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3. QUADRATIC FUNCTIONS TYPE OF POWER FUNCTION

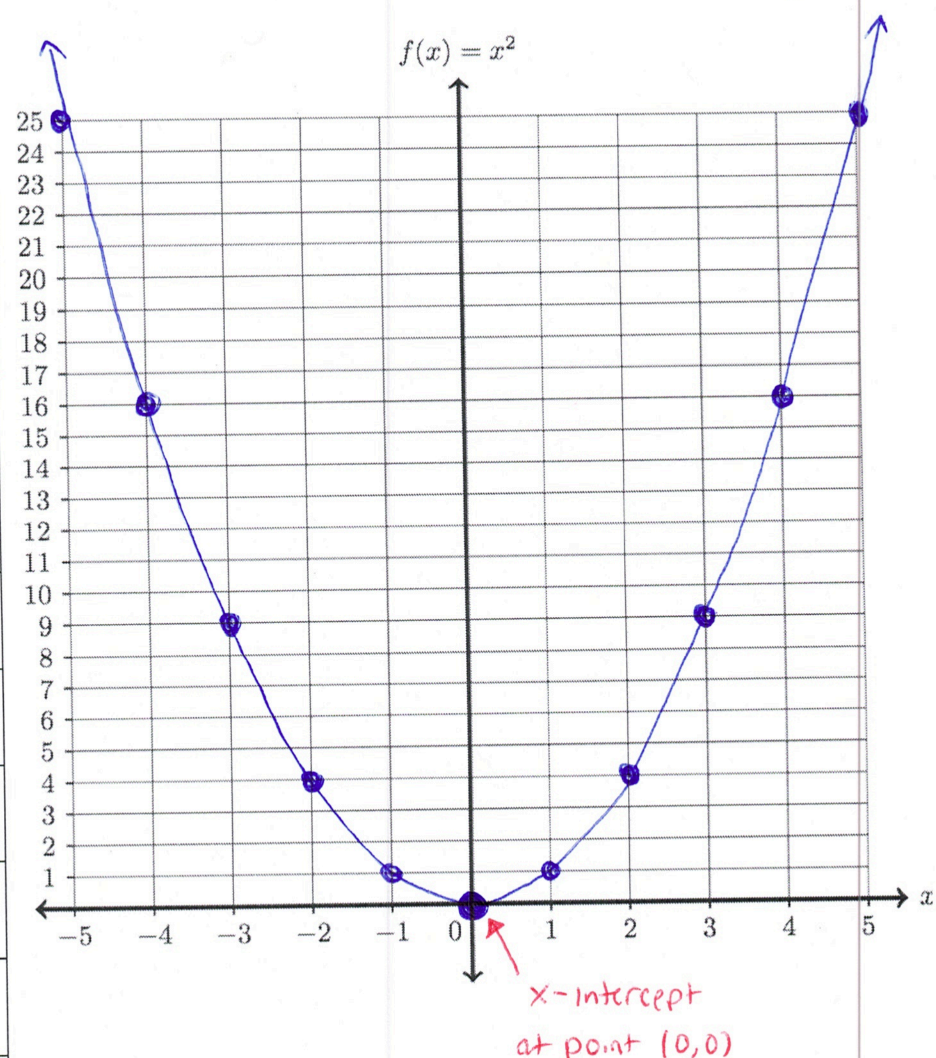
Consider the quadratic function

$$f(x) = x^2$$

Fill out the table below. Then use that table to graph the quadratic function.

A. Fill in the table below
B. Plot these points on the axis provided
C. Interpolate between the points you plotted to create the graph of this function

Input	*Output*
x	$f(x) = x^2$
-5	+25
-4	+ 16
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9
4	16
5	25



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3D. What is the x–intercept of the quadratic function $f(x) = x^2$?

(Write about how the x-intercept shows up in your graph from parts 3A – 3C).

- The x-intercept is the point where the graph touches the x-axis (where the y-value is zero).
- This x-intercept point shows up on the graph where our curve goes through the $y = 0$ horizontal line
- In the case of the function $y = x^2$, we see that the x-intercept is at point $(0, 0)$ since when $x = 0 \Rightarrow x^2 = 0^2 = 0 = y$

3E. What is the y–intercept of the quadratic function $f(x) = x^2$?

(Write about how the y-intercept shows up in your graph from parts 3A – 3C).

- The y-intercept is the point where the graph touches the y-axis (where the x-value is zero)
- This y-intercept point shows up on the graph where our curve goes through the $x = 0$ vertical line
- For $y = x^2$, this happens at point $(0, 0)$.

3F. Why does the graph of $f(x) = x^2$ never go below the x-axis?

- The output values y of the function $y = f(x) = x^2$ come from squaring the input value x: this output will ALWAYS be nonnegative and thus the graph never dips below $y=0$ line.
- Notice, we have only three situations

i. Input $x > 0$ (input is positive) $\Rightarrow x \cdot x = x^2 > 0$ ← a positive times a positive is positive

ii. Input $x < 0$ (input is negative) $\Rightarrow x \cdot x = x^2 > 0$ ← a negative times a negative is a positive

iii. Input $x = 0$ (Input is zero) $\Rightarrow x \cdot x = x^2 = 0$ ← zero times any number is always zero

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3G. What is the domain of the quadratic function $f(x) = x^2$?
(Write about how the domain shows up in your graph from parts 3A – 3C).

- The domain of any function $y = f(x)$ is the set of all valid input values x.

- for the function $f(x) = x^2$, we notice that we can input any real number x and square it. In other words, we encounter no problem input values x in $f(x) = x^2$.

- Thus, the domain for $f(x) = x^2$ is all real numbers.

- symbollically, we can write this as

$$\text{domain}(f(x)) = \underbrace{\mathbb{R}}_{\text{set notation}} = \underbrace{(-\infty, \infty)}_{\text{interval notation}}$$

3H. What is the range of the quadratic function $f(x) = x^2$?
(Write about how the range shows up in your graph from parts 3A – 3C).

- The range of a function is all values of the output that are achieved by the function.

- In other words, if we take all possible input values x and evaluate $f(x)$ for each one, the collect all output values we achieve, we'll get our range.

- For the function $f(x) = x^2$, we see the output of this function is all nonngative numbers $y \geq 0$.

- Thus, the range for $f(x) = x^2$ is all nonnegative real numbers. symbollically, we write

$$\text{Rng}(f(x)) = \text{Range}(x^2) = \underbrace{[0, \infty)}_{\text{interval notation}} = \overbrace{\{y \in \mathbb{R} : y \geq 0\}}^{\text{set notation}}$$

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4. SQUARE ROOT FUNCTION — TYPE OF ROOT FUNCTION

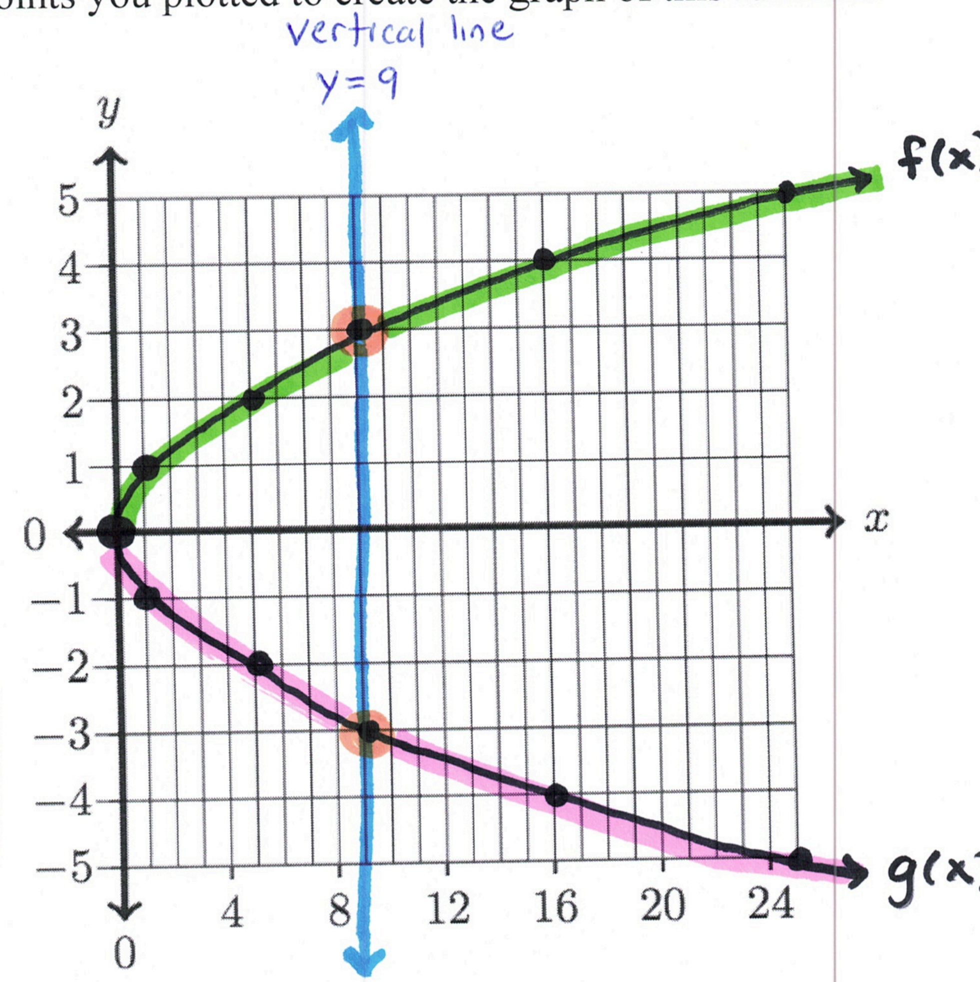
Consider the absolute value function

$$f(x) = \sqrt[2]{x} = \boxed{+\sqrt[2]{x}}$$

Fill out the table below. Then use that table to graph the quadratic function.

A. Fill in the table below
B. Plot these points on the axis provided
C. Interpolate between the points you plotted to create the graph of this function

x^2	x: −	x: +
−1	Not Possible	
0	0	
1	−1	+1
4	−2	+2
9	−3	+3
16	−4	+4
25	−5	+5



$$g(x) = \boxed{-\sqrt{x}}$$

fails the vertical line test

⇒ graph represents a relation, not a function

Formally speaking, when we write

$$\sqrt[2]{25} = x$$

$$\Rightarrow \left(\sqrt[2]{25}\right)^2 = x^2$$

$$\Rightarrow 25 = x^2$$

$$\Rightarrow x = +5 \quad \text{or} \quad x = -5$$

Then, it should be the case

that $\sqrt[2]{25} \overset{?}{=} +5$ or -5

Note: Mathematicians say $\sqrt[2]{25} = +5$ and choose the nonnegative answer to these questions in general.

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4D. What is the x–intercept of the square root function $f(x) = \sqrt[2]{x}$?
(Write about how the x-intercept shows up in your graph from parts 4A – 4C).

- The x-intercept of a function is the point where the graph of that function crosses through the x-axis (touches the horizontal line $y=0$).
- Since the function $y = \sqrt[2]{x}$ is the nonnegative branch, we see this graph touches the x-axis with $y=0$ at point $(0,0)$.
- this is the x-intercept. $(0,0)$

4E. What is the y–intercept of the square root function $f(x) = \sqrt[2]{x}$?
(Write about how the y-intercept shows up in your graph from parts 4A – 4C).

- The y-intercept of a function is the point where the graph of $y=f(x)$ crosses through the y-axis (touches the vertical line $x=0$)
- We see that for $y=\sqrt[2]{x}$, when $x=0$, $y=\sqrt{x}=\sqrt{0}=0$ so our y-intercept is at the point $(0,0)$.

4F. Why does the graph of $f(x) = \sqrt[2]{x}$ never go below the x-axis?

- For the function $y=\sqrt[2]{x}$ we are trying to "undo" a square operation so that we find an y-value where $y^2 = x$.
- For any positive x-value, we can find two possible y-values so that $y^2=x$: one will be negative and the other positive
- In order to ensure $f(x)=\sqrt[2]{x}$ is a function where each input maps to a unique output, we say $y=\sqrt[2]{x} \geq 0$ are the nonegative values s.t. the square equals x with $y^2=x$ and $y \geq 0$
- This assumption means we never dip below the x-axis.

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4G. What is the domain of the square root function $f(x) = \sqrt[2]{x}$?
(Write about how the domain shows up in your graph from parts 3A – 3C).

- The domain of a function $y = f(x)$ is the set of all valid input x-values.

- For $f(x) = \sqrt[2]{x}$, we see x can never be negative:

$\Rightarrow$ Domain of $\sqrt[2]{x}$ is all nonnegative x-values with $x \geq 0$

$\Rightarrow$ Domain$(\sqrt[2]{x}) = [0, \infty)$
$= \{x \in \mathbb{R} : x \geq 0\}$

Side note
Let's pretend $x = -1$ is valid
$\Rightarrow y = \sqrt[2]{-1}$
$\Rightarrow y^2 = -1$ ← Not possible this is
But y^2 will always be nonnegative with $y^2 \geq 0$

4H. What is the range of the square root function $f(x) = \sqrt[2]{x}$?
(Write about how the range shows up in your graph from parts 4A – 4C).

- The Range of the function $y = f(x)$ is the set of all possible output y-values. In other words, if we run through all possible input x-values and collect all output values, we produce the range.

- For $y = \sqrt[2]{x}$ we see that $y \geq 0$ so that the Range is given as $[0, \infty) = \{y \in \mathbb{R} : y \geq 0\}$