Name : ______________________ **Class Number**: ______

Math 2B: Applied Linear Algebra
Sample Exam 1

How long is this exam?

- This exam is scheduled for a 135 minute period.
- Make sure you have 5 sheets of paper (10 pages front and back) including this cover page.
- There are a total of 8 separate questions on this exam including:
  - 8 Free-response questions (with NO subproblems) (50 points)
  - 1 Optional, extra credit challenge problem (5 points)

How will your written work be graded on these questions?

- Your work should show evidence of original thought and deep understanding. Work that too closely resembles the ideas presented in Jeff's lesson notes will likely NOT earn top scores. Work that does not demonstrate individualized, nuanced understanding will likely NOT earn top scores.
- Read the directions carefully. Your work will be graded based on what you are being asked to do.
- In order to earn a top score, please show all your work. In most cases, a correct answer with no supporting work will NOT earn top scores. What you write down and how you write it are the most important means of getting a good score on this exam.
- Neatness and organization are IMPORTANT! Do your best to make your work easy to read.
- You will be graded on accurate use of the notation we studied in this class.

What can you use on this exam?

- You may use no more than six note sheets (double-sided) or twelve note sheets (single-sided).
- Each note sheet is to be no larger then 11-inches by 8.5-inches (standard U.S. letter-sized paper).
- You must be the author of your own notes. and your note sheets must be handwritten (in YOUR OWN handwriting).
- PLEASE SUBMIT ALL OF YOUR NOTE SHEETS WITH YOUR EXAM.
- You are allowed to use calculators for this exam. Examples of acceptable calculators include TI 83, TI 84, and TI 86 calculators. You are not allowed to use any calculator with a Computer Algebra System including TI 89 and TI NSpire. If you have a question, please ask your instructor about this.

What other rules govern your participation during this exam?

- PLEASE DO NOT TURN THIS PAGE UNTIL TOLD TO DO SO!
- It is a violation of the Foothill Academic Integrity Code to, in any way, assist another person in the completion of this exam. Please keep your own work covered up as much as possible during the exam so that others will not be tempted or distracted. Thank you for your cooperation.
- No notes (other than your note sheets), books, or classmates can be used as resources for this exam.
- Please turn off your cell phones during this exam. No cell phones will be allowed on your desk.

1. (6 points) Let

$$A = \{a \in \mathbb{Z} : a^3 - 4a^2 + 3a = 0\} \text{ and } B = \{\ b : b + 1 = 2^n \text{ for } n \in \{0, 1, 2\}\ \}.$$

Show $A = B$.

2. (6 points) Consider the following four sets:

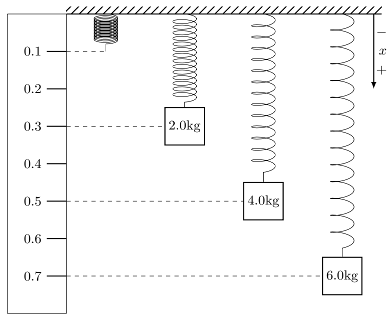
$$\begin{aligned} S &= \{(x,y) \,:\, y = x^2\} \subseteq \mathbb{R}\times\mathbb{R}. \\ R &= \{(y,x) \,:\, y = x^2\} \subseteq \mathbb{R}\times\mathbb{R}. \\ R_+ &= \{(y,x) \,:\, y = x^2 \text{ with } x \geq 0\} \subseteq \mathbb{R}\times\mathbb{R}. \\ R_- &= \{(y,x) \,:\, y = x^2 \text{ with } x \leq 0\} \subseteq [0,\infty)\times\mathbb{R}. \end{aligned}$$

For each these sets, determine the following

i. The domain space
ii. The domain
iii. The codomain
iv. The range

Then, classify each set as either a relation or function. Justify your decisions using the appropriate formal, set-theoretic definitions from Lesson 2.

3. (6 points) Consider the ideal version of a Hooke's law experiment depicted below. Suppose we hang three masses on the same ideal extension spring and record the position data for that spring using a metric ruler so that all positions are measured in meters. Assume also that the acceleration due to earth's gravity is $g = 9.8N/kg$. Finally, suppose that the mass of the spring is zero and that this spring satisfies an ideal version of Hooke's law.



Create a vector model that describes Hooke's law by forming each of the following vectors

i. Mass vector $\mathbf{m}$

ii Raw position vector $\mathbf{x}$

iii. Spring force vector $\mathbf{f}_s$

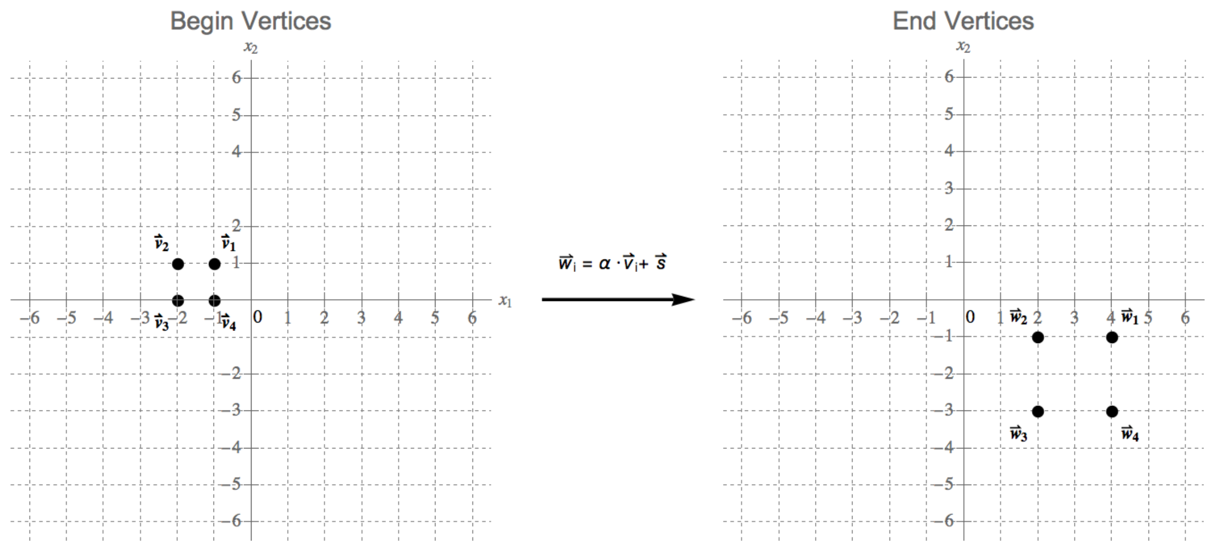
iv. Displacement vector $\mathbf{u}$

Explain your work and demonstrate how to use scalar-vector multiplication and vector-vector addition in this modeling exercise.

4. (6 points) Suppose we model a square using a set of four *begin vertices* given by the four vertices

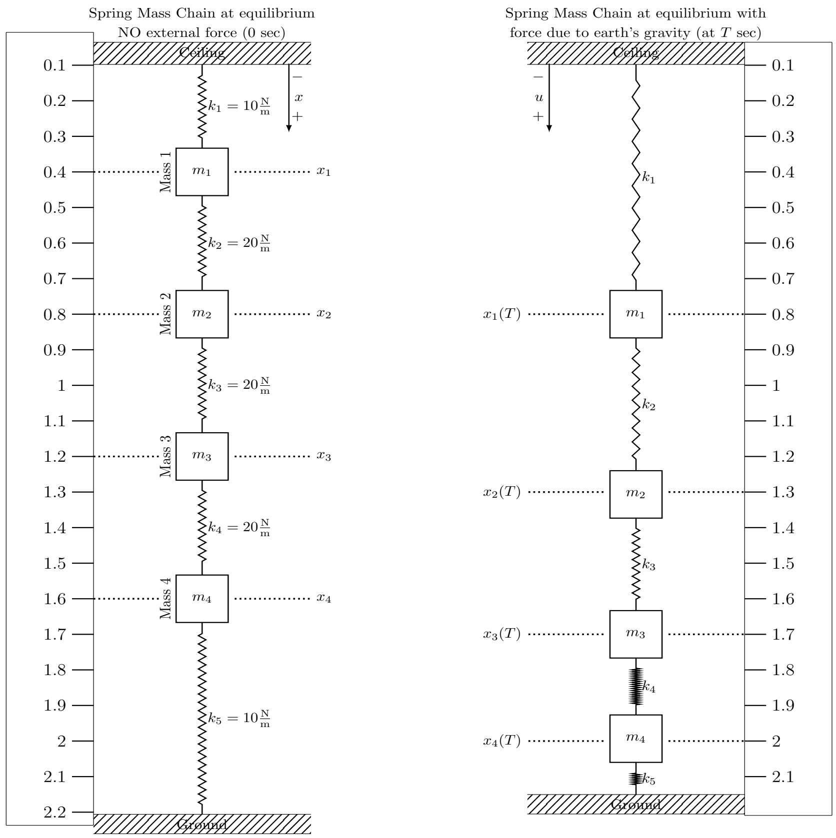
$$\mathbf{v}_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \qquad \mathbf{v}_2 = \begin{bmatrix} -2 \\ 1 \end{bmatrix}, \qquad \mathbf{v}_3 = \begin{bmatrix} -2 \\ 0 \end{bmatrix}, \qquad \mathbf{v}_4 = \begin{bmatrix} -1 \\ 0 \end{bmatrix}.$$

Define a set of four *end vertices* with the property that $\mathbf{w}_i = \alpha \cdot \mathbf{v}_i + \mathbf{s}$ for a special scalar $\alpha \in \mathbb{R}$ and special vector $\mathbf{s} \in \mathbb{R}^2$, where $i \in [4]$. Under these assumptions, we graph both the begin and end vertices for this problem in the figure below.



Using this information, set up a system of equations that might help you figure out the specific values of α and $\mathbf{s}$ used in this problem. Then, find these values and explain your work.

5. (8 points) Consider the following model for a 4-mass, 5-spring chain drawn below. Note that positive positions and positive displacements are marked in the downward direction. Assume the ruler gives position measurements in meters.



Use this diagram and the given position data, set up the vectors $\mathbf{x}_0, \mathbf{x}(T), \mathbf{u}(T)$ that we discussed in class. Describe the significance of each vector. Also, show how we can use scalar-vector multiplication and vector-vector addition to form $\mathbf{u}(T)$ from the vectors $\mathbf{x}_0$ and $\mathbf{x}(T)$.

6. (6 points) Describe as much as you can about Problem 0: The Applied Math Modeling Process. In order to earn full credit on this problem, your answer should

   i. Identify the stages of the Applied Math Modeling Process (including a diagram with descriptions)
   ii. Explain where and how vector and matrix modeling arises in this process
   iii. Describe how Problem 0 is related to our use of the word "given" in Problems 1, 2A, 2B, 3, and 4
   iv. Be written in your own voice with as much unique thought as possible.

For problems 7 - 8, describe each of the given problems in detail. In order to earn full credit on each problem, your answer should accurately address each of the following:

i. Identify the problem statement
ii. Identify the given and unknown quantities (explicitly identify relevant dimensions)
iii. Identify the function description of this problem (explicitly discuss domain, codomain and range)
iv. Describe in as much detail as you can how each problem is similar to and different from the other two problems in the list below.
v. Write your solutions in your own voice with as much unique thought as possible.

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7. (6 points) Problem 1: The Matrix-Vector Multiplication Problem

8. (6 points) Problem 2B: The General Linear-Systems Problem

Challenge Problem

9. (5 points) Optional, Extra Credit, Challenge Problem: Suppose $n \in \mathbb{N}$ and let $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$ with $x_i, y_i \in [26]$ for all $i \in [n]$. Prove or disprove the following conjecture:

   Conjecture: If $\|\mathbf{x}\|_2 = \|\mathbf{y}\|_2$, then the entries of $\mathbf{y}$ are identical to the entries of $\mathbf{x}$ through some permutation $\pi : [n] \longrightarrow [n]$ with $y_i = x_{\pi(i)}$ for all $i \in [n]$.