Name: Solutions Class #:

Math 48B, Quiz 3, Lessons 6 - 10: Zeros of a Polynomial and Rational Functions

In your first draft solutions to this quiz, I encourage you to take extra space and make your work very easy to read. I might encourage you to write one solution per page. I want to focus your mind here on two goals. First, this is designed to help build understanding of the material. Second, as you write your solutions, think about creating a document that you can look back on and understand years into the future. In this way, your solutions can become a so-called second brain where you store math knowledge for future reference. For more about ideas on how to format your solutions, please take a look at Jeff's Conquering College Study Skills Activity 5.

1.	POLYNOMIAL DIVISION

✓ 1A. Find all zeros of the polynomial $f(x) = 2x^3 - 13x^2 + 3x + 18$. Show your steps and explain how you solve this problem.

✓ 1B. Use polynomial long division to solve the following problem:

$$q(x) = \frac{2x^3 - 13x^2 + 3x + 18}{(2x - 3)}$$

✓ 1C. Find the complete zero factorization form of the polynomial $f(x) = 2x^3 - 13x^2 + 3x + 18$.

2.	ZEROS OF A POLYNOMIAL

✓ 2A. Use polynomial long division to solve the following problem:

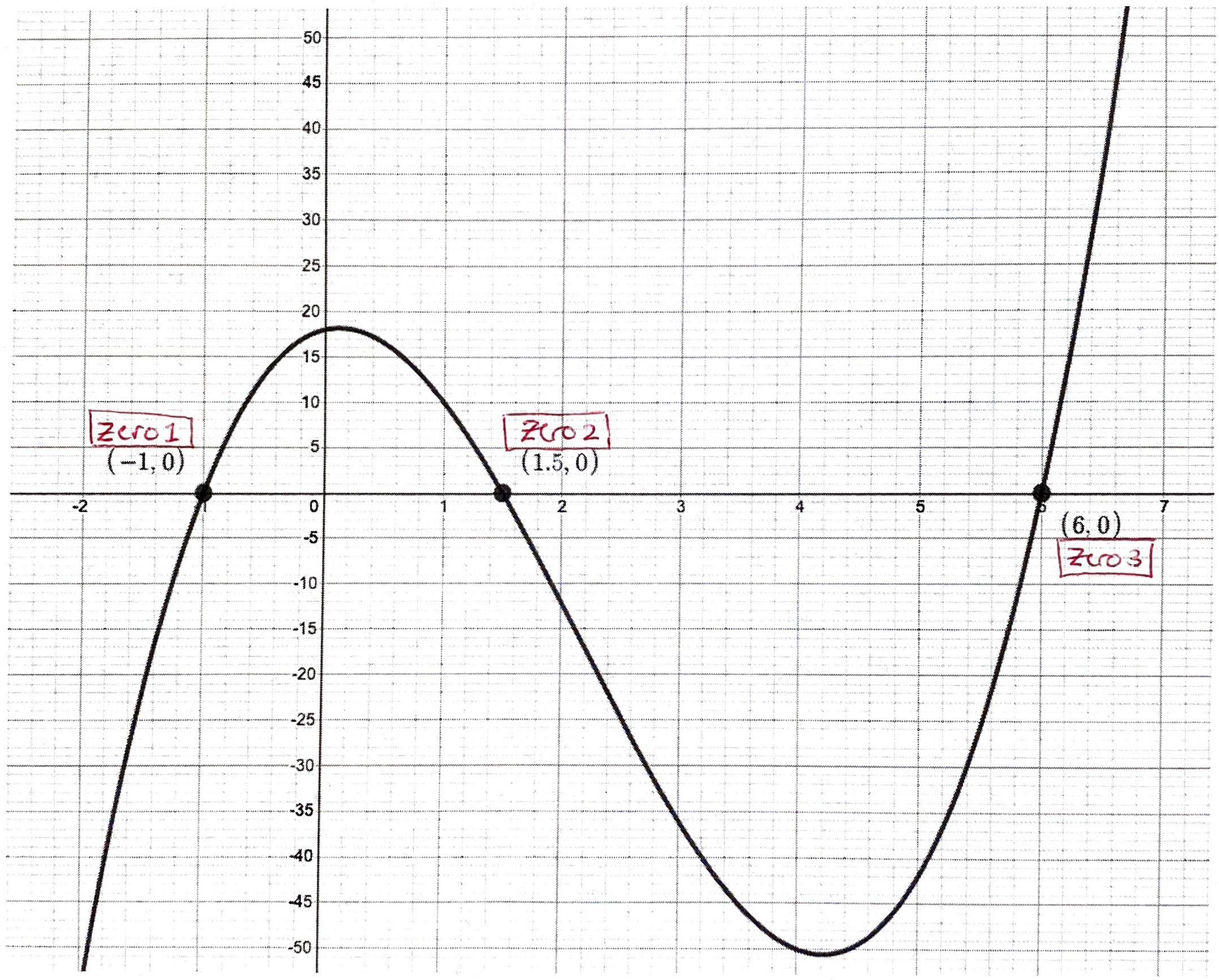
$$f(x) = \frac{x^3 - 6x^2 + 3x + 10}{(x - 2)}$$

✓ 2B. How are the function $f(x)$ from Problem 2A above related to the function $g(x) = x^2 - 4x - 5$? Please graph each function. Also include a discussion of the domain of each function.

Problem 1A) To find the zeros of the polynomial functio

$$f(x) = 2x^3 - 13x^2 + 3x + 18$$

let's graph this function on the axis below.



Notice that we find three zeros at points

Zero 1: (-1, 0)

Zero 2: (1.5, 0)

Zero 3: (6, 0)

Problem 1A, continued...

For each of these zeros, we can identify the x-values. We note

Zero 1: $(-1, 0) \Rightarrow x = -1$

Zero 2: $(1.5, 0) \Rightarrow x = 1.5 = \frac{3}{2}$

Zero 3: $(6, 0) \Rightarrow x = 6$

For each of these zeros, we can find a unique factor of our polynomial

Zero 1: $x = -1 \Rightarrow x + 1 = 0$

Zero 2: $x = \frac{3}{2} \Rightarrow 2x = 3$

$\Rightarrow 2x - 3 = 0$

Zero 3: $x = 6 \Rightarrow x - 6 = 0$

Problem 1 A, continued...

We can check that these are the zeros of our polynomial :

$$(x+1)(2x-3)(x-6)$$

$$= (2x^2 - 3x + 2x - 3)(x-6)$$

$$= (2x^2 - x - 3)(x-6)$$

$$= 2x^3 - x^2 - 3x - 12x^2 + 6x + 18$$

$$= 2x^3 - 13x + 3x + 18 \checkmark$$

Thus we have both forms of our polynomial

$$f(x) = 2x^3 - 13x + 3x + 18 \leftarrow$$ standard form

$$= \underbrace{(x+1)}_{\text{zero 1}} \underbrace{(2x-3)}_{\text{zero 2}} \underbrace{(x-6)}_{\text{zero 3}} \leftarrow$$ complete zero factorization form

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Problem 1B) Let's do polynomial long division:

$$
\begin{array}{r|l}
 & x^2 - 5x - 6 \\
\hline
2x - 3 & 2x^3 - 13x^2 + 3x + 18 \\
 & -2x^3 + 3x^2 \\
\hline
 & 0 - 10x^2 \\
 & \quad + 10x^2 - 15x \\
\hline
 & \quad 0 - 12x \\
 & \qquad + 12x - 18 \\
\hline
 & \qquad 0 + \boxed{0}
\end{array}
$$

remainder

Then we have:

$$\frac{2x^3 - 13x^2 + 3x + 18}{2x-3} = x^2 - 5x - 6 + \frac{\boxed{0}}{2x-3}$$

Problem 1B, continued ...

Using polynomial long division, we see

$$2x^3 - 13x^2 + 3x + 18 = (2x-3)(x^2-5x-6)$$

Problem 1C) we can confirm our previous statement about the complete zero factorization form by factoring $x^2 - 5x - 6$:

$$x^2 - 5x - 6 = (x+1)(x-6)$$

$$\Rightarrow \underbrace{2x^3 - 13x^2 + 3x + 18}_{\text{standard form}} = \underbrace{(x+1)(2x-3)(x-6)}_{\text{complete zero factorization form}}$$

Problem 2A Let's use polynomial division to find

$$\frac{x^3 - 6x^2 + 3x + 10}{x - 2}$$

To that end, consider:

$$
\begin{array}{r|l}
 & x^2 - 4x - 5 \\
\hline
x - 2 & x^3 - 6x^2 + 3x + 10 \\
 & -x^3 + 2x^2 \\
\cline{2-2}
 & 0 - 4x^2 \\
 & \quad + 4x^2 - 8x \\
\cline{2-2}
 & \quad 0 - 5x \\
 & \qquad + 5x - 10 \\
\cline{2-2}
 & \qquad 0 + \boxed{0}
\end{array}
$$

remainder

Notice that we've found

$$\frac{x^3 - 6x^2 + 3x + 10}{x - 2} = x^2 - 4x - 5 + \frac{\boxed{0}}{x - 2}$$

Problem 2A, continued...

Notice that we can further factor our quotient

$$x^2 - 4x - 5 = (x-5)\cdot(x+1)$$

This gives us a complete zero factorization form

$$x^3 - 6x^2 + 3x + 10 = (x+1)(x-2)(x-5)$$

We can use this form to write

$$f(x) = \frac{x^3 - 6x^2 + 3x + 10}{(x-2)}$$

$$= \frac{(x+1)(x-2)(x-5)}{(x-2)}$$

Recall:

$\frac{x-2}{x-2} = 1$ if $x \neq 2$

Denominator cannot be zero

$$= (x+1)(x-5) \quad \text{if} \quad x \neq 2$$

Problem 2B

We notice that functions $f(x)$ and $g(x)$ look very similar but have different domains:

For $f(x) = \dfrac{x^3 - 6x^2 + 3x + 10}{x-2}$, we note that $f(x) = x^2 - 4x - 5$ if $x \neq 2$ with domain $(f) = (-\infty, 2) \cup (2, \infty)$. In other words, $f(x)$ has a whole at $x = 2$.

On the other hand, for

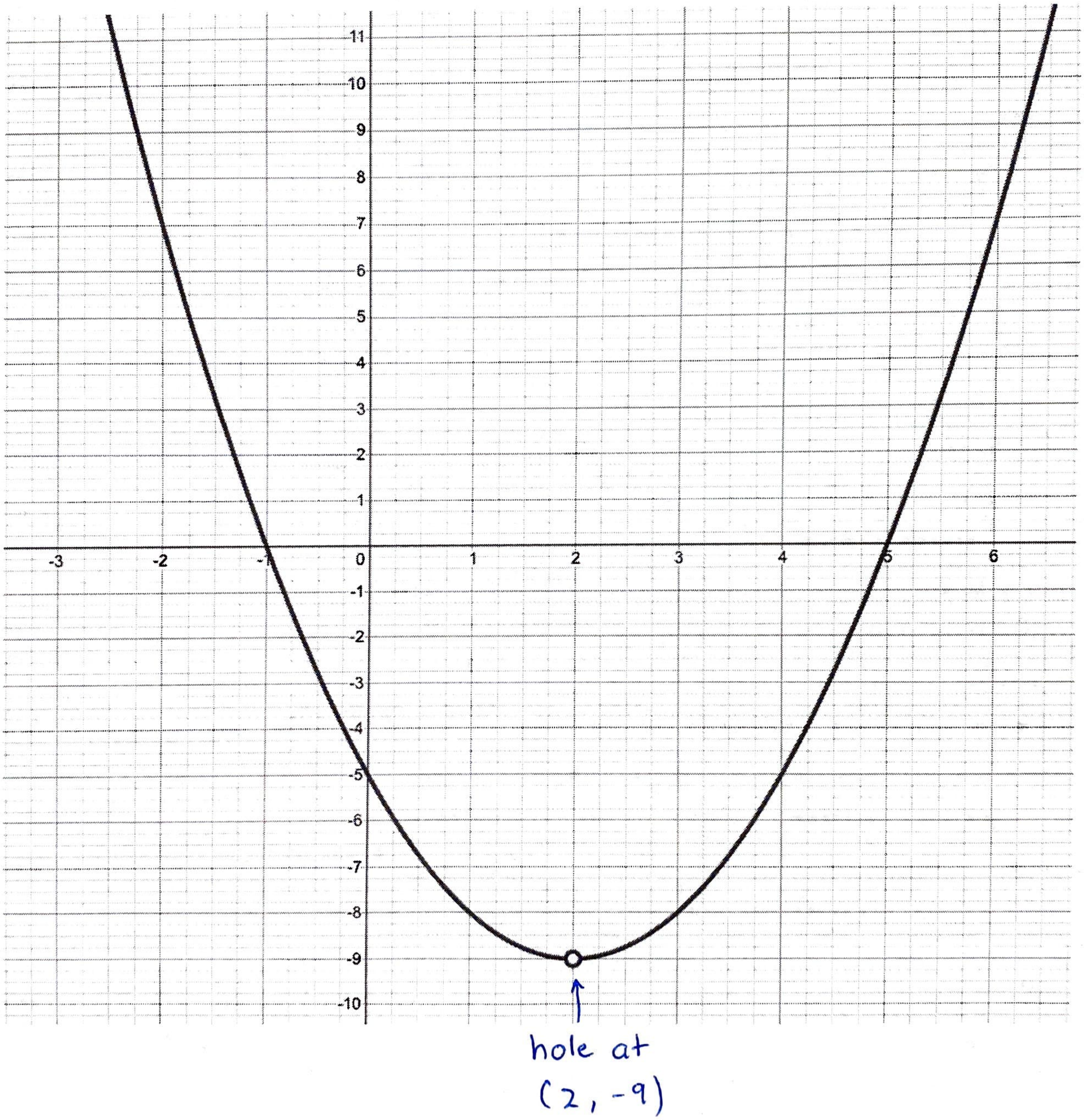
$$g(x) = x^2 - 4x - 5$$

We see $g(x)$ has domain $(-\infty, \infty)$.

Problem 2B, continued...

We can see the differences between these functions in the graph. Below we graph

$$f(x) = \frac{x^3 - 6x + 3x + 10}{x - 2} = x^2 - 4x - 5 \quad \text{for } x \neq 2$$

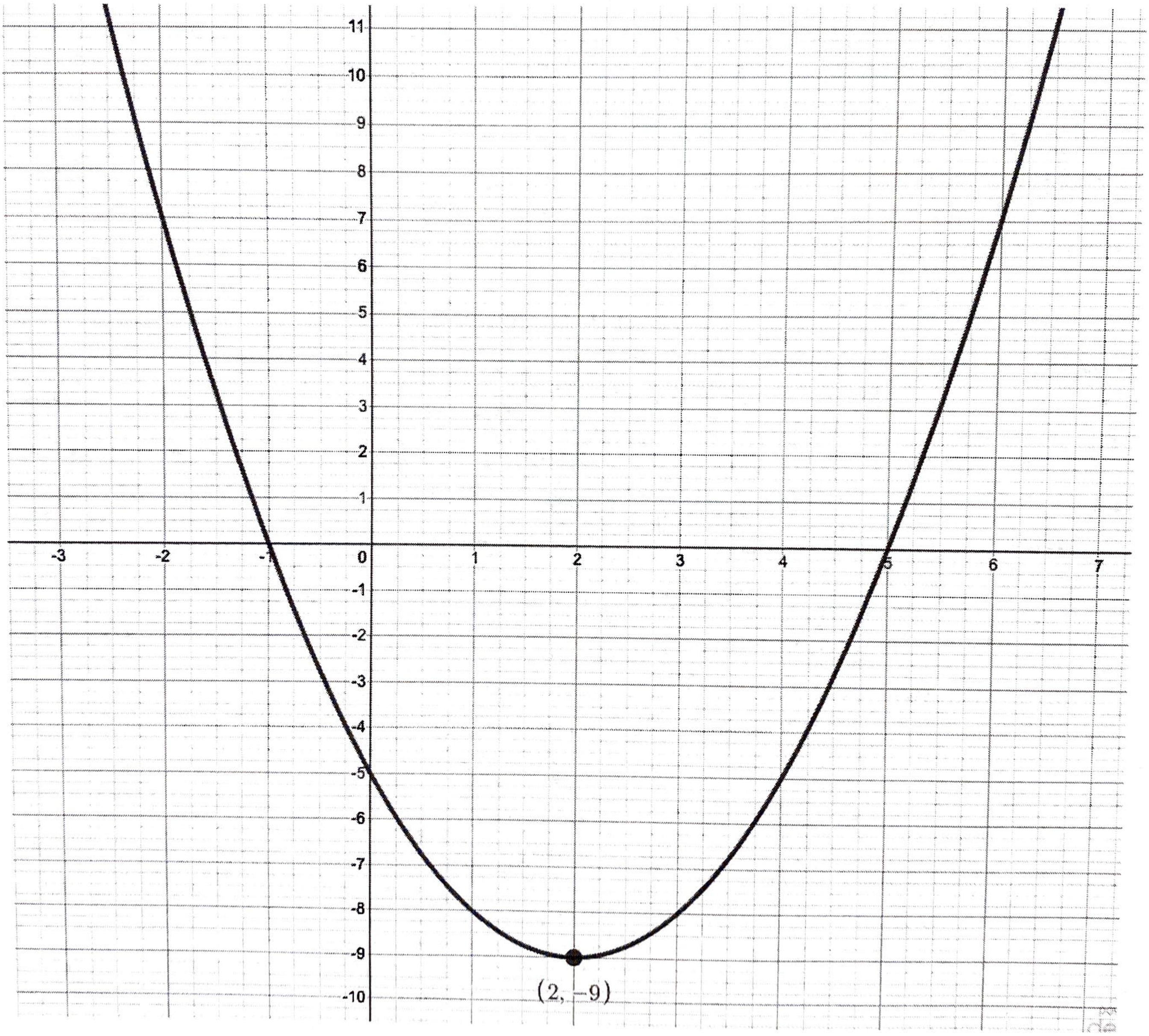


Notice, there is a hole in this graph at $x = 2$.

Problem 2B, continued ...

Now, we compare the graph of $f(x)$ with the graph of function

$$g(x) = x^2 - 4x - 5$$

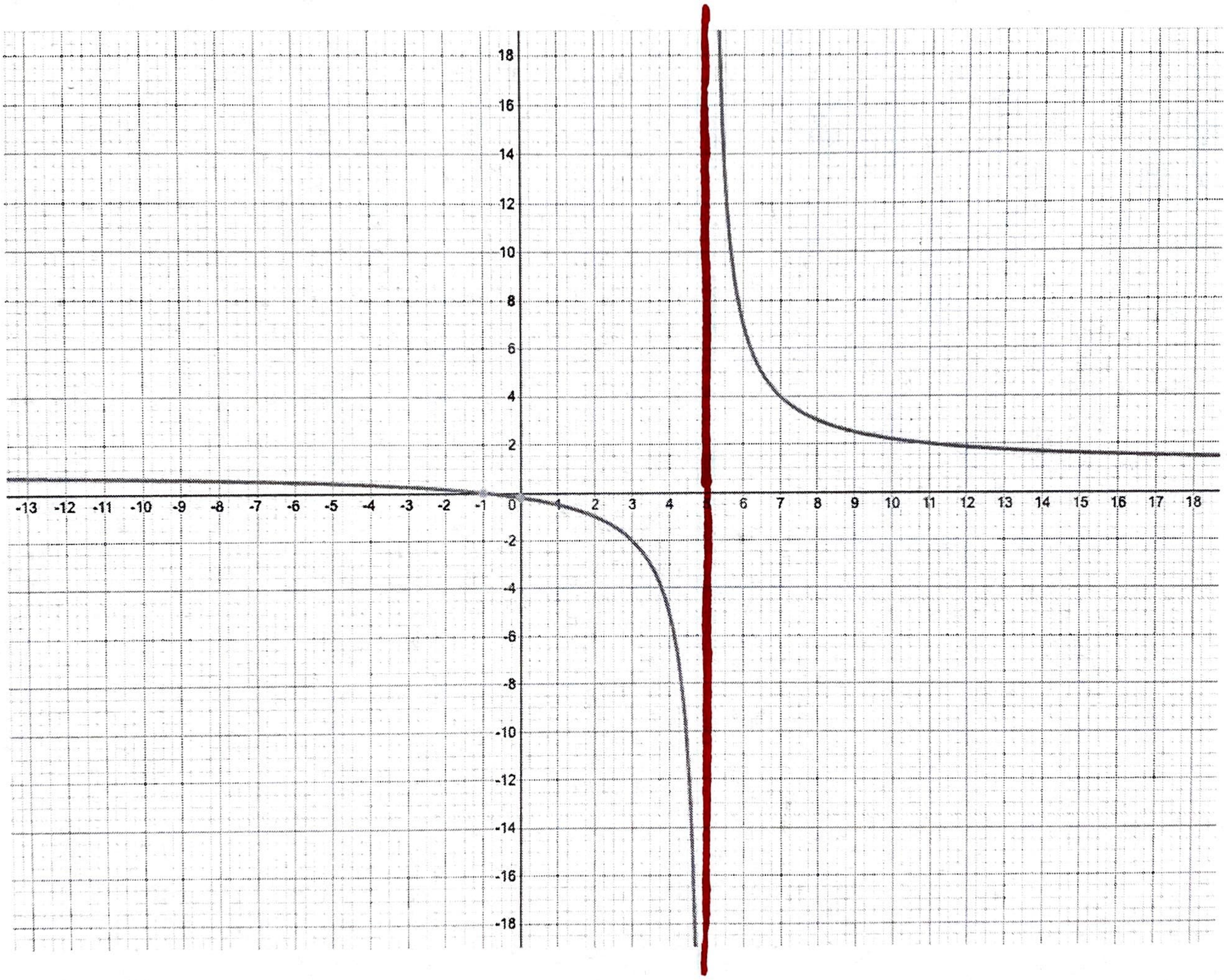


In this case, we see that $g(x)$ does not have a hole at $x = 2$ with Domain $(g) = \mathbb{R} = (-\infty, \infty)$.

Problem 3

Below we graph the function

$$g(x) = \frac{x+1}{x-5}$$



Problem 3A

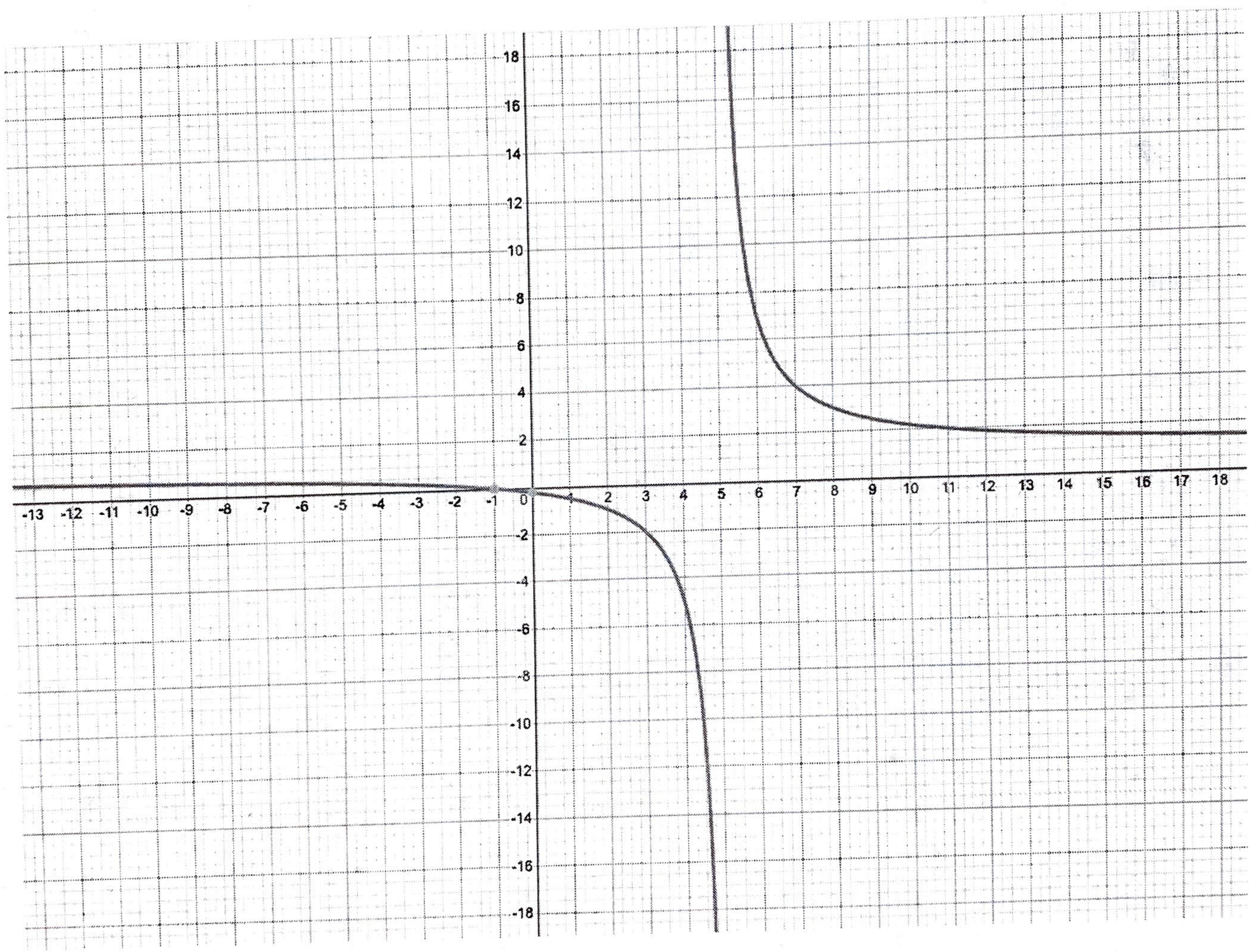
There are no holes in this function. We remember that a hole happens when we have an identical factor in both the numerator and the denominator in the form

$$\frac{x-c}{x-c} = 1 \quad \text{if} \quad x \neq c$$

← this factor has a hole at $x = c$

Denominator can't be zero

Problem 3, continued ...



$\boxed{\text{Problem 3F}}$ We want to write this function

$$g(x) = \frac{x+1}{x-5}$$

as a transformation of $f(x) = \frac{1}{x}$.

To this end, let's consider the polynomial long division

$$\begin{array}{r|l} & \quad 1 \\ \hline x-5 & \; x+1 \\ & -x+5 \\ \hline & \; 0 \; \boxed{+6} \end{array}$$

remainder

$$\Rightarrow g(x) = \frac{x+1}{x-5}$$

$$= 1 + \frac{6}{x-5}$$

$$= 1 + 6 \cdot \frac{1}{x-5}$$

$$= 1 + 6 \cdot f(x-5)$$

$\Rightarrow g(x) = a \cdot f(x-h) + k$

where $a = 6$, $h = 5$, $k = 1$

$\Rightarrow$ we shift $f(x)$ $h = 5$ units to the left, $k = 1$ unit upward and stretch the graph using scaler factor $a = 6$ with no reflection around the x-axis since $a > 0$.