.

$$x = -2$$

3D. What is the solution to the equation $-2x - 3 = \frac{1}{2}x + 2$ that you found in Problem 2? How does that answer compare to your answer to problem 3C?

It is identical !

Finding points of intersection
solves equations

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3E. Use the graph you drew in Problem 5A to solve the inequality: $-2x - 3 \leq \frac{1}{2}x + 2$

$$\boxed{-2x - 3} \leq \boxed{\frac{1}{2}x + 2}$$

This is true where

$$-2 \leq x < \infty$$

In interval notation

$$[-2, \infty)$$

Closed bracket
(it's allowed
to be equal)

Open
parenthesis
(Never touches
this value)