

1. (6 points) Let the vectors $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$ with $n \in \mathbb{N}$. Suppose $\mathbf{x} \cdot \mathbf{y} = 0$, $\|\mathbf{x}\|_2 = 15$, and $\|\mathbf{y}\|_2 = 16$. Find

$$\|\mathbf{x} - \frac{1}{2}\mathbf{y}\|_2$$

Consider $\|\tilde{\mathbf{x}} - \frac{1}{2}\tilde{\mathbf{y}}\|_2^2 = (\tilde{\mathbf{x}} - \frac{1}{2}\tilde{\mathbf{y}}) \cdot (\tilde{\mathbf{x}} - \frac{1}{2}\tilde{\mathbf{y}})$

$$= \tilde{\mathbf{x}} \cdot (\tilde{\mathbf{x}} - \frac{1}{2}\tilde{\mathbf{y}}) - \frac{1}{2}\tilde{\mathbf{y}} \cdot (\tilde{\mathbf{x}} - \frac{1}{2}\tilde{\mathbf{y}})$$

$$= \tilde{\mathbf{x}} \cdot \tilde{\mathbf{x}} - \tilde{\mathbf{x}} \cdot (\frac{1}{2}\tilde{\mathbf{y}}) - (\frac{1}{2}\tilde{\mathbf{y}}) \cdot \tilde{\mathbf{x}} + \frac{1}{4}\tilde{\mathbf{y}} \cdot \tilde{\mathbf{y}}$$

$$= \|\tilde{\mathbf{x}}\|_2^2 - \frac{1}{2}(\tilde{\mathbf{x}} \cdot \tilde{\mathbf{y}}) - \frac{1}{2}\tilde{\mathbf{y}} \cdot \tilde{\mathbf{x}} + \frac{1}{4}\|\tilde{\mathbf{y}}\|_2^2$$

$$= \|\tilde{\mathbf{x}}\|_2^2 + \frac{\|\tilde{\mathbf{y}}\|_2^2}{4}$$

$$= 15^2 + \frac{16^2}{4}$$

$$= 225 + \frac{16 \cdot 16}{4}$$

$$= 225 + 4 \cdot 16$$

$$= 225 + 64$$

$$= 289 \Rightarrow \|\tilde{\mathbf{x}} - \frac{1}{2}\tilde{\mathbf{y}}\|_2 = \sqrt{289} = \boxed{17}$$

2. (8 points) Let $n \in \mathbb{N}$ with $i, k \in [n]$ and $i \neq k$. Suppose $c \in \mathbb{R}$ is nonzero. Use the definition of the shear matrix $S_{ik}(c)$ to find the output of the product $\vec{e}_i^T \cdot S_{ik}(c) \cdot \vec{e}_k$. Show your work.

Let's begin with a special case: $n = 4, i = 3, k = 1, c \in \mathbb{R}$

$$\Rightarrow \vec{e}_3^T \cdot S_{31}(c) \cdot \vec{e}_1 = \begin{bmatrix} \vec{e}_3^T \\ 1 \times 4 \end{bmatrix} \cdot \begin{bmatrix} S_{31}(c) \\ 4 \times 4 \end{bmatrix} \cdot \begin{bmatrix} \vec{e}_1 \\ 4 \times 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 \times 4 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ c & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4 \times 4} \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}_{4 \times 1}$$

Note:

$$\vec{e}_3^T \cdot S_{31}(c) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 \times 4 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ c & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4 \times 4}$$

$$\begin{aligned} &= 0 \cdot [0 \ 0 \ 0 \ 0] \\ &+ 0 \cdot [0 \ 1 \ 0 \ 0] \\ &+ 1 \cdot [c \ 0 \ 1 \ 0] \\ &+ 0 \cdot [0 \ 0 \ 0 \ 1] \\ &= [c \ 0 \ 1 \ 0] \end{aligned}$$

$$= \begin{bmatrix} c & 0 & 1 & 0 \\ 1 \times 4 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} =$$

conjecture:

$$= \boxed{C}$$

Problem 2 continued ...

For the general $n \in \mathbb{N}$ case w.t.n $i, k \in [n]$ and $i \neq k$,

Notice that the product

$$\vec{e}_i^\top \cdot S_{ik}(c) = \sum_{j=1}^n \text{Entry}_{ij}(\vec{e}_i^\top) \cdot \text{Row}_j(S_{ik}(c))$$

But, by construction we know

$$\text{Entry}_{ij}(\vec{e}_i^\top) = \begin{cases} 0 & \text{if } i \neq j \\ 1 & \text{if } j = i \end{cases}$$

$$\Rightarrow \vec{e}_i^\top \cdot S_{ik}(c) = \text{Row}_i(S_{ik}(c))$$

But, by construction we have

$$S_{ik}(c) = I_n + c \cdot \vec{e}_i \vec{e}_k^\top$$

\uparrow
 $m \times n$ Identity matrix
with all main diagonal
entries equal 1 and
all off-diagonal entries equal zero

\nwarrow an $n \times n$ matrix unit with $n-1$ zero
entry & whose single non-zero
entry is in row i
column k and has
value of c (note since $i \neq k$,
this is not in diagonal position)

Then $\text{Row}_i(S_{ik}(c))$ is a $1 \times n$ vector with $n-2$ zero entries and whose 2 non zero coefficients are in entry ii ($\text{Row}_i(S_{ik}(c)) = 1$

$$\square \text{Entry}_{ik}(\text{Row}_i(S_{ik}(c))) = c$$

$$\Rightarrow [\vec{e}_i^T \cdot S_{ik}(c)] \cdot \vec{e}_k = \text{Row}_i(S_{ik}(c)) \cdot \vec{e}_k$$

$$= \sum_{j=1}^n \text{Entry}_j(\text{Row}_i(S_{ik}(c))) \cdot \text{Entry}_j(\vec{e}_k)$$

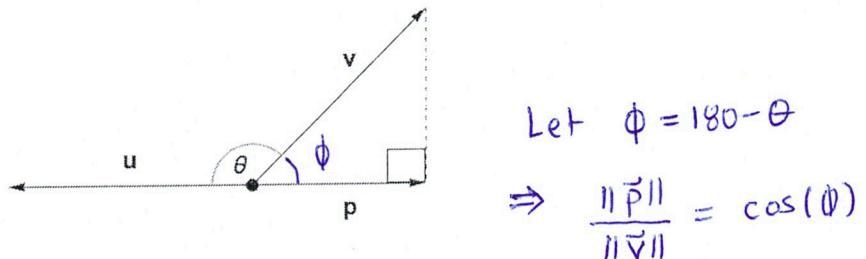
We know that

$$\text{Entry}_j(\vec{e}_k) = \begin{cases} 0 & \text{if } j \neq k \\ 1 & \text{if } j = k \end{cases}$$

$$\begin{aligned} \Rightarrow [\vec{e}_i^T \cdot S_{ik}(c)] \cdot \vec{e}_k &= \text{Entry}_k(\text{Row}_i(S_{ik}(c))) \cdot \text{Entry}_k(\vec{e}_k) \\ &= \text{Entry}_{ik}(S_{ik}(c)) \\ &= c \quad \checkmark \end{aligned}$$

This is what we wanted to show.

3. (6 points) Let $n \in \mathbb{N}$ and suppose that $\mathbf{u}, \mathbf{v} \in \mathbb{R}^n$, with $\|\mathbf{u}\|_2 = \|\mathbf{v}\|_2 = 1$. Let θ be the angle between \mathbf{u} and \mathbf{v} . Suppose $\mathbf{p} \in \mathbb{R}^n$ is the vector shown in the diagram below:



Then, find $\|\mathbf{p}\|_2$. Explain how this problem is related to the cosine formula for the inner product.

Recall from cosine formula for inner product

$$\vec{u} \cdot \vec{v} = \|\vec{u}\|_2 \cdot \|\vec{v}\|_2 \cdot \cos(\theta)$$

$$\Rightarrow \vec{u} \cdot \vec{v} = \cos(\theta)$$

where $\vec{u} \cdot \vec{v}$ measures the amount of vector \vec{v} in the direction of \vec{u}

We see from above that

$$\frac{\|\vec{p}\|_2}{\|\vec{v}\|_2} = \cos(\phi)$$

$$\Rightarrow \|\vec{p}\|_2 = \cos(180 - \theta)$$

$$\Rightarrow \|\vec{p}\|_2 = -\cos(\theta) \quad \text{assuming } \theta > 90^\circ$$

4. (8 points) Here Consider the list set of vectors

$$\mathbf{a}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 2 \end{bmatrix},$$

$$\mathbf{a}_2 = \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix},$$

$$\mathbf{a}_3 = \begin{bmatrix} 4 \\ 2 \\ 3 \\ 6 \end{bmatrix}$$

A. (2 points) Show that these vectors are linear dependent by demonstrating that you can write one of these vectors as a linear combination of the other two.

$$\vec{\mathbf{a}}_3 = \begin{bmatrix} 4 \\ 2 \\ 3 \\ 6 \end{bmatrix} = 3 \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \\ 2 \end{bmatrix} + -1 \cdot \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} = 3 \cdot \vec{\mathbf{a}}_1 + -1 \cdot \vec{\mathbf{a}}_2$$

$$\text{Check: } 3 \begin{bmatrix} 1 \\ 1 \\ 1 \\ 2 \end{bmatrix} + -1 \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 3 \\ 6 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 3 \\ 6 \end{bmatrix} \checkmark$$

B. (2 points) For $k \in [3]$, define $A \in \mathbb{R}^{4 \times 3}$ with $A(:, k) = \mathbf{a}_k$. Find a nonzero vector $\mathbf{x} \in \mathbb{R}^3$ such that $A \cdot \mathbf{x} = \mathbf{0}$

$$\text{Note } \vec{\mathbf{a}}_3 = 3 \cdot \vec{\mathbf{a}}_1 + -1 \cdot \vec{\mathbf{a}}_2$$

$$\Rightarrow -3 \cdot \vec{\mathbf{a}}_1 + 1 \cdot \vec{\mathbf{a}}_2 + 1 \cdot \vec{\mathbf{a}}_3 = \vec{0} \in \mathbb{R}^{4 \times 1}$$

$$\Rightarrow [\vec{\mathbf{a}}_1 \mid \vec{\mathbf{a}}_2 \mid \vec{\mathbf{a}}_3]_{4 \times 3} \cdot \begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix}_{3 \times 1} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}_{4 \times 1}$$

$$\Rightarrow \vec{\mathbf{x}}^\top = [-3 \ 1 \ 1] \text{ produces linear combo of columns of } A$$

C. (2 points) Is the vector $\mathbf{x} \in \mathbb{R}^3$ that you found in part B above unique? Explain your reasoning. that map to 0

No. Let $\vec{\mathbf{y}} = \alpha \cdot \vec{\mathbf{x}} = \begin{bmatrix} -3\alpha \\ 1\alpha \\ 1\alpha \end{bmatrix}$ also goes to zero for

all $\alpha \in \mathbb{R}$.

D. (2 points) Find a vector in \mathbb{R}^4 that is not in $\text{Span}\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$. Justify your answer.

Note, for every $\vec{\mathbf{b}} \in \text{Span}\{\vec{\mathbf{a}}_k\}_{k=1}^3$ we see

$$2 \cdot \text{Entry}_3(\vec{\mathbf{b}}) = \text{Entry}_4(\vec{\mathbf{b}})$$

Thus the vector $\vec{\mathbf{z}}^\top = [0 \ 0 \ 0 \ 1] \notin \text{span}\{\vec{\mathbf{a}}_k\}_{k=1}^3$

5. (6 points) Suppose we let $n = 4$. Recall that if $i, j, k \in [n]$ with $i \neq k$, we defined the elementary matrices $S_{ik}(c)$, $D_j(c)$, and P_{ik} . Using these definitions, find the matrix sum given by

$$D_4(2) + S_{32}(5) - S_{23}(-2) + S_{41}(1) - S_{24}(4) + P_{13}$$

$$\text{Note } \square \begin{bmatrix} D_4(2) \\ 4 \times 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix} \quad \square P_{13} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

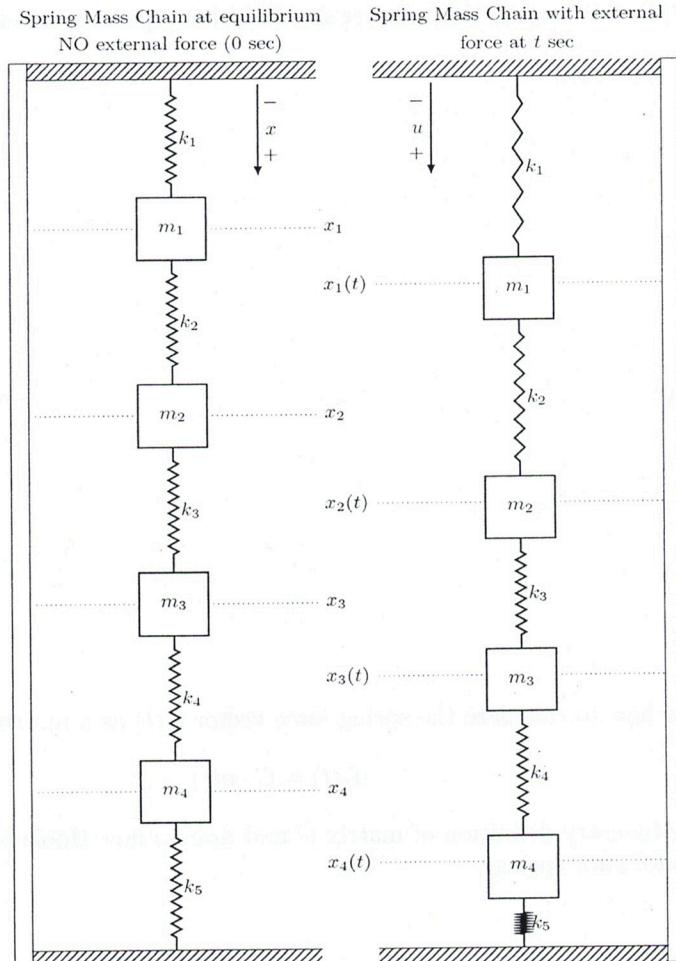
$$\square \begin{bmatrix} S_{32}(5) \\ 4 \times 4 \end{bmatrix} - \begin{bmatrix} S_{23}(-2) \\ 4 \times 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 5 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\square S_{41}(1) - S_{24}(4) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -4 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

Then our desired sum is given as

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 2 & 2 & -4 \\ 1 & 5 & 1 & 0 \\ 1 & 0 & 0 & 3 \end{bmatrix}$$

6. (10 points) For the following problems, consider the following model for a 4-mass, 5-spring chain. Note that positive positions and positive displacements are marked in the downward direction. Assume the acceleration due to earth's gravity is $g = 9.8m/s^2$. Also assume that the mass of each spring is zero and that these springs satisfy the ideal version of Hooke's law exactly.



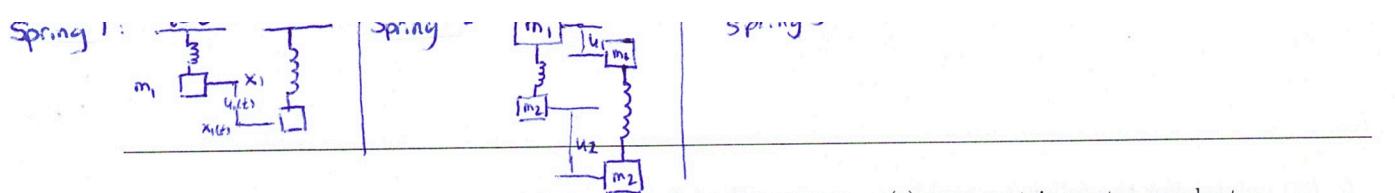
This solution for Problem 6
is D-level work!
Please see other exam solutions for a
more thorough solution

- A. (2 points) Generate vector models (using appropriate matrices and vectors) to define

$$\mathbf{x}_0, \mathbf{x}(t), \text{ and } \mathbf{u}(t)$$

where these vectors represent the equilibrium position vector, the positions of each mass at time t , and the displacement vector, respectively (as discussed in class and in our lesson notes).

$$\mathbf{x}_0 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}_{4 \times 1}, \quad \mathbf{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix}, \quad \mathbf{u}(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \\ u_4(t) \end{bmatrix}$$



B. (2 points) Show how to calculate the elongation vector $\mathbf{e}(t)$ as a matrix-vector product

$$\mathbf{e}(t) = A \cdot \mathbf{u}(t)$$

Write the entry-by-entry definition of matrix A and explain how you derived the equation for each coefficient $e_i(t)$ in this vector. Your answer should include specific references to the diagrams below.

From diagrams above, we see

$$e_1(t) = u_1(t), e_2(t) = u_2(t) - u_1(t), e_3(t) = u_3(t) - u_2(t), e_4(t) = u_4(t) - u_3(t)$$

$$e_5(t) = -u_4(t)$$

$$\Rightarrow \vec{\mathbf{e}}(t) = \begin{bmatrix} e_1(t) \\ e_2(t) \\ e_3(t) \\ e_4(t) \\ e_5(t) \end{bmatrix}_{5 \times 1} = \begin{bmatrix} u_1(t) \\ u_2(t) - u_1(t) \\ u_3(t) - u_2(t) \\ u_4(t) - u_3(t) \\ -u_4(t) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & -1 \end{bmatrix}_{5 \times 4} \begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \\ u_4(t) \\ \end{bmatrix}_{4 \times 1} = A \cdot \vec{\mathbf{u}}(t)$$

C. (2 points) Show how to calculate the spring force vector $\mathbf{f}_s(t)$ as a matrix-vector product

$$\mathbf{f}_s(t) = C \cdot \mathbf{e}(t)$$

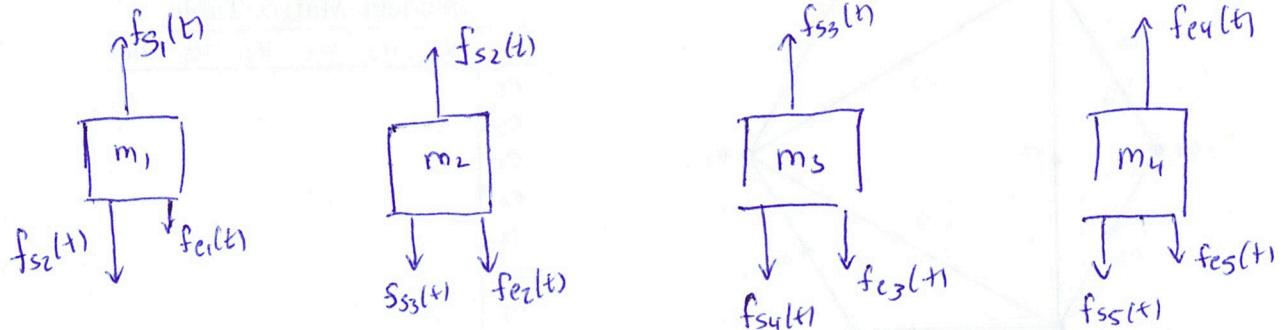
Write the entry-by-entry definition of matrix C and discuss how Hooke's law is used to create the vector of forces for each spring.

$$\mathbf{f}_s(t) = \begin{bmatrix} f_{s1}(t) \\ f_{s2}(t) \\ f_{s3}(t) \\ f_{s4}(t) \\ f_{s5}(t) \end{bmatrix}_{5 \times 1} = \begin{bmatrix} k_1 & 0 & 0 & 0 & 0 \\ 0 & k_2 & 0 & 0 & 0 \\ 0 & 0 & k_3 & 0 & 0 \\ 0 & 0 & 0 & k_4 & 0 \\ 0 & 0 & 0 & 0 & k_5 \end{bmatrix}_{5 \times 5} \begin{bmatrix} e_1(t) \\ e_2(t) \\ e_3(t) \\ e_4(t) \\ e_5(t) \end{bmatrix}_{5 \times 1} = C \cdot \vec{\mathbf{e}}(t)$$

- D. (2 points) Create "free-body" diagrams that show all forces acting on each mass m_i . Use these diagrams to derive the vector

$$\ddot{\mathbf{y}}(t) = -A^T \cdot \mathbf{f}_s(t)$$

of internal forces. Also, show how to combine your equation for $\ddot{\mathbf{y}}(t)$ with equations from parts B and C to form the stiffness matrix K . You should also find the entry-by-entry definition of K .



$$\ddot{\mathbf{y}}(t) = \begin{bmatrix} \ddot{y}_1(t) \\ \ddot{y}_2(t) \\ \ddot{y}_3(t) \\ \ddot{y}_4(t) \end{bmatrix} = - \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} f_{s1}(t) \\ f_{s2}(t) \\ f_{s3}(t) \\ f_{s4}(t) \\ f_{s5}(t) \end{bmatrix}$$

- E. (2 points) Use Newton's second law to derive the matrix equation

$$M \cdot \ddot{\mathbf{u}}(t) + K \cdot \dot{\mathbf{u}}(t) = \mathbf{f}_e(t)$$

where $\mathbf{f}_e(t)$ represents the vector of external forces on each mass. Show the entry-by-entry definition of the mass matrix M .

$$\sum \vec{F}(t) = \begin{bmatrix} \sum F_1(t) \\ \sum F_2(t) \\ \sum F_3(t) \\ \sum F_4(t) \\ \sum f_{st}(t) \end{bmatrix} = \begin{bmatrix} m_1 \cdot \ddot{u}_1(t) \\ m_2 \cdot \ddot{u}_2(t) \\ m_3 \cdot \ddot{u}_3(t) \\ m_4 \cdot \ddot{u}_4(t) \\ f_{e1}(t) \end{bmatrix}$$

$$M = \begin{bmatrix} m_1 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 \\ 0 & 0 & m_3 & 0 \\ 0 & 0 & 0 & m_4 \end{bmatrix}$$

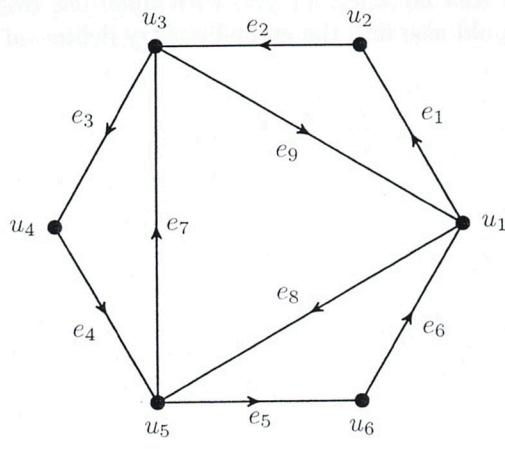
$$\Rightarrow \ddot{\mathbf{y}}(t) + \vec{f}_e(t) = M \ddot{\mathbf{u}}(t)$$

$$\Rightarrow -\underbrace{A^T C A}_{K} \ddot{\mathbf{u}}(t) + \vec{f}_e(t) = M \ddot{\mathbf{u}}(t)$$

K

$$\Rightarrow \boxed{M \ddot{\mathbf{u}}(t) + K \ddot{\mathbf{u}}(t) = \vec{f}_e(t)}$$

7. (6 points) Consider the directed graph given below. Use this graph to fill in the corresponding incidence matrix table. Let $A \in \mathbb{R}^{9 \times 6}$ be the incidence matrix corresponding to this directed graph.



Directed Graph
Incident Matrix Table

	u_1	u_2	u_3	u_4	u_5	u_6
e_1	1	-1	0	0	0	0
e_2	0	1	-1	0	0	0
e_3	0	0	1	-1	0	0
e_4	0	0	0	1	-1	0
e_5	0	0	0	0	1	-1
e_6	-1	0	0	0	0	1
e_7	0	0	-1	0	1	0
e_8	1	0	0	0	-1	0
e_9	-1	0	1	0	0	0

Suppose that $B = A^T$. Find a vector $\mathbf{y} \in \mathbb{R}^6$ such that

$$\mathbf{y}^T \cdot B = \mathbf{0}$$

Explain your choice.

Note : all entries in a single row of A add to zero

If we set $\bar{\mathbf{y}} = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}$ then $\text{Row}_i(A) \cdot \bar{\mathbf{y}} = A(i,:) \cdot \bar{\mathbf{y}} = 0$

$$\Rightarrow A \cdot \bar{\mathbf{y}} = \vec{0} \in \mathbb{R}^9$$

$$\Rightarrow \bar{\mathbf{y}}^T \cdot B = \vec{0} \in \mathbb{R}^{1 \times 9}$$

\Rightarrow all entries in a single column of B sum to zero.

Challenge Problem

8. (Optional, Extra Credit, Challenge Problem) Let $m, n \in \mathbb{N}$. Suppose $A \in \mathbb{R}^{m \times n}$ and $C \in \mathbb{R}^{m \times m}$ where the diagonal elements of C are positive $c_{ii} > 0$ for all $i \in \{1, 2, \dots, m\}$. Let $K = A^T \cdot C \cdot A$. Prove that

$$K \cdot \vec{x} = \mathbf{0} \text{ if and only if } A \cdot \vec{x} = \mathbf{0}.$$

If $A \cdot \vec{x} = \mathbf{0}$, then

$$\begin{aligned} K \cdot \vec{x} &= (A^T \cdot C \cdot A) \cdot \vec{x} \\ &= A^T \cdot C \cdot (A_{m \times n} \cdot \vec{x}_{n \times 1}) \\ &= A^T (C \cdot \vec{0}_{m \times 1}) \\ &= [A^T]_{n \times m} \cdot \vec{0}_{m \times 1} \\ &= \vec{0}_{n \times 1} \quad \checkmark \end{aligned}$$

If $K \cdot \vec{x} = \mathbf{0} \Rightarrow A^T \cdot C \cdot A \cdot \vec{x} = \mathbf{0}$

$$\Rightarrow \text{if } \vec{y} = A \cdot \vec{x}, \text{ then } \vec{y}^T = \vec{x}^T \cdot A^T$$

and $\vec{x}^T \cdot K \vec{x} = \mathbf{0}$

$$\Rightarrow \vec{x}^T A^T \cdot C \cdot A \cdot \vec{x} = \mathbf{0}$$

$$\Rightarrow \vec{y}^T \cdot C \cdot \vec{y} = \mathbf{0}$$

$$\Rightarrow c_{11} \cdot y_1^2 + c_{22} \cdot y_2^2 + \dots + c_{nn} \cdot y_n^2 = \mathbf{0}$$

$$\Rightarrow y_k = 0 \text{ for all } k \Rightarrow A \cdot \vec{x} = \mathbf{0} \quad \checkmark$$