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Math 48A, Lesson 2: Graphs Popular Functions

1. GRAPH LINEAR FUNCTION (TYPE OF POWER FUNCTION)

Consider the linear function

$$f(x) = -1 - \frac{5}{2}x$$

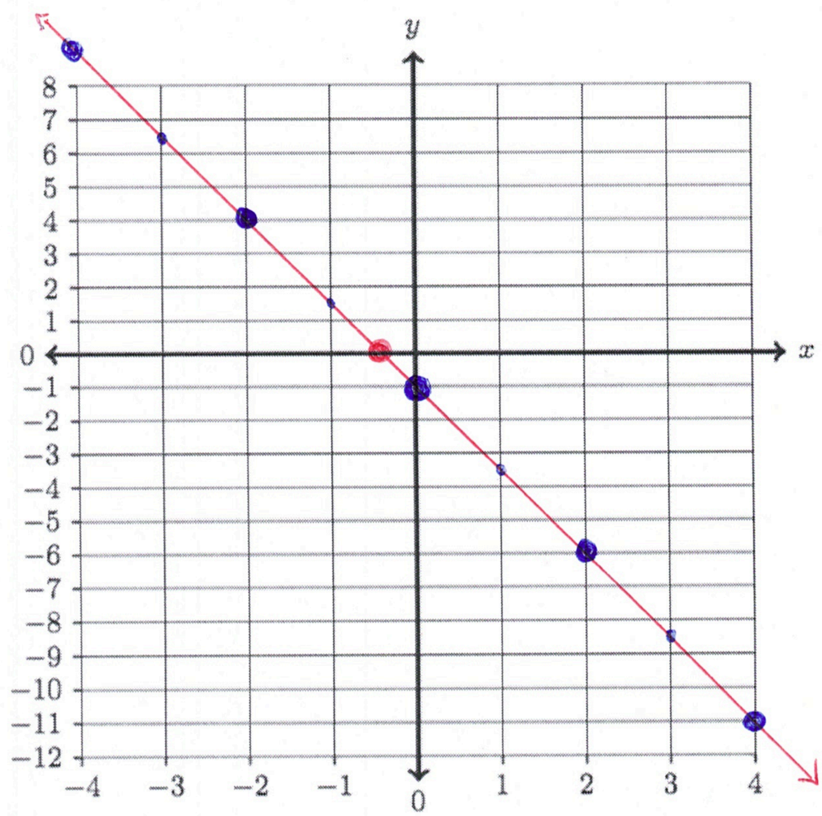
Fill out the table below. Then use that table to graph the given linear function.

A. Fill in the table below ← (see next pages for more details)

B. Plot these points on the axis provided

C. Interpolate between the points you plotted to create the graph of this function

Input	*Output values*
x	$f(x) = -1 - \frac{5}{2}x$
-4	+9
-3	+6.5
-2	+4
-1	+1.5
0	-1
1	-3.5
2	-6
3	-8.5
4	-11



Slope: $m = \frac{-5}{2}$

y-intercept: $(0, b) = (0, -1)$
(with $b = -1$)

(point where line crosses the up & down axis)

Let's do this old school (by paper and pencil) to warm ourselves up.

$$\boxed{\text{Input} \quad x = -4} \Rightarrow f(-4) = +9$$

$$f(x) = -1 - \frac{5}{2} \cdot x$$

If $x = -4$, then we have

$$f(x) = f(-4)$$

$$= -1 - \frac{5}{2} \cdot (-4)$$

$$= -1 + \boxed{\frac{-5}{2} \cdot \frac{-4}{1}}$$

for more here see the next page

$$= -1 + \boxed{10} = \boxed{-9}$$

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Recall that when we multiply two fractions, we can multiply the numerators and also multiply the denominators:

$$\frac{A}{B} \cdot \frac{C}{D} = \frac{A \cdot C}{B \cdot D} = \frac{AC}{BD}$$

For this case, we see

negative times a negative becomes positive

$$\frac{-5}{2} \cdot \frac{-4}{1} = \frac{(-5)\cdot(-4)}{2 \cdot 1} = \frac{+20}{2} = 10$$

$\boxed{\text{Input } x = -3}$ $\Rightarrow$ $f(-3) = 6.5 = \frac{13}{2}$

$$f(x) = f(-3)$$

$$= -1 - \frac{5}{2} \cdot (-3)$$

$$= -1 + \boxed{\frac{-5}{2} \cdot \frac{-3}{1}}$$

← when multiplying a fraction by another fraction multiply the numbers on top and also multiply the numbers on the bottom (bring multiplication into both numerator AND into the denominator)

$$= -1 + \boxed{\frac{(-5)\cdot(-3)}{2 \cdot 1}}$$

for more here see next page

$$= \boxed{-1} + \frac{15}{2}$$

$$= \boxed{\frac{-2}{2}} + \frac{15}{2}$$

$$= \frac{-2+15}{2} = \frac{13}{2} = 6.5 = 6\tfrac{1}{2}$$

When adding two fractions, we want to find a common denominator. If we add two fractions with the same denominator, we can add numerators and leave the denominator alone:

add the two numerators together

$$\frac{A}{D} + \frac{B}{D} = \frac{A+B}{D}$$

the same denominator

denominator stays the same (only one copy)

In our case, we want to find:

$$-1 + \frac{15}{2} = -\frac{1}{1} + \frac{15}{2}$$

we can always put a 1 in the denominator since division by one does not change the value we can always multiply by one and maintain equality

$$\Rightarrow \frac{-1}{1} + \frac{15}{2} = \frac{-1}{1} \cdot \boxed{1} + \frac{15}{2}$$

these denominators are different...

(we want to find a common denominator $\frac{A}{D} + \frac{B}{D} = \frac{A+B}{D}$)

we can write one in creative form to get a common denominator

$$= \frac{-1}{1} \cdot \boxed{\frac{2}{2}} + \frac{15}{2}$$

to multiply two fractions multiply stuff on top and also stuff on bottom

$$\boxed{\frac{A}{B} \cdot \frac{C}{D} = \frac{AC}{BD}}$$

$$= \boxed{\frac{-1 \cdot 2}{1 \cdot 2}} + \frac{15}{2}$$

$$= \boxed{\frac{-2}{2}} + \frac{15}{2}$$

these denominators are the same... (Now we can add)

$\boxed{\text{Input } x = -2}$ $\Rightarrow$ $f(-2) = +4$

$$f(x) = f(-2)$$

$$= -1 - \frac{5}{2} \cdot (-2)$$

$$= -1 + \frac{-5}{2} \cdot \frac{-2}{1}$$

$$= -1 + \frac{(-5)\cdot(-2)}{2 \cdot 1}$$

$$= -1 + \frac{10}{2}$$

$$= -1 + 5$$

$$= 4$$

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number next to variable x

constant not near x

1D. What is the slope of the line $f(x) = -1 - \frac{5}{2}x$?
(Write more about how the slope shows up in your graph from parts 1A – 1C).

□ Recall the general slope intercept form: $y = \boxed{m}x + \boxed{b}$

$$f(x) = -1 - \frac{5}{2}x = -1 + \frac{-5}{2}x = \boxed{\frac{-5}{2}}x + \boxed{-1}$$

$$\Rightarrow \text{slope} = \boxed{m = \frac{-5}{2}} = \frac{\text{rise}}{\text{run}}$$

Go down five and go to the right by two units

1E. What is the x–intercept of the line $f(x) = -1 - \frac{5}{2}x$?
(Write about how the x-intercept shows up in your graph from parts 1A – 1C).

□ Recall that the x-intercept is the point where the graph crosses the horizontal axis. We can also say that the x-intercept is the place where line touches x-axis.

□ Each point has two coordinates (x, y). On the horizontal axis, we know $y=0$. we want to find x where $y=0$ on line $\Rightarrow$ x-intercept is $\boxed{(-2/5, 0)}$

1E. What is the y–intercept of the line $f(x) = -1 - \frac{5}{2}x$?
(Write about how the y-intercept shows up in your graph from parts 1A – 1C).

□ The y-intercept is the point where the curve crosses the vertical axis. We might also state that the y-intercept is where the line touches the y-axis (where $x=0$)

□ The y-intercept is a point with two coordinates (x,y) On the vertical axis, we know $x=0$. To find the y-intercept, we set $x=0$ and find output at this point: $f(0) = -1 \Rightarrow$ y-intercept is $\boxed{(0,-1)}$

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To find x-intercept, we set output $f(x) = 0$ and try to find x:

$$f(x) = 0 = -1 - \frac{5}{2} \cdot x$$

$$\Rightarrow \quad -1 - \frac{5}{2} x = 0$$

$$+1 \qquad\qquad +1$$

$$\Rightarrow \quad -\frac{5}{2} x = 1$$

$$\Rightarrow \quad \frac{-2}{5} \cdot \frac{-5}{2} \cdot x = \frac{-2}{5} \cdot 1$$

$$\Rightarrow \quad \boxed{x = \frac{-2}{5}}$$

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$\Rightarrow$ x-intercept is at point $(-2/5, 0) = (-0.4, 0)$

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1G. What is the domain of the line $f(x) = -1 - \frac{5}{2}x$?
(Write about how the domain shows up in your graph from parts 1A – 1C).

The domain of f(X) is the set of all valid input values for variable X.

For our linear function $f(x) = -1 - \frac{5}{2}x$ we can input any real number

$\Rightarrow$ "doman of our line is all real numbers"

$\Rightarrow$ $dom(f) = (-\infty, \infty)$ or $dom(f) = \mathbb{R}$

"the domain of function f is all real numbers from negative to positive infinity"

1H. What is the range of the line $f(x) = -1 - \frac{5}{2}x$?
(Write about how the range shows up in your graph from parts 1A – 1C).

The range of function f(X) is the set of all achieved output values when we evaluate f(X) at all doman values.

For our linear function $f(x) = -1 - \frac{5}{2}x$, we see our line outputs all possible real numbers

$\Rightarrow$ range of our line is all real numbers" or $Rng(f) = \mathbb{R}$

$\Rightarrow$ $Rng(f) = (-\infty, \infty)$

"the range of our function f is all real numbers between negative infinity and positive infinity"

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2. ABSOLUTE VALUE FUNCTIONS

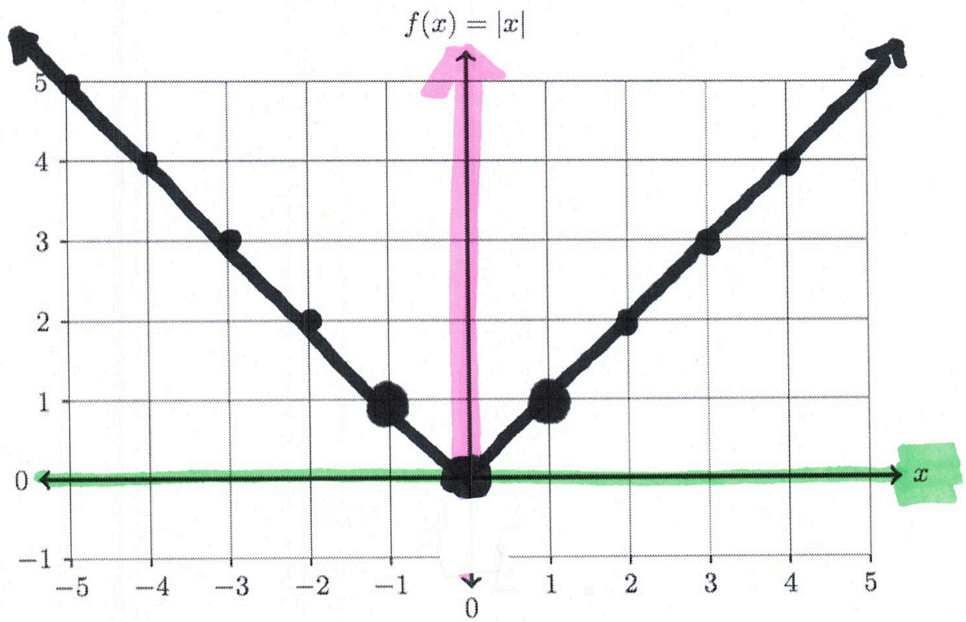
Consider the absolute value function

$$f(x) = |\, x \,|$$

Fill out the table below. Then use that table to graph the absolute value function.

A. Fill in the table below
B. Plot these points on the axis provided
C. Interpolate between the points you plotted to create the graph of this function

Input	*Output*
x	$f(x) = \|\, x \,\|$
-5	$+5$
-4	$+4$
-3	$+3$
-2	$+2$
-1	$f(-1) = \|-1\| = 1$
0	$f(0) = \|0\|$ 0
1	$f(1) = \|1\| = 1$
2	2
3	3
4	4
5	5



$0 \Leftrightarrow (0, 0)$ 2nd coordinate is output y-value

$(1, 1)$

1st coordinate of ordered pair corresponds to the x-value

- input is horizontal
  output is vertical
- output is nonnegative
  on or above green line
  (x-axis)

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2D. What is the x–intercept of the absolute value function $f(x) = |x|$?
(Write about how the x-intercept shows up in your graph from parts 2A – 2C).

The x-intercept of our graph is where our curve touches the horizontal x-axis. Since any point on the x-axis has an output value of $y=0$, we can find this point by setting $f(x)=0$ and finding all x that satisfy this. For $f(x)=|x|$, we see our x-intercept is $\boxed{(0,0)}$

2E. What is the y–intercept of the absolute value function $f(x) = |x|$?
(Write about how the y-intercept shows up in your graph from parts 2A – 2C).

The y-intercept of our graph is where the absolute value function touches the vertical y-axis. This must happen when $x=0$ since all points on y-axis have x-value $x=0$. Thus, we see our y-intercept is at $\boxed{(0,0)}$ since $f(0)=|0|=0$

2F. Why does the graph of $f(x) = |x|$ never go below the x-axis?

Absolute value function can never be negative

or $|x| \geq 0$

$\Rightarrow$ the graph never dips below the horizontal axis

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2G. What is the domain of the absolute value function $f(x) = |x|$?
(Write about how the domain shows up in your graph from parts 2A – 2C).

domain : • all valid inputs for a function

• all x-values we can use

ANY Number

domain of $|x|$:
- any positive #
- zero works
- any negative #

$\text{dom}(|x|) = (-\infty, \infty) = \mathbb{R} =$ all real numbers

2H. What is the range of the absolute value function $f(x) = |x|$?
(Write about how the range shows up in your graph from parts 2A – 2C).

Range • all valid outputs for a function

• all y-values we can use

$\text{Rng}(|x|) = [0, \infty)$ ← "the range of $|x|$ is all nonnegative real numbers"

(interval notation)

Arithmetic and Algebra Tools

- ☐ Figure out the sign of a product of two real numbers

- ☐ $$\frac{A}{B} \cdot \frac{C}{D} = \frac{A \cdot C}{B \cdot D} = \frac{AC}{BD}$$

- ☐ $$\frac{A}{D} + \frac{B}{D} = \frac{A+B}{D}$$

- ☐ WARNING: $$\frac{A}{B} + \frac{C}{D} \neq \frac{A+C}{B+D}$$

- ☐ For any real number, we have

  - ☐ $1 \cdot A = A = A \cdot 1$ (multiplication by 1 does nothing)

  - ☐ $\frac{A}{1} = A$ (division by 1 does nothing)

☐ For any nonzero real number $A \neq 0$ we have

$$\frac{A}{A} = 1$$

(any number divided by itself equals one as long as we're not talking about the number zero)

☐ For any nonzero real number $A \neq 0$, we have that

$$\frac{0}{A} = 0$$

zero divided by any real number is zero