$\boxed{\text{Math 48A, Lesson 11 Solutions}}$

Recall from Lesson 10 : if $c > 0$

input
inside parethesis

$\boxed{\text{vertical upward shift}}$ $\quad g(x) = \underbrace{f(x)}_{\text{output}} + c$

this graph shifts up vertically by c units

addition happens outside parenthes (shifts the output)

- Addition/subtraction outside the parenthesis effects y-axis placement

input
inside parenthesis

$\boxed{\text{vertical downward shift}}$ $\quad g(x) = \underbrace{f(x)}_{\text{output}} - c$

the graph of f(x) is shifted vertically downward by c units

subtraction happens outside parenthesis

Horizontal Shift:

$\boxed{\text{Horizontal Right Shift}}$

$$g(x) = f(x - c)$$

subtraction happens inside parenthesis (effects the input)

inside the parenthesis, subtract moves graph horizontally towards the right

$\boxed{\text{Horizontal Left Shift}}$

inside the parenthesis, addition moves the graph horizontally toward the left

$$g(x) = f(x + c) = f(x \underbrace{-}\ -c)$$

addition happens inside the parenthesis

Note: $+c = -(-c)$

If the plus or minus are inside the parenthesis, then that determines whether the curve goes left or right

Name: Solutions Class #:

Math 48A, Lesson 11: Transformations of Functions (Part 2)

1. COMBINE HORIZONTAL AND VERTICAL SHIFTS

1A. Consider the following quadratic functions

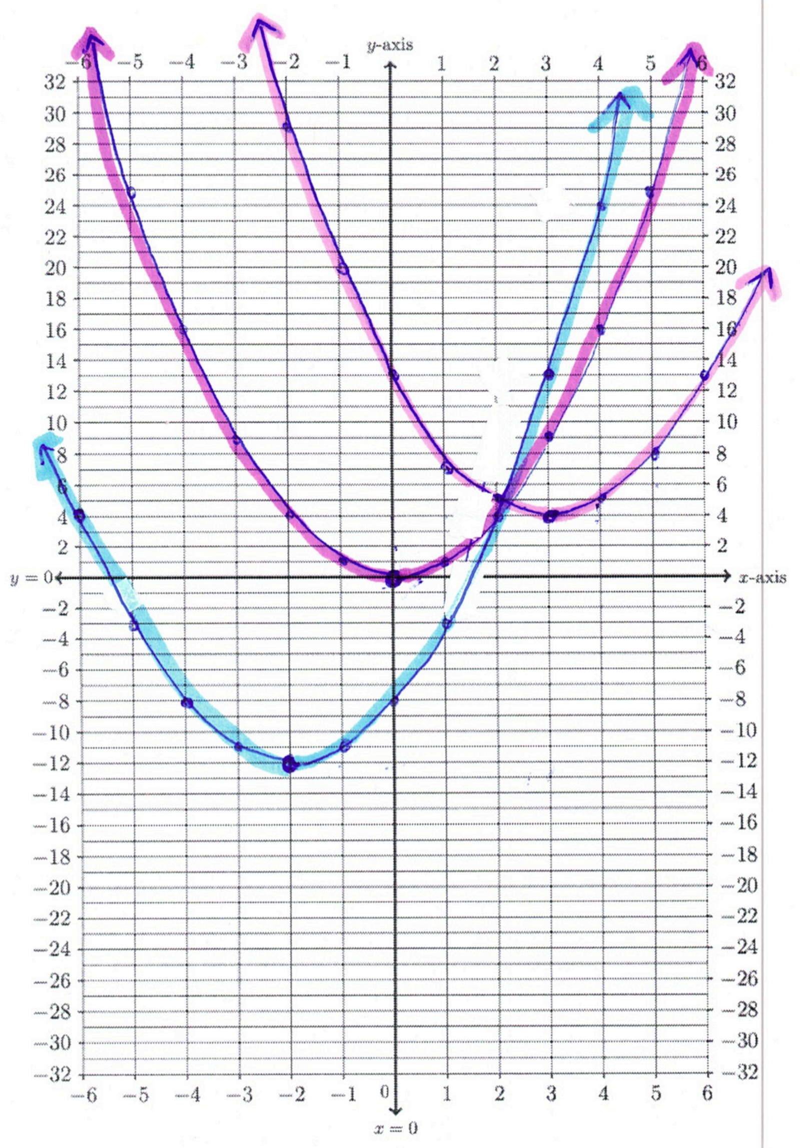
up shift left shift

$$f(x) = x^2, \qquad g(x) = f(x - 3) + 4, \qquad h(x) = f(x + 2) - 12$$

right shift down shift

Create a table of values and graph the resulting parabolas on these axes below.

Input	*Output*		
x	$f(x)$	$g(x)$	$h(x)$
−6	36	85	4
−5	25	68	−3
−4	16	53	−8
−3	9	40	−11
−2	4	29	−12 vertex
−1	1	20	−11
0	0 vertex	13	−8
1	1	8	−3
2	4	5	4
3	9	4 vertex	13
4	16	5	24
5	25	8	37
6	36	13	52



 calcu

Problem 1A) Let $f(x) = x^2$

we take the given input and square it

Then $f(\boxed{}) = (\boxed{})^2$

whatever is on the inside

addition on outside of parenthesis shifts graph 4 units upward

Consider $g(x) = f(\boxed{x-3}) + 4$

subtraction on inside of parenthesis shifts the graph of f(x) 3 units the the right

$$\Rightarrow g(x) = f(\underbrace{\boxed{x-3}}) + 4$$

we take this expression inside the parenthesis and square it

$$= (\boxed{x-3})^2 + 4$$

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1B. Make a conjecture (a mathematical guess) about what happens in the following scenario:

Assume we have a function $f(x)$ and constants $h, k > 0$.
Suppose we define functions

addition outside

$$g(x) = f(x - h) + k$$

subtraction inside

What is the relationship between $f(x)$ and $g(x)$? What happens if constant h is positive or negative? What happens if constant k is positive or negative?

Let's define $h, K > 0$ and let

$$g(x) = f(x - h) + K$$

addition on the outside of the parenthe shifts graph of $f(x)$ up by K units

subtraction on the inside shifts graph of $f(x)$ to the right by h units

- If $h < 0$, then we move $f(x)$ right by a negative number which is equivalent to moving $f(x)$ left $|h|$ units (where we take absolute value $|h|$ to transform $-$ to $+$).

- If $K < 0$, then we move $f(x)$ up by a negative number or, in other words, we move $f(x)$ down by $|K|$ units.

here we take the absolute value to turn a $-$ into a $+$

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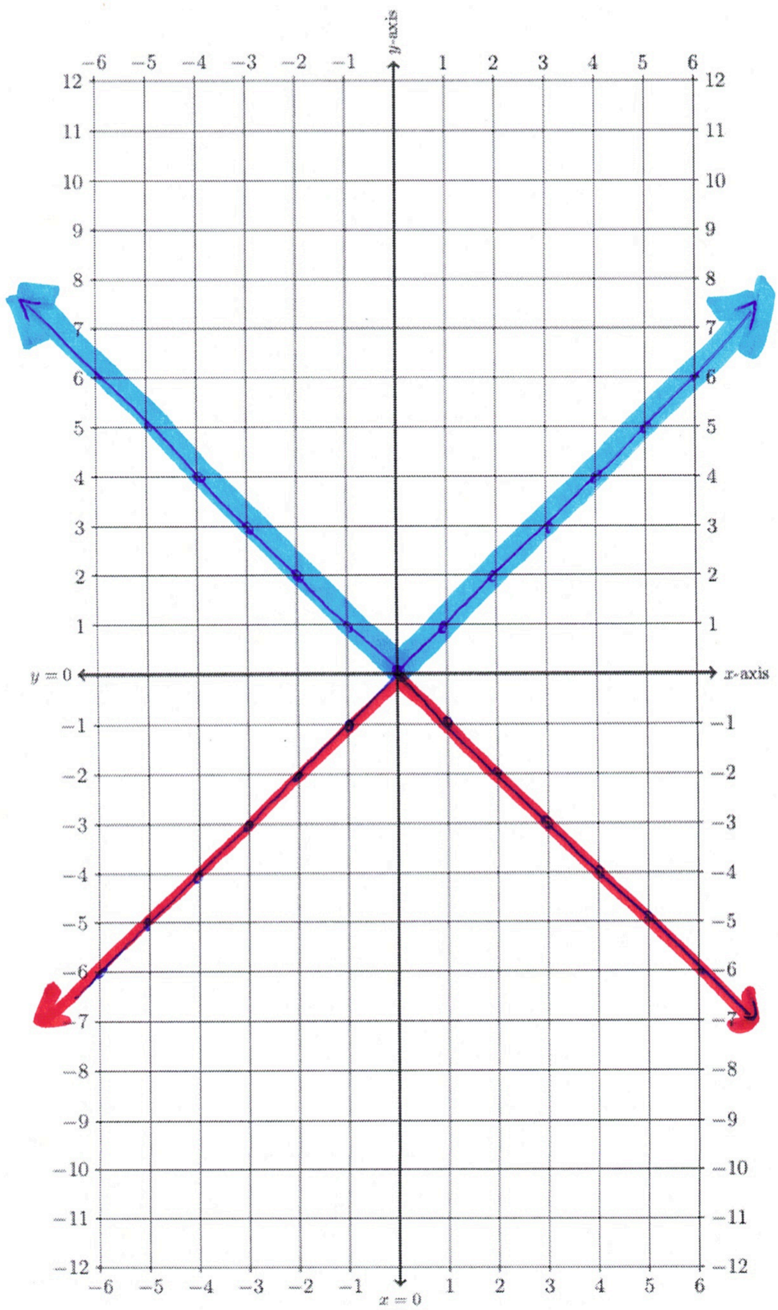
2. REFLECTING GRAPHS ABOUT THE X-AXIS

2A. Let's consider how to "reflect" the graph of a function about the x −axis. To do so, consider the following functions

$\boxed{f(x) = |x|}$ and $\boxed{g(x) = -f(x) = -|x|}$

Create a table of values and graph the resulting curves on these axes below.

Input	*Output*	
x	$f(x)$	$g(x)$
−6	6	-6
−5	5	-5
−4	4	-4
−3	3	-3
−2	2	-2
−1	1	-1
0	0 vertex	0 vertex
1	1	-1
2	2	-2
3	3	-3
4	4	-4
5	5	-5
6	6	-6



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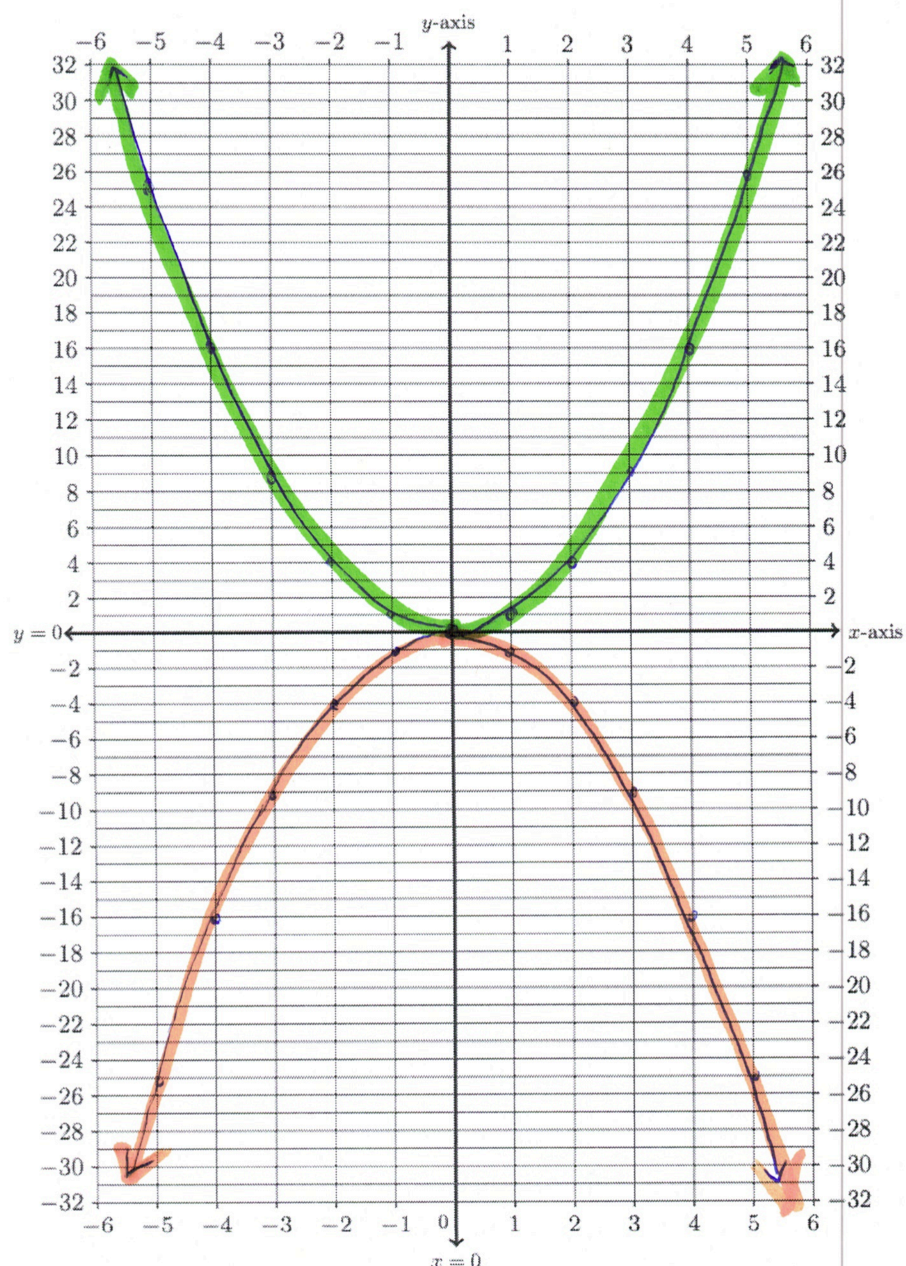
2B. Let's consider how to "reflect" the graph of a function about the x –axis. To do so, consider the following functions

$$\boxed{f(x) = x^2} \quad \text{and} \quad g(x) = -f(x).$$

Create a table of values and graph the resulting curves on these axes below.

Input	*Output*	
x	$f(x)$	$g(x)$
-6	36	-36
-5	25	-25
-4	16	-16
-3	9	-9
-2	4	-4
-1	1	-1
0	0	0
1	1	-1
2	4	-4
3	9	-9
4	16	-16
5	25	-25
6	36	-36



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2C. Suppose we have a function $f(x)$ and we define a new function $g(x) = -f(x)$. Based on your work in Problems 2A and 2B, make a conjecture about the relationship between the graphs of $f(x)$ and $g(x)$. Explain why you think your conjecture might be true.

Notice that if we define

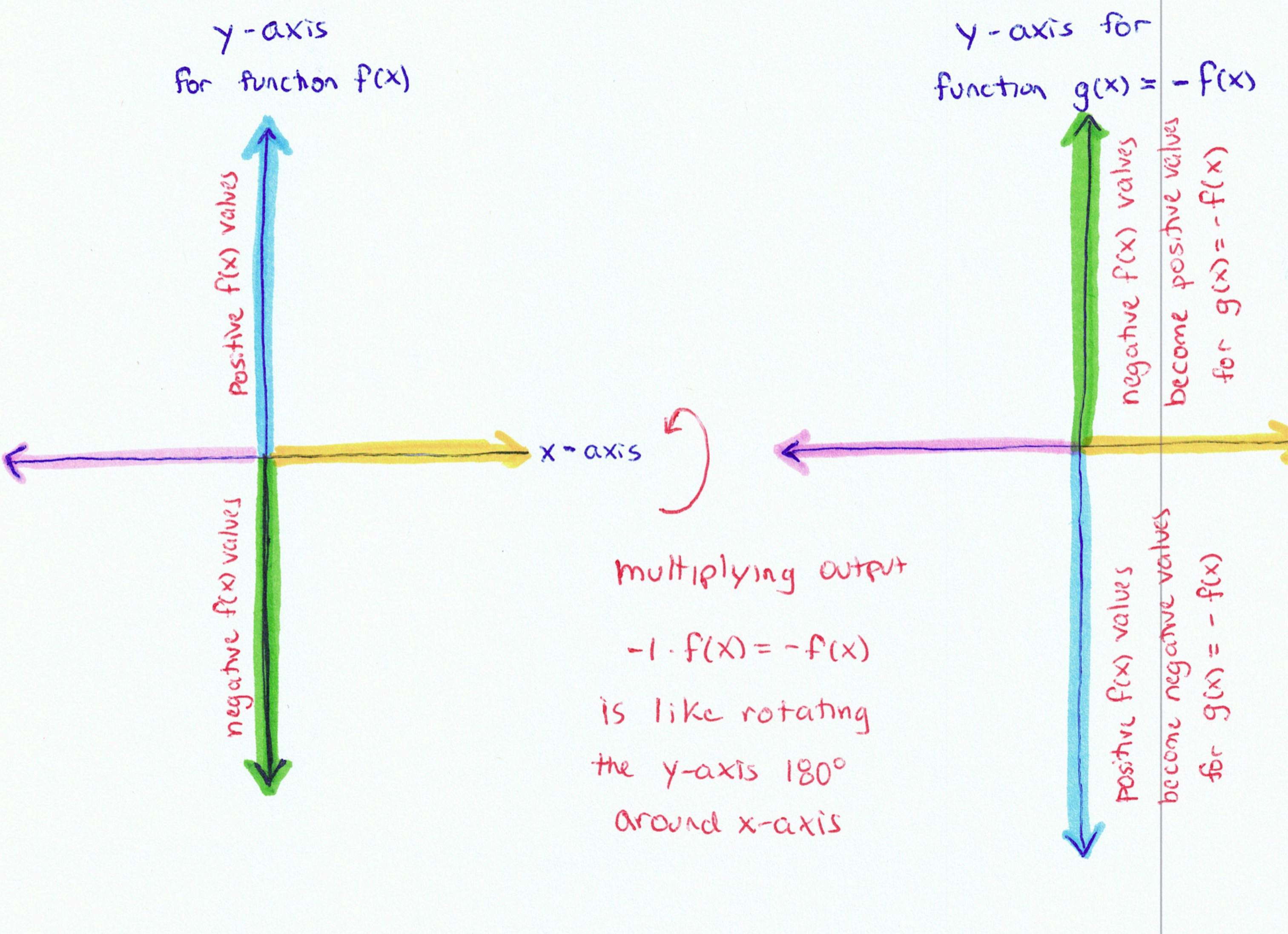
$$\underbrace{g(x)}_{\text{the output of } g} = - \underbrace{f(x)}_{\text{the output of } f}$$

then the negative sign on f(x) flips the sign so that the output of g(x) has the exact opposite sign as the corresponding output of f(x).

	sign of output of f(x)	sign of output of g(x) = -f(x)	
positive	+	−	negative
zero	0	0	zero
negative	−	+	positive

In other words, when we tranform the output of function f(x) by multiplying by -1, we effectively rotate the y-axis 180° so that the negatives become positive and vice versa.

Let's visualize this on a graph



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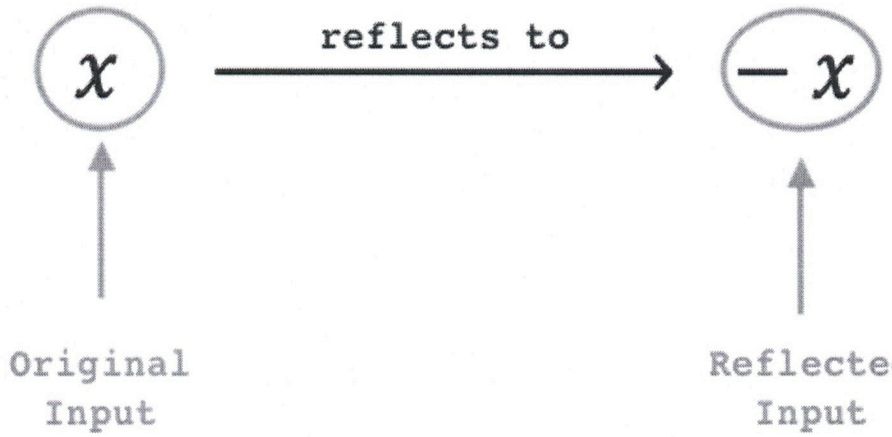
3. REFLECTION ABOUT VERTICAL AXIS

Homework

Consider the following shifts of the input variable



Draw the effect of this shift on the real number line (x −axis) below:

Shifted values:	10	9	8	7	6	5	4	3	2	1	0	-1	-2	-3	-4	-5	-6	-7	-8	-9	-10
x−values:	−10	−9	−8	−7	−6	−5	−4	−3	−2	−1	0	1	2	3	4	5	6	7	8	9	10

What do you notice about the reflexed input $-x$ versus the original input x?

when we transform the input x into input −x, we flip the sign of these inputs:

sign of original input value x		sign of transformed input −x	
positive	+	−	negative
zero	0	0	zero
negative	−	+	positive

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4. REFLECTING GRAPHS ABOUT THE Y-AXIS

4A. Let's consider how to "reflect" the graph of a function about the x –axis. To do so, consider the following functions

$$f(x) = \sqrt{x} \qquad \text{and} \qquad g(x) = f(-x).$$

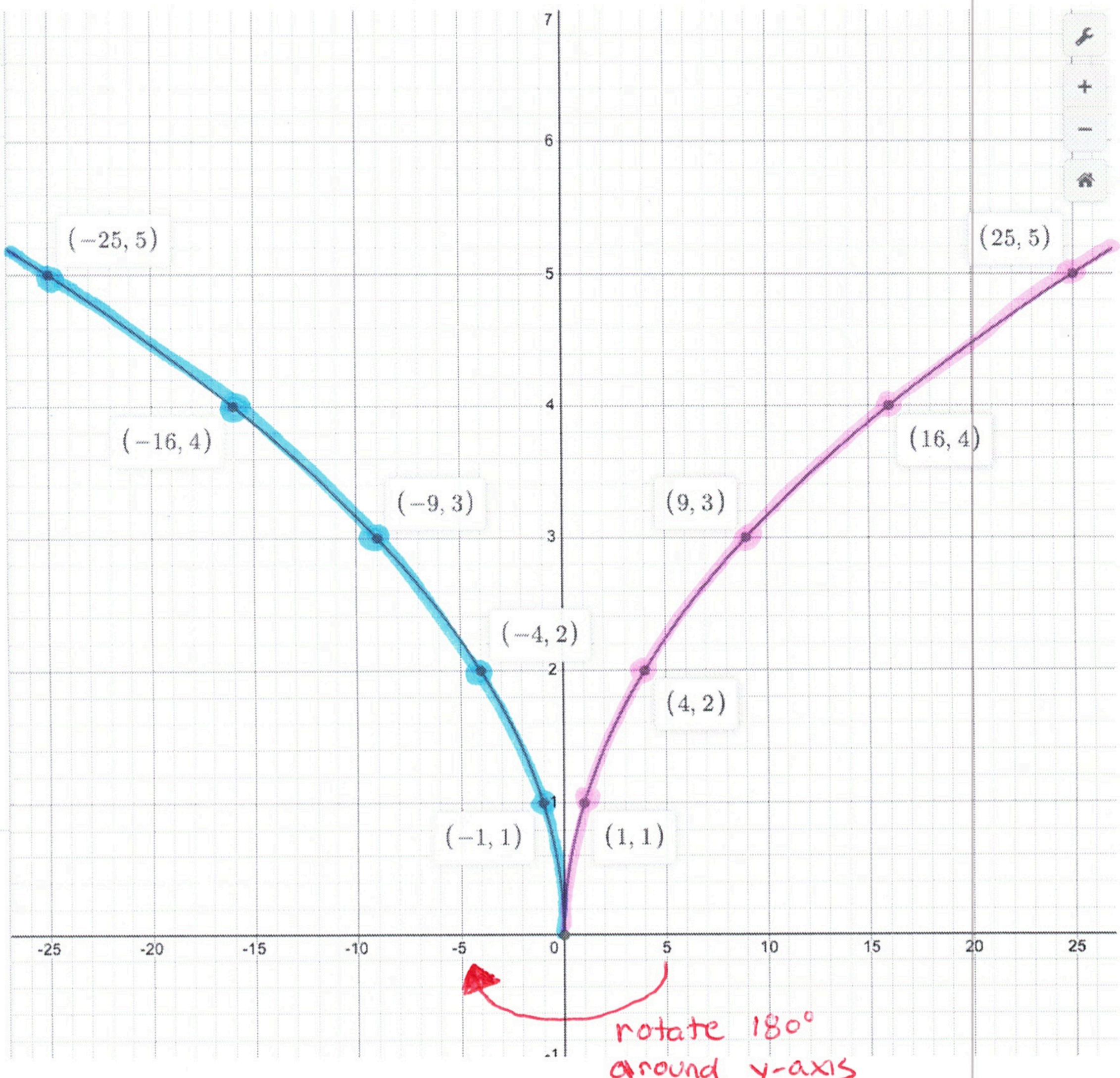
Create a table of values and graph the resulting curves using Desmos.com

see next page for desmos graph and table.

$f(x) = \sqrt{x}$

$g(x) = f(-x)$

x	$f(x)$	$g(x)$
−25	undefined	5
−16	undefined	4
−9	undefined	3
−4	undefined	2
−1	undefined	1
0	0	0
1	1	undefined
4	2	undefined
9	3	undefined
16	4	undefined
25	5	undefined



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For $g(x) = f(-x)$, we flip the sign for input to $f(x)$ which, geometrically, is identical to rotating the graph of $f(x)$ 180° around y-axis. Look at the graph above and rotate the red curve $\sqrt{x}$ around y-axis 180° to produce the blue curve for $\sqrt{-x}$

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4B. Let's consider how to "reflect" the graph of a function about the x −axis. To do so, consider the following functions

$$f(x) = x^3 \qquad \text{and} \qquad g(x) = f(-x).$$

Create a table of values and graph the resulting curves using Desmos.com

See next page for desmos graph and table

spin 180° this way

When we define $g(x) = f(-x)$ it's as if we rotate the x-axis 180° around the y-axis. Indeed, if you take the green curve and rotate 180° around y-axis (spin horizontally.), you'd get the orange curve.

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4C. Suppose we have a function $f(x)$ and we define a new function $g(x) = f(-x)$. Based on your work in Problems 4A and 4B, make a conjecture about the relationship between the graphs of $f(x)$ and $g(x)$. Explain why you think your conjecture might be true.

When we transform the input to function $f(x)$ by multiplying by -1, we can think of this as a rotation of the x-axis 180° around the y-axis so that negative inputs become positive and vice versa.

Let's visualize this on a graph: