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8.	ROOT FUNCTIONS	AKA RADICAL FUNCTIONS INVERSE OF POWER FUNCTIONS

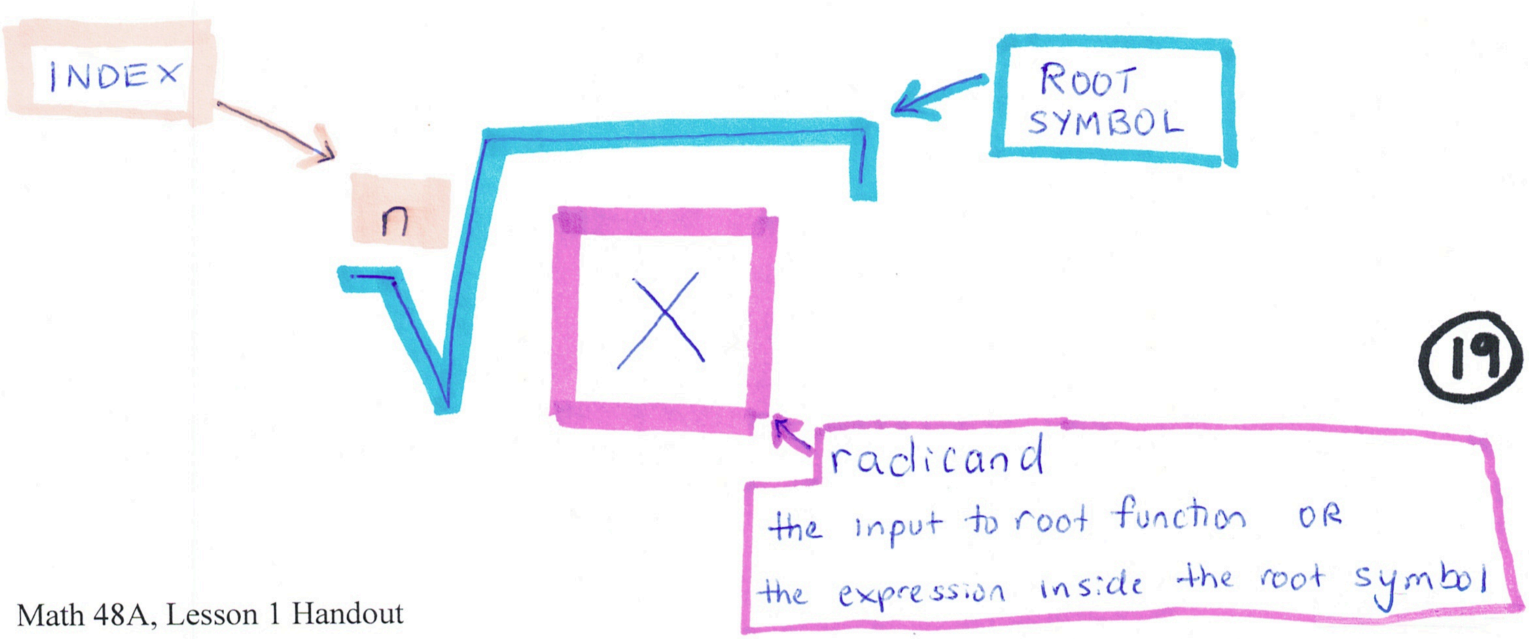
Let's explore root notation (also known as radical notation). How might we read the following statement:

$$f(x) = \sqrt[n]{x} \quad \text{for} \quad n \in \mathbb{N}$$

"f of x equals the nth root of x for n in the natural numbers."

What does this notation mean? What are the special features of this notation?

Recall from our discussion in class, radical notation has a number of interesting features



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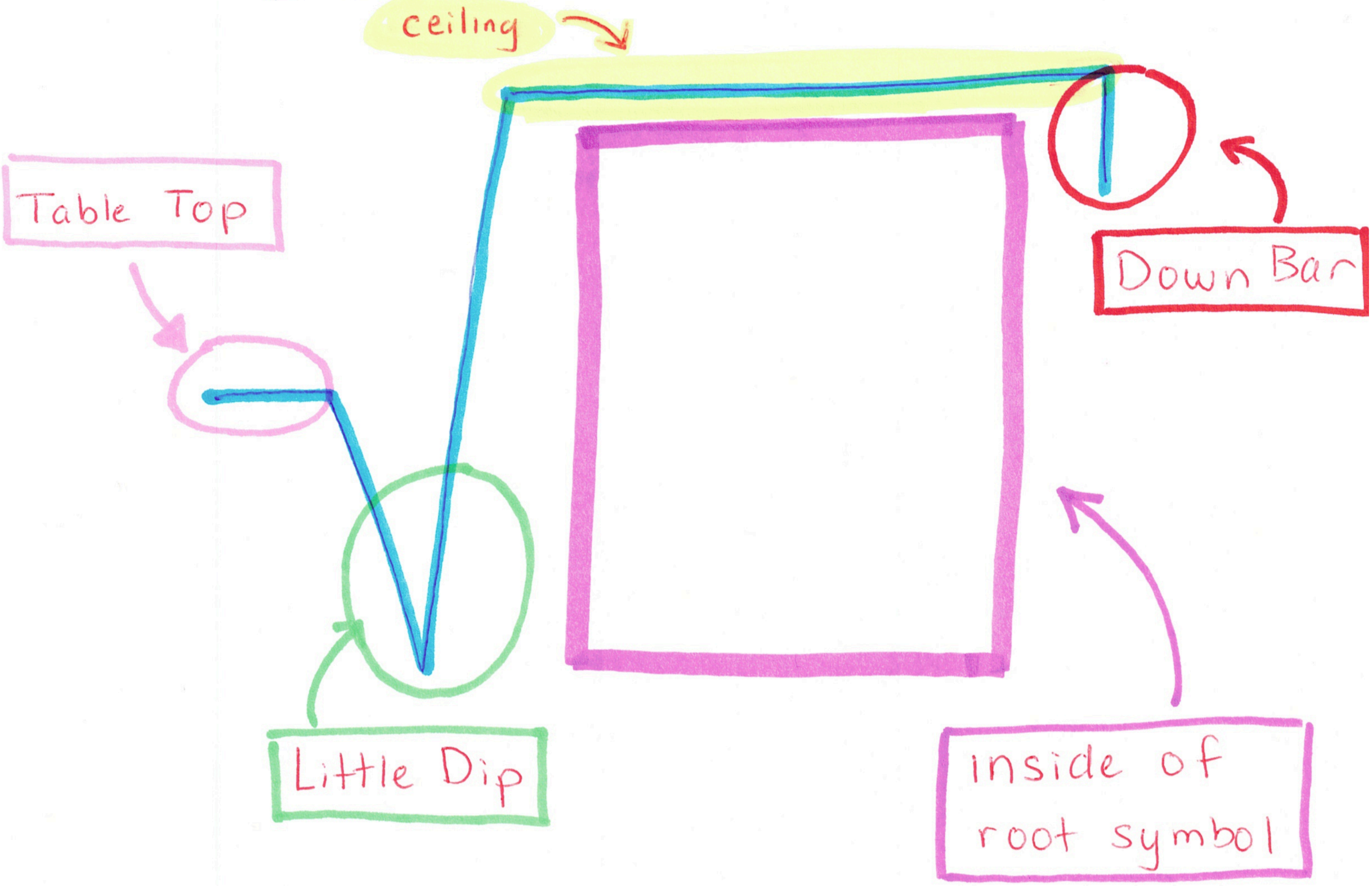
When drawing the root symbol, I recommend paying special attention to each of the following features:

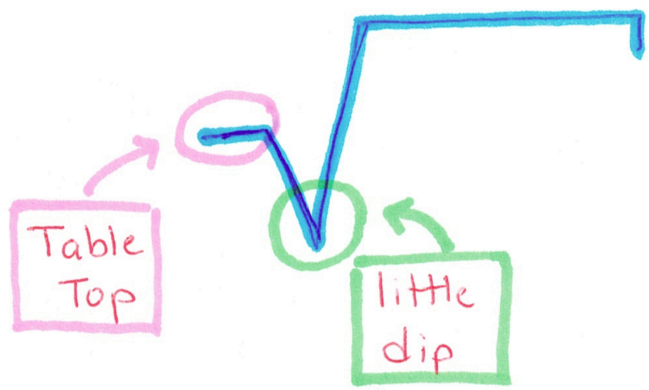
Anatomy of Root Symbol



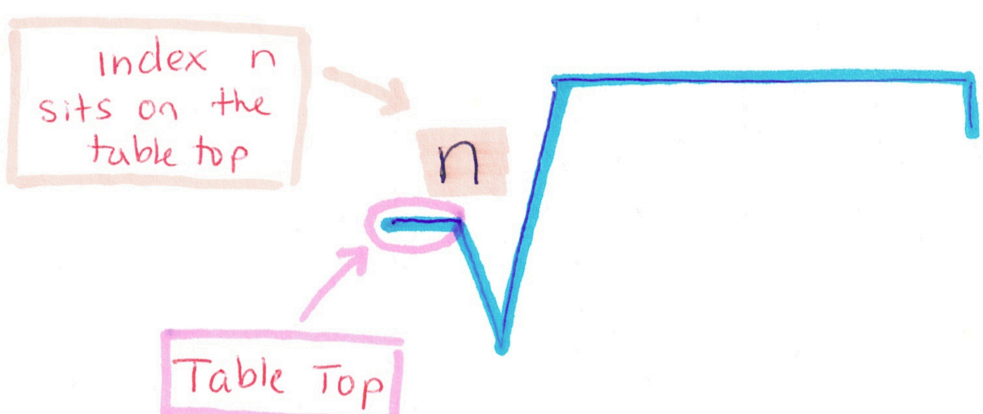
Anatomy of Root Symbol

Table top

- small horizontal line sticking out to the left of the little dip



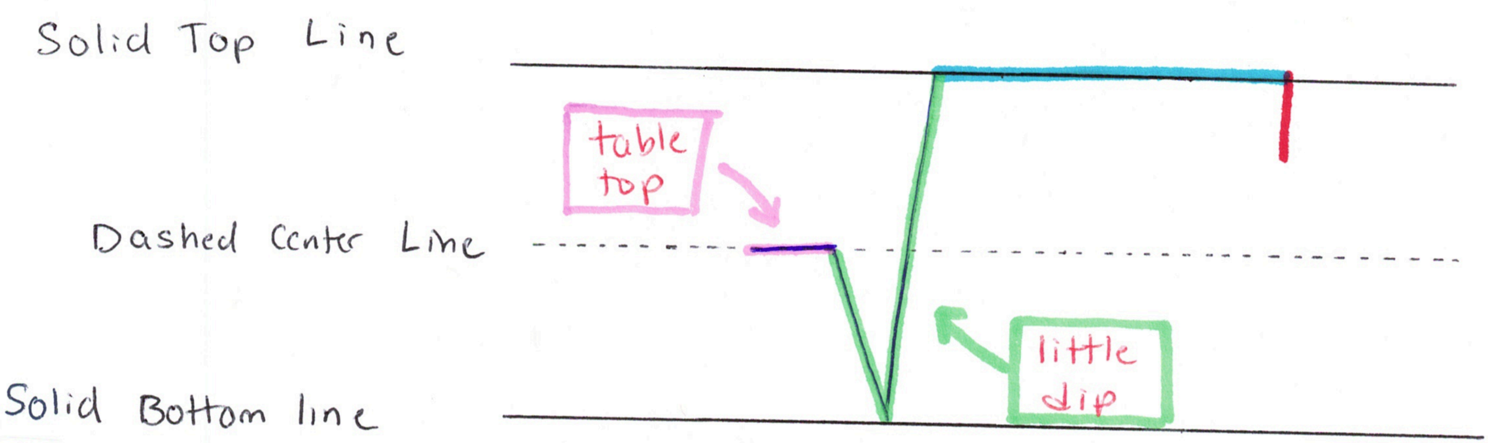
- the table top holds the $\boxed{\text{index}}$ value (the value of n for the nth root)



Anatomy of Root Symbol

Little dip

- the check-mark shaped portion on left-hand side of the root symbol

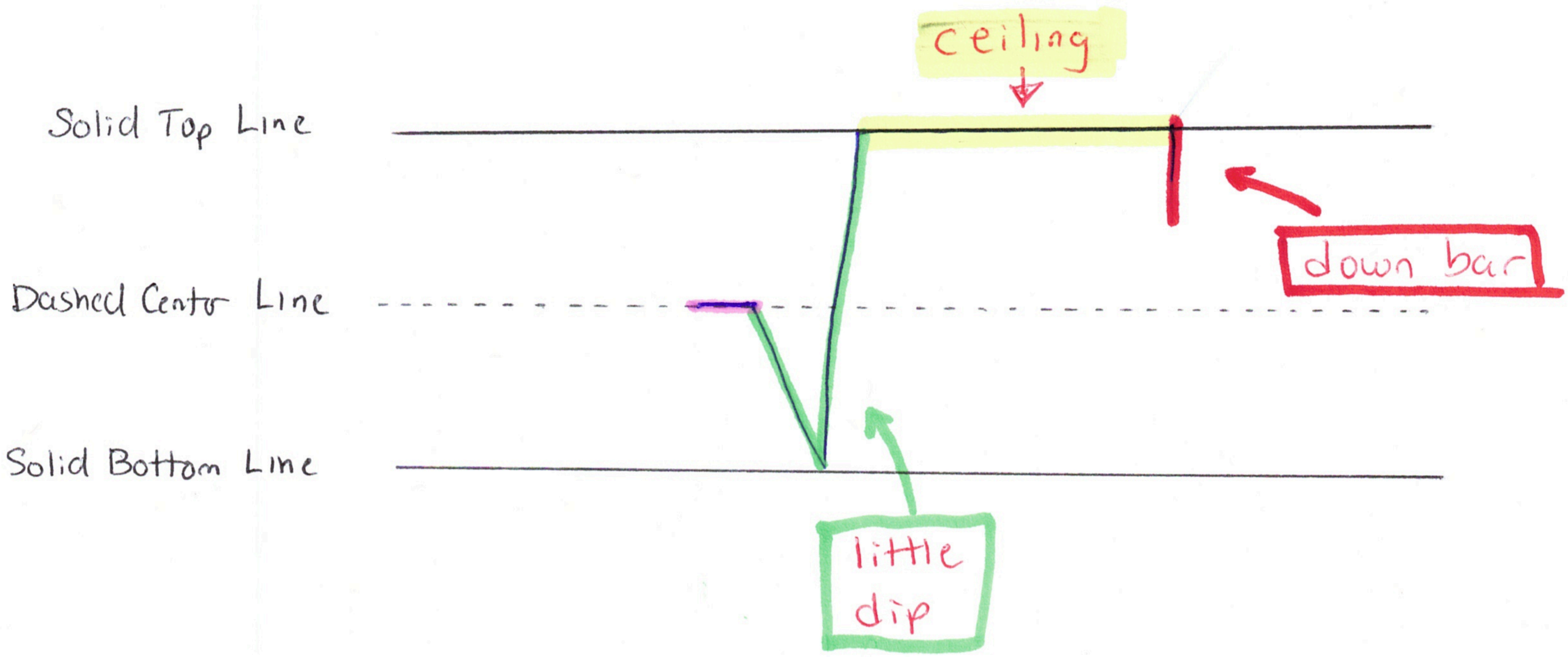


As we draw root symbol, we can use Hand writing grid lines to guide our use of this notation

- table top starts on dashed center line and then the little dip drops, in check-mark style, down to solid bottom line and then all the way up to solid top line.

Anatomy of Root Symbol

Ceiling
- the horizontal line across the top of root symbol



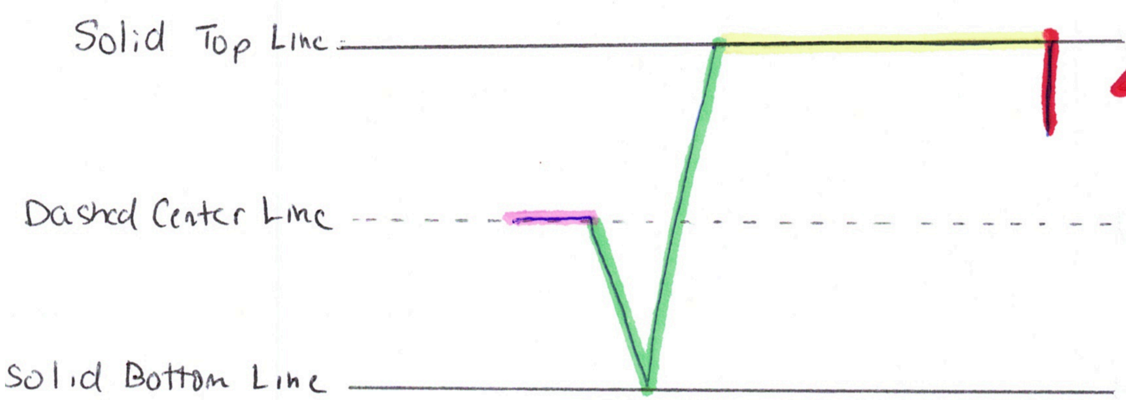
- the ceiling line segment runs between the top part of the little dip on the left side and the top part of the down bar on the right

- the ceiling helps us identify what is inside our root.

Anatomy of Root Symbol

Down bar

- the short vertical line on the right-hand side of the root symbol



The length of Down bar is about half of distance between solid top line and dashed center line

- the $\boxed{\text{down bar}}$ helps to identify the stuff that is supposed to be inside the root versus stuff that is supposed to be outside the root.

Anatomy of Root Symbol

Consider the following statement

$$\sqrt{\;}\; x + 2$$

This statement does not include a down bar and leaves some room for confusion:

Do we mean for the + 2 to be inside the root symbol or outside the root?

$$\sqrt{\;}\; x + 2 \quad \text{OR}$$

$$\sqrt{x + 2}$$

plus two inside root symbol

plus two outside root symbol

$$\sqrt{x} + 2$$

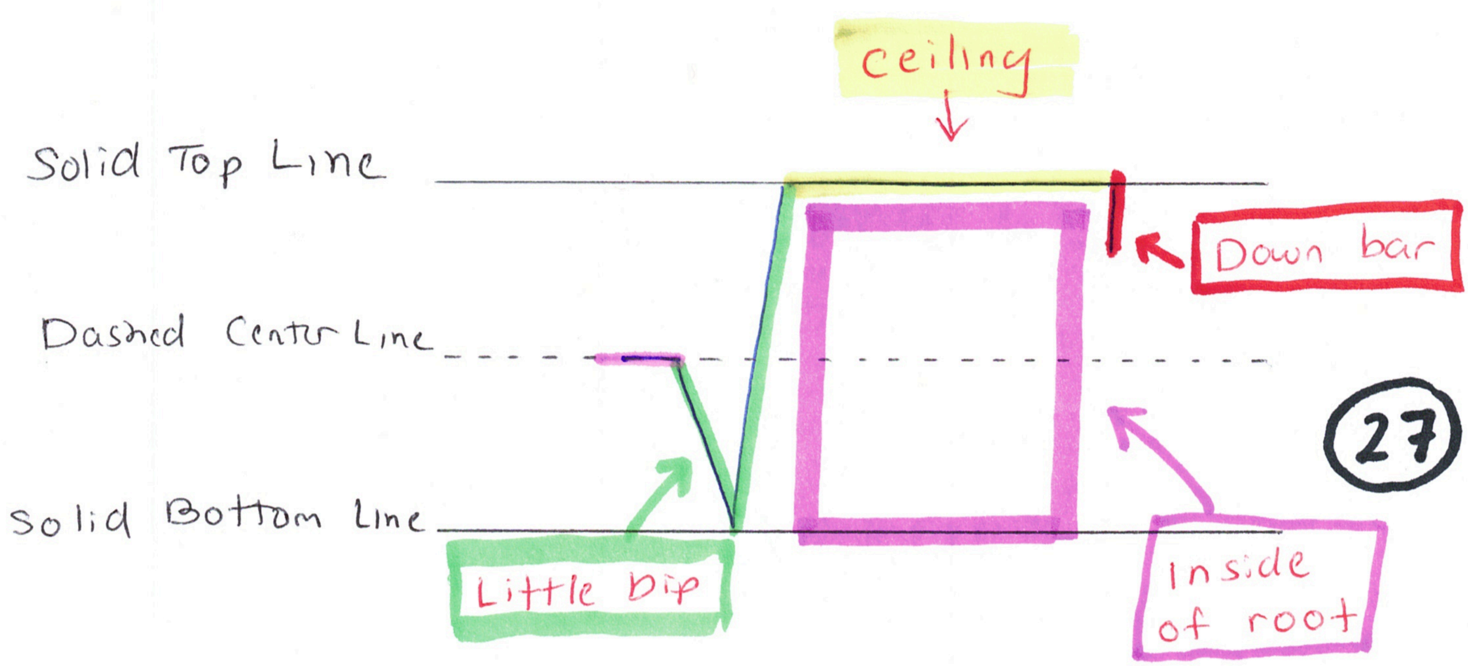
- The down bar forever fixes this problem of trying to figure out what we mean to put inside the root and what we want to be outside the root symbol.

- We use the down bar to indicate the end of the inside of the root.

Anatomy of Root Symbol

inside of Root symbol

- also known as radicand
- the space between the little dip on the left, the ceiling on top, the down bar on the right and the solid bottom line below



When working with roots, we have two equivalent ways to think about our nth root problem

Root Notation (or Radical Notation)

index

root symbol

$$\sqrt[n]{\boxed{x}} = \boxed{y}$$

radicand (input to root function or expression inside root symbol)

output of root function

is equivalent to trying to find value of y such that

Power Notation OR inverse notation)

$$\boxed{x} = \boxed{y}^{n}$$

In math symbols, we write

$$\sqrt[n]{\boxed{x}} = \boxed{y} \iff \boxed{x} = \boxed{y}^{n}$$

"the nth root of x equals y if and only if x equals y to the nth power"

This problem is designed to find a number y such that when we multiply y by itself n times we get back to x.

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9. How might we read the following statements? What does the notation mean? If possible, try to name the type of function using nerdy language. The first one is done for you 😁.

Statement	How to read?	What does it mean?
$f(x) = \frac{1}{x}$	"f of x equals one divided by x" "f of x equals one over x" "f of x is a reciprocal function with one in numerator and x in the denominator"	To get output y take the number one and divide that number by the input x $y = \frac{1}{x}$ My inner nerd 🤓 would call this a rational function.
$f(x) = \frac{1}{x^2}$	"f of x equals one divided by x squared" "f of x equals one over x squared" "f of x is a rational function with one in numerator and x squared in denominator"	To find the output y, we take the number one and divide by the square of input x.
$f(x) = \frac{1}{x^3}$	"f of x equals one divided by x to the third power" "f of x is equal to one over x cubed" "f of x is a rational function with one in numerator and x cubed in denominator"	To find output y, we take the number one and divide by input x raised to third power

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10. RATIONAL FUNCTIONS AKA RECIPROCAL FUNCTIONS

Let's explore rational function notation (also known as reciprocal notation). How might we read the following statement:

$$f(x) = \frac{1}{x^n} \quad \text{for} \quad n \in \mathbb{N}$$

"f of x equals one divided by x to the power of n for n an element of the natural numbers"

What does this notation mean? What are the special features of this notation?

To find the output value, we take the number one and divide by input value x to the nth power

$$y = \underset{\text{dividend}}{1} \div \underset{\text{divisor}}{x^n} = \frac{1}{x^n}$$

Numerator

denominator

Name:_________________________ Class #:______

11. What does the word "ratio" mean? Can you give an example of a ratio of two numbers? Look back at your work on problem 10. How does this relate to the idea of a *ratio*nal function?

- A ratio is a quotient of two mathematical expressions
- A ratio results when we divide one math statement by another math statement
- The first five letters of the phrase

  ratio nal function

  spell the word "ratio".
- Rational functions are quotients of polynomial functions.

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12. How might we read the following statements? What does the notation mean? If possible, try to name the type of function using nerdy language. The first one is done for you 😁.

Statement	How to read?	What does it mean?
$f(x) = 2^x$	"f of x equals two to the x" "f of x is an exponential function with base two and exponent x" "f of x equals two multiplied by itself x times"	Find an output y created by multiplying based 2 by itself x times $y = 2^x$ My inner nerd 🤓 would call this an exponential function.
$f(x) = 3^x$	"f of x equals three to the x" "f of x is an exponential function with base three and exponent x" "f of x equals three multiplid by itself x times"	Find output y that results from taking base three to the exponent x $y = 3^x$
$f(x) = 4^x$	"f of x equals four to the x" "f of x is an exponential function with base four and variable input x in the exponent."	Find the value that pops out when we take base four to the exponent x $y = 4^x$

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13. EXPONENTIAL FUNCTIONS	NOT POWER FUNCTIONS INVERSE OF LOGS

Let's explore root notation (also known as radical notation). How might we read the following statement:

$$f(x) = a^x \quad \text{for} \quad a \in \mathbb{R} \ \text{ with } a > 0$$

we Read these symbols verbally as follows:

" f of x is equal to a to the exponent x for a element of the real numbers with a greater than zero"

What does this notation mean? What are the special features of this notation?

This is exponent notation.

variable input exponent value

$$\boxed{a}^{\boxed{x}} = \boxed{y}$$

constant base a

(the value of a is constant and positive)

output of an exponential function

Let's explore the similarities and differences between exponential functions and power functions:

Exponent Function

variable input goes into exponent (can be any real number)

output of an exponential function

$$a^x = y$$

constant base a

(the value of base a is constant and positive)

Power Function

constant power n

(the value of power n must be a positive whole number)

$$x^n = y$$

variable input goes into the base

output to a power function

Both exponential and power functions share some common features

Input in exponent

$$a^{\boxed{x}} = y$$

constant base

output

Exponential Functions

constant power

$$\boxed{x}^{n} = y$$

input in base

output

Power Functions

In each case, we have the following:

- variable input
- value of the base
- value in the exponent
- value of the output

variable input: • when evaluating a function

$$\boxed{y} = f(\boxed{x})$$

output input

We can choose many values for input x. In other words, the input values "vary" or x is an input variable.

• for each single input value we choose, there is a single output value of our function

• we group the input and output together to form an ordered pair:

input output

$$(\boxed{x}, \boxed{y})$$

Exponential Functions

input in exponent

$$\boxed{a^{\boxed{x}} = y}$$

constant base

output

- To evaluate an exponential function, we take the constant value of base a to the exponent x.
- The variable input x for exponential functions is inside the exponent: a to the exponent x.

Power Functions

constant power

$$\boxed{x}^{n} = y$$

input in base

output

- To evaluate a power function, we take the variable input x and raise x to the constant power of n.
- The variable input x for power functions is inside the base: x to the power of n.

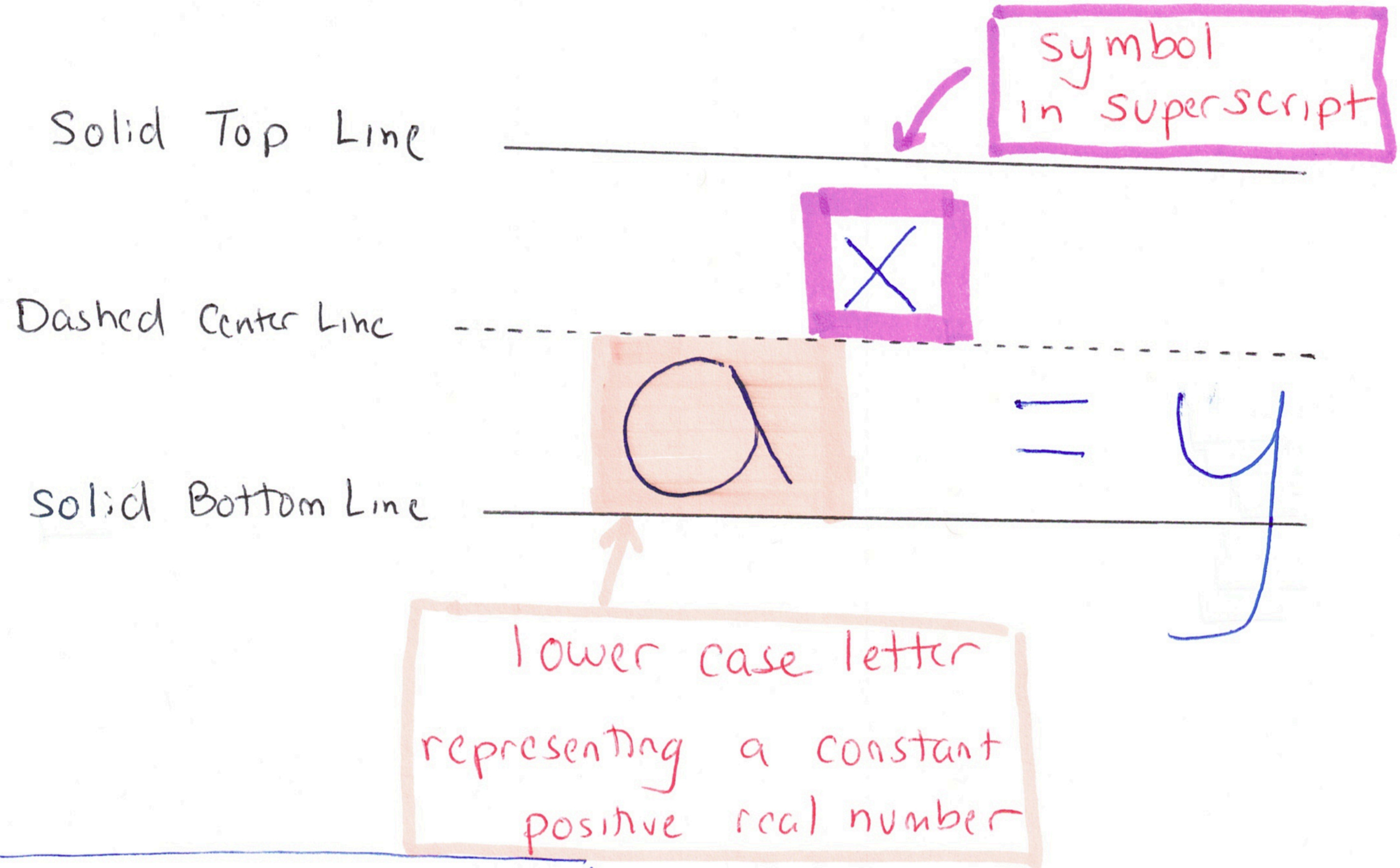
Exponential Functions	Power Functions
input in exponent	constant power
$a^{\boxed{x}} = y$	$\boxed{x}^{n} = y$
constant base; output	input in base; output

These differences lead to the following notation:

- ☐ When we have a constant base and the variable input x is in the exponent, we say x is the "exponent" on base a.

- ☐ When we have a variable input x in the base and we take that input x to a constant power n, we say that we take x to the "power" of n.

As we write in exponential notation, we can use handwriting grid lines to guide our work



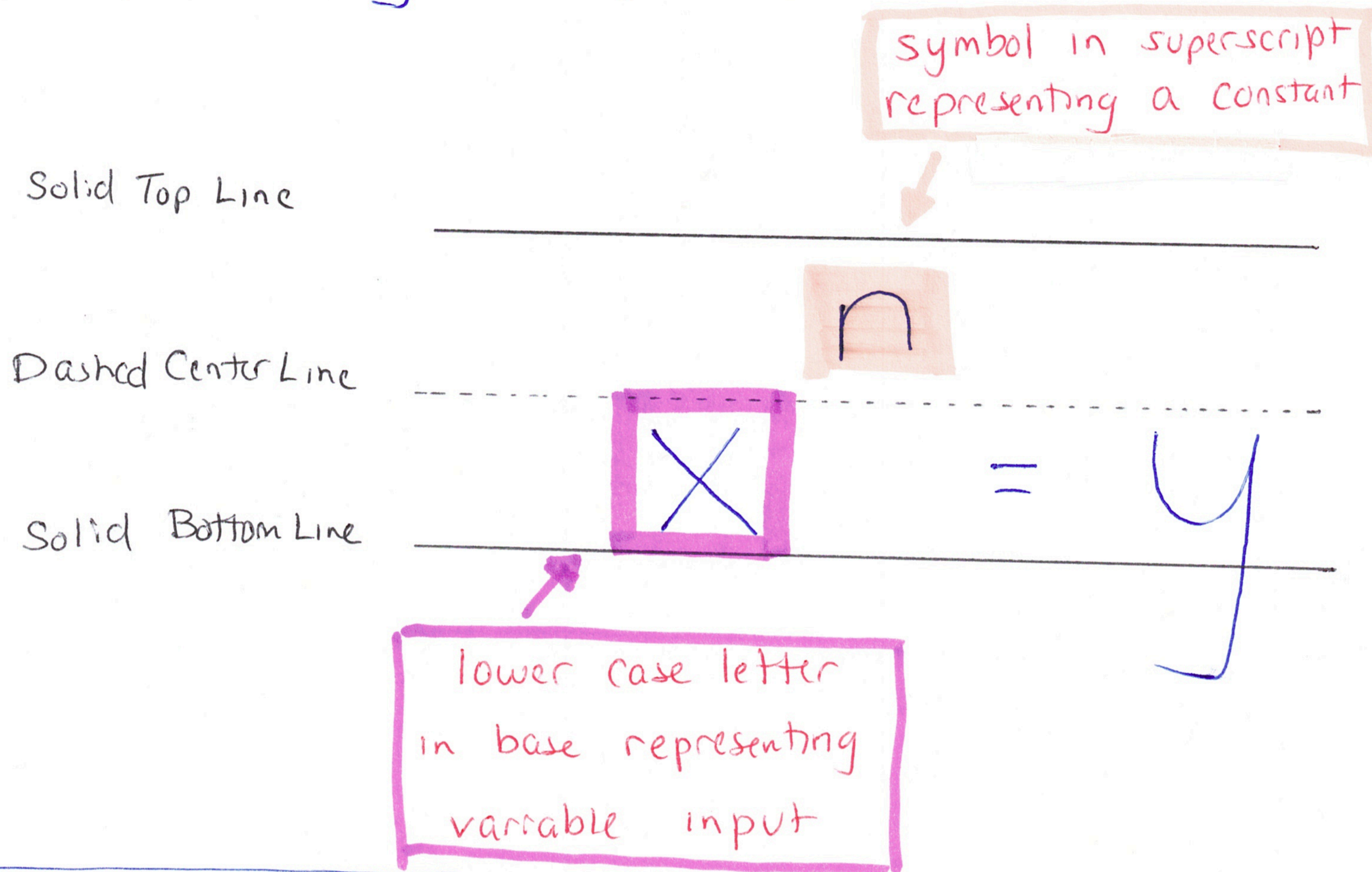
For exponential Functions

☐ The base is a constant, positive real number

☐ The variable input is written as a superscript (to the "North-east of base) on the base and called an exponent

☐ When the value written in superscript is understood to be variable input, we call that an "exponent"

We can also use handwriting grid lines to guide our understanding of power functions



For Power Functions

- ☐ The base is a variable input value X
- ☐ This variable input is in the base (NOT superscript)
- ☐ The value in superscript is understood to be constant and thus is called a "power"

We can summarize these observations as follows:

- ☐ When base is constant and superscript contains a variable, we call the superscript an "exponent" (exponents = variable superscript)

- ☐ When base is variable and superscript contains a constant, we call the superscript a "power" (power = constant superscripts)

- ☐ When the base is constant and the exponent is variable, we call that an "exponential" function

- ☐ When the base is variable and the value in superscript is constant, we call that a "power" function.

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14. How might we read the following statements? What does the notation mean? If possible, try to name the type of function using nerdy language. The first one is done for you 😁.

Statement	How to read?	What does it mean?
$f(x) = \log_2(x)$	"f of x equals log base 2 of x" "f of x is a logarithmic function with base two and input x" "f of x equals log to the base two of x"	Find a output y such that when we take base two to the value of that output we get x: $y = \log_2(x) \Leftrightarrow 2^y = x$ My inner nerd 🤓 would call this a log base 2 function.
$f(x) = \log_3(x)$	"f of x equals log base 3 of x" "f of x is a logarithm with base three and input x" "f of x equals log to the base three of x"	Find an output y such that when when we take base three to this output we get back to x $y = \log_3(x) \Leftrightarrow 3^y = x$ This is a log base three function.
$f(x) = \log_4(x)$	"f of x equals log base four of x" "f of x is a logarithmic function with base four and input x" "f of x equals log to the base four of x"	Find an output y such that when we take base four to the exponent value of output y, we get back to input value x $y = \log_4(x) \Leftrightarrow 4^y = x$

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15. LOGARITHMIC FUNCTIONS	INVERSE OF EXPONENTIALS

Let's explore root notation (also known as radical notation). How might we read the following statement:

$$f(x) = \log_a(x) \quad \text{for} \quad a \in \mathbb{R} \text{ with } a > 0$$

We read this verbally as: "f of x equals log base a of x for a an element of the real numbers with a strictly greater than zero."

What does this notation mean? What are the special features of this notation?

This is logarithm notation

variable input in log function x

$$\log_{a}(\boxed{x}) = \boxed{y}$$

constant base a (the value of a is constant and positive)

output of a logarithmic function

When working with logarithms, we have two equivalent ways to think about our log problems:

Logarithm notation

$$\log_a\left(\boxed{x}\right) = \boxed{y}$$

is equivalent to trying to find the value of y such that

Exponential Notation

$$\boxed{x} = a^{y}$$

In math symbols, we write

$$\log_{a}(x) = y \iff x = a^{y}$$

"log base a of x equals y if and only if x equals a to the exponent y"

This problem is designed to find the value of exponent y necessary so that when we take a to the exponent y we get back to value of input x.

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Lesson 1 Quiz: Test Your Memory

In this class, we will focus on our energy on learning to describe different classes of functions known as elementary functions. The foundations you set in this class will support you for the rest of your career in mathematics. Using the work you did on questions 1 – 15, try to match the given algebraic statements with the nerdy names for these statements.

Linear function___F.___	A. $f(x) = 2 + \log_{10}(x-1)$
Quadratic function___E.___ (Hint: in Spanish, the word Cuadro means square)	B. $f(x) = \frac{8}{x^2-4}$
Absolute value function___H.___	C. $f(x) = 3 + 2^x$
Rational function___B.___ (aka reciprocal function)	D. $f(x) = 3 + \sqrt[2]{x-1}$
Power function___G.___	E. $f(x) = x^2 + 6x + 9$
Root function___D.___	F. $f(x) = 3x - 2$
Exponential function___C.___	G. $f(x) = 2x^5 - 4$
Logarithmic function___A.___	H. $f(x) = 2 \cdot \lvert 1-2x \rvert - 4$

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Please see the next pages for some ideas about in-class timed exams.