Name : ____________________ **Class Number**: ______

Exam 1, Version 1A

Math 2B: Linear Algebra

What are the rules of this exam?

- PLEASE DO NOT TURN THIS PAGE UNTIL TOLD TO DO SO!
- It is a violation of the Foothill Academic Integrity Code to, in any way, assist another person in the completion of this exam. Please keep your own work covered up as much as possible during the exam so that others will not be tempted or distracted. Thank you for your cooperation.
- No notes (other than your note card), books, or classmates can be used as resources for this exam.
- Please turn off your cell phones or put your cell phones into airplane mode during this exam. Please place your cell phones inside your bag. No cell phones will be allowed on your desk.
- Close your bag and put it under your seat.

How long is this exam?

- This exam is scheduled for a 100 minute class period.
- Make sure you have 8 sheets of paper (16 pages front and back) including this cover page.
- There are a total of 24 separate questions (100 points) on this exam including:
  - 5 True/False Questions (10 points)
  - 15 Multiple Choice Questions (60 points)
  - 3 Free-Response Questions (30 points)
  - 1 Optional, Extra Credit Challenge Problem (10 points)

What can I use on this exam?

- You may use one note card that is no larger than 8.5 inches by 11 inches This note card must be handwritten. You may write on both sides of this note card. PLEASE SUBMIT YOUR NOTECARD WITH YOUR EXAM.
- You are allowed to use calculators for this exam. Examples of acceptable calculators include TI 83, TI 84, and TI 86 calculators. You are not allowed to use any calculator with a Computer Algebra System including TI 89 and TI NSpire. If you have a question, please ask your instructor about this.

How will I be graded on the Free-Response Questions?

- Read the directions carefully. **Show all your work for full credit.** In most cases, a correct answer with no supporting work will NOT receive full credit. What you write down and how you write it are the most important means of getting a good score on this exam. Neatness and organization are IMPORTANT!
- You will be graded on proper use of vector notation and matrix notation on this exam.
- You will be graded on proper use of theorems and definitions from in-class discussions and homework.

True/False (10 points: 2 points each) For the problems below, circle T if the answer is true and circle F is the answer is false. After you've chosen your answer, mark the appropriate space on your Scantron form. Notice that letter A corresponds to true while letter B corresponds to false.

1. T F Let $A \in \mathbb{R}^{m \times n}$ and $\mathbf{x} \in \mathbb{R}^n$. Then the matrix-vector product $A \cdot \mathbf{x}$ represents a linear combination of the rows of A with scalar multiples defined by the entries of $\mathbf{x}$.

2. T F All functions are relations.

3. T F Any set of vectors that contains the zero vector must be linearly dependent.

4. T F Any two matrices that are conformable for matrix multiplication must have the same number of rows.

5. T F Since all entries of the vectors $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ are zero, these vectors are equal.

Multiple Choice

(60 points: 4 points each) For the problems below, circle the correct response for each question. After you've chosen, mark your answer on your Scantron form.

6. Define vectors

$$\mathbf{x} = \begin{bmatrix} t \\ -4 \\ 2 \\ t \end{bmatrix}, \qquad \mathbf{y} = \begin{bmatrix} -t \\ t \\ 5 \\ 1 \end{bmatrix}$$

Find <u>all</u> values of scalar t so that the inner product $\mathbf{x} \cdot \mathbf{y} = 0$

A. $t = -2$ B. $t = 5$ and $t = -2$ C. $t = 5$ and $t = 2$ D. $t = -5$ and $t = 2$ E. $t = 5$

7. Suppose that $\mathbf{e}_k \in \mathbb{R}^3$ is the 3×1 elementary basis vector with $\mathbf{e}_k = I_3(:, k)$ for $k = 1, 2, 3$. Let

$$A = -2 \cdot \mathbf{e}_3 \cdot \mathbf{e}_1^T + 4 \cdot \mathbf{e}_2 \cdot \mathbf{e}_2^T + 3 \cdot \mathbf{e}_3 \cdot \mathbf{e}_3^T - \mathbf{e}_1 \cdot \mathbf{e}_2^T$$

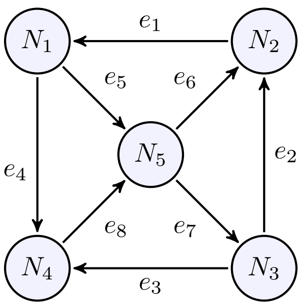
Then, which of the following gives $A(:, 2) \cdot A(1, :)$?

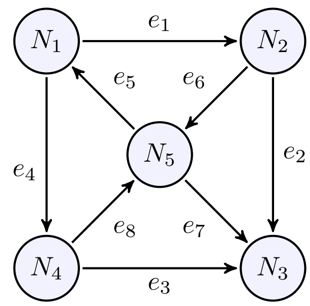
A. $\begin{bmatrix} 0 & -1 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ B. 4 C. 1 D. $\begin{bmatrix} 0 & 1 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ E. $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

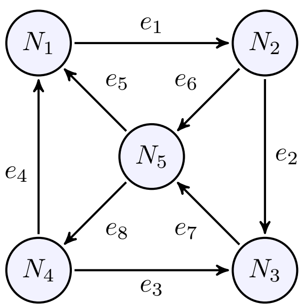
8. Let the following matrix $A \in \mathbb{R}^{8\times 5}$ be the incidence matrix for a directed graph:

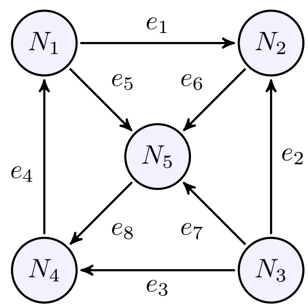
$$A = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ -1 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}$$

Then, this is the incidence matrix for which of the following directed graphs:

A. 

B. 

C. 

D. 

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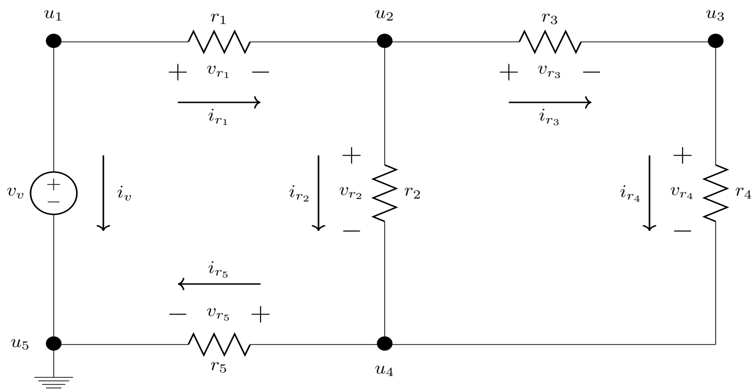
9. Suppose that we define ellipse $E = \left\{(x,y) \,:\, \dfrac{x^2}{25} + \dfrac{y^2}{9} = 1\right\}$. Find the range, $\mathrm{Rng}(E)$, of this relation.

A. $(-5,5)$ B. $[-5,5]$ C. $[-3,3]$ D. $(-3,3)$ E. $\mathbb{R}$

Consider the following ideal circuit diagram. Use this figure to answer questions 10, 11, and 12 below.



10. Which of the following matrix-vector products is used to calculate the voltage across each circuit element.

A. $\begin{bmatrix} v_{r_1} \\ v_{r_2} \\ v_{r_3} \\ v_{r_4} \\ v_{r_5} \\ v_v \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ 1 & 0 & 0 & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{bmatrix}$

B. $\begin{bmatrix} v_{r_1} \\ v_{r_2} \\ v_{r_3} \\ v_{r_4} \\ v_{r_5} \\ v_v \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 \\ -1 & 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{bmatrix}$

C. $\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & -1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & -1 \end{bmatrix} \cdot \begin{bmatrix} v_{r_1} \\ v_{r_2} \\ v_{r_3} \\ v_{r_4} \\ v_{r_5} \\ v_v \end{bmatrix}$

D. $\begin{bmatrix} v_{r_1} \\ v_{r_2} \\ v_{r_3} \\ v_{r_4} \\ v_{r_5} \\ v_v \end{bmatrix} = \begin{bmatrix} \frac{1}{r_1} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{r_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{r_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{r_4} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{r_5} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} i_{r_1} \\ i_{r_2} \\ i_{r_3} \\ i_{r_4} \\ i_{r_5} \\ v_v \end{bmatrix}$

11. Which matrix-vector multiplication problems gives Kirchoff's Current Laws for the entire circuit?

A. $\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & -1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & -1 \end{bmatrix} \cdot \begin{bmatrix} i_{r_1} \\ i_{r_2} \\ i_{r_3} \\ i_{r_4} \\ i_{r_5} \\ i_v \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

B. $\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & -1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & -1 \end{bmatrix} \cdot \begin{bmatrix} v_{r_1} \\ v_{r_2} \\ v_{r_3} \\ v_{r_4} \\ v_{r_5} \\ v_v \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -1 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} i_{r_1} \\ i_{r_2} \\ i_{r_3} \\ i_{r_4} \\ i_{r_5} \\ i_v \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

D. $\begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ 1 & 0 & 0 & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} i_{r_1} \\ i_{r_2} \\ i_{r_3} \\ i_{r_4} \\ i_{r_5} \\ i_v \end{bmatrix}$

12. Which of the following matrix-vector multiplication problems can be used to state Ohm's Law for each resistor in the circuit?

A. $\begin{bmatrix} v_{r_1} \\ v_{r_2} \\ v_{r_3} \\ v_{r_4} \\ v_{r_5} \end{bmatrix} = \begin{bmatrix} r_1 & 0 & 0 & 0 & 0 \\ 0 & r_2 & 0 & 0 & 0 \\ 0 & 0 & r_3 & 0 & 0 \\ 0 & 0 & 0 & r_4 & 0 \\ 0 & 0 & 0 & 0 & r_5 \end{bmatrix} \cdot \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{bmatrix}$

B. $\begin{bmatrix} i_{r_1} \\ i_{r_2} \\ i_{r_3} \\ i_{r_4} \\ i_{r_5} \end{bmatrix} = \begin{bmatrix} r_1 & 0 & 0 & 0 & 0 \\ 0 & r_2 & 0 & 0 & 0 \\ 0 & 0 & r_3 & 0 & 0 \\ 0 & 0 & 0 & r_4 & 0 \\ 0 & 0 & 0 & 0 & r_5 \end{bmatrix} \cdot \begin{bmatrix} v_{r_1} \\ v_{r_2} \\ v_{r_3} \\ v_{r_4} \\ v_{r_5} \end{bmatrix}$

C. $\begin{bmatrix} v_{r_1} \\ v_{r_2} \\ v_{r_3} \\ v_{r_4} \\ v_{r_5} \end{bmatrix} = \begin{bmatrix} r_1 & 0 & 0 & 0 & 0 \\ 0 & r_2 & 0 & 0 & 0 \\ 0 & 0 & r_3 & 0 & 0 \\ 0 & 0 & 0 & r_4 & 0 \\ 0 & 0 & 0 & 0 & r_5 \end{bmatrix} \cdot \begin{bmatrix} i_{r_1} \\ i_{r_2} \\ i_{r_3} \\ i_{r_4} \\ i_{r_5} \end{bmatrix}$

D. $\begin{bmatrix} v_{r_1} \\ v_{r_2} \\ v_{r_3} \\ v_{r_4} \\ v_{r_5} \end{bmatrix} = \begin{bmatrix} \frac{1}{r_1} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{r_2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{r_3} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{r_4} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{r_5} \end{bmatrix} \cdot \begin{bmatrix} i_{r_1} \\ i_{r_2} \\ i_{r_3} \\ i_{r_4} \\ i_{r_5} \end{bmatrix}$

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13. Let 10110110 be an 8−bit binary integer. What is the decimal representation of this number?

A. 109 B. 364 C. 218 D. 80880880 E. 182

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14. Define three vectors in $\mathbb{R}^4$ as

$$\mathbf{a}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \qquad \mathbf{a}_2 = \begin{bmatrix} 0 \\ 5 \\ 10 \\ 15 \end{bmatrix}, \qquad \mathbf{a}_3 = \begin{bmatrix} 0 \\ 25 \\ 100 \\ 225 \end{bmatrix}, \qquad \mathbf{a}_4 = \begin{bmatrix} 5 \\ -45 \\ -45 \\ 5 \end{bmatrix}$$

We can confirm that $\mathbf{a}_4 = 5 \cdot \mathbf{a}_1 - 15 \cdot \mathbf{a}_2 + 1 \cdot \mathbf{a}_3$. Choose the vector $\mathbf{x} \in \mathbb{R}^4$ such that

$$\left[\ \mathbf{a}_1 \ \middle| \ \mathbf{a}_2 \ \middle| \ \mathbf{a}_3 \ \middle| \ \mathbf{a}_4 \ \right] \cdot \mathbf{x} = \mathbf{0}$$

A. $\begin{bmatrix} 45 \\ -1 \\ 1 \\ -1 \end{bmatrix}$ B. $\begin{bmatrix} 5 \\ -15 \\ 1 \\ 1 \end{bmatrix}$ C. $\begin{bmatrix} -5 \\ 15 \\ -1 \\ -1 \end{bmatrix}$ D. $\begin{bmatrix} 5 \\ -15 \\ 1 \\ -1 \end{bmatrix}$ E. The product will never be zero

15. Let $A \in \mathbb{R}^{8\times 4}$, $B \in \mathbb{R}^{4\times 7}$, and $C \in \mathbb{R}^{7\times 5}$. Let the matrix D be formed by the product

$$D = (A \cdot B \cdot C)^T$$

What are the dimensions of the matrix $[D(:,4)]^T$?

A. 8×5 B. 5×8 C. 1×5 D. 1×8 E. 5×1

16. Consider the set of vectors given by

$$\mathbf{a}_1 = \begin{bmatrix} 2 \\ 0 \\ 2 \\ 0 \end{bmatrix}, \qquad \mathbf{a}_2 = \begin{bmatrix} -1 \\ 0 \\ -1 \\ 0 \end{bmatrix}, \qquad \mathbf{a}_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix},$$

Which of the following vectors sets is equivalent to the span of these three vectors?

A. $\mathbb{R}^4$ B. $\left\{ \begin{bmatrix} x_1 \\ x_1 \\ x_1 \\ x_1 \end{bmatrix} : x_1 \in \mathbb{R} \right\}$ C. $\left\{ \begin{bmatrix} x_1 \\ x_1 \\ x_2 \\ x_2 \end{bmatrix} : x_i \in \mathbb{R} \text{ for } i = 1, 2 \right\}$

D. $\left\{ \begin{bmatrix} 2 \\ 0 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \right\}$ E. $\left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_1 \\ x_2 \end{bmatrix} : x_i \in \mathbb{R} \text{ for } i = 1, 2 \right\}$

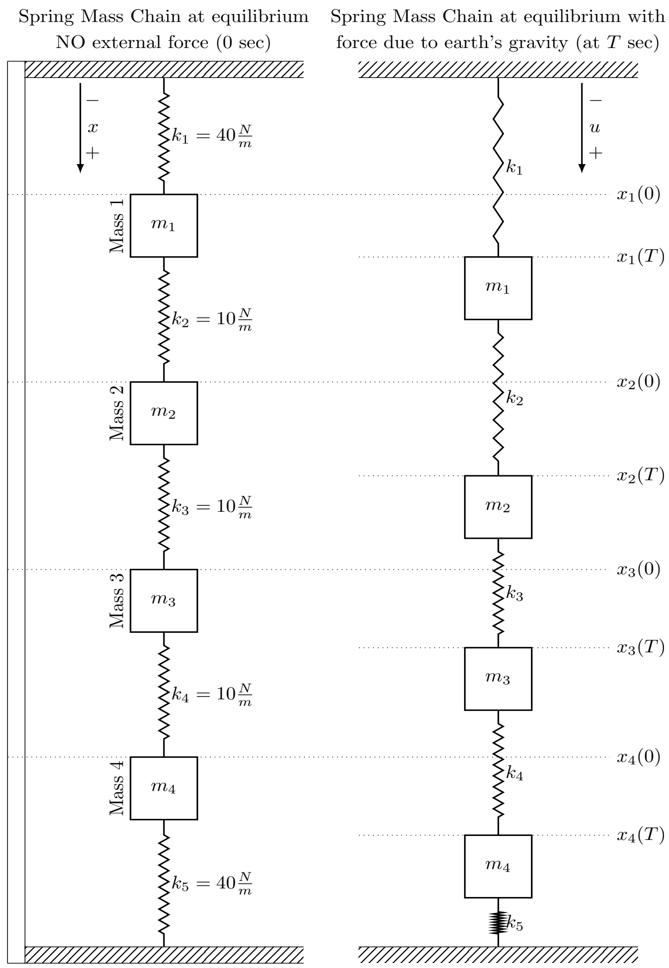
17. Define the matrix $B \in \mathbb{R}^{3\times 3}$ as a sum of elementary matrices given by

$$B = D_1(2) + S_{21}(2) + S_{31}(3) - S_{13}(-4).$$

Which of the following matrices is equivalent to B?

A. $\begin{bmatrix} 3 & 0 & 4 \\ 2 & 2 & 0 \\ 3 & 0 & 2 \end{bmatrix}$ B. $\begin{bmatrix} 2 & 0 & 4 \\ 2 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix}$ C. $\begin{bmatrix} 2 & 0 & -4 \\ 2 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix}$ D. $\begin{bmatrix} 4 & 0 & 4 \\ 2 & 2 & 0 \\ 3 & 0 & 2 \end{bmatrix}$ E. $\begin{bmatrix} 3 & 0 & -4 \\ 2 & 2 & 0 \\ 3 & 0 & 2 \end{bmatrix}$

For problems 17 and 18, consider the following 4-mass, 5-spring chain presented below. Notice that positive positions and positive displacements are marked in the downward direction. Assume the acceleration due to earth's gravity is $g = 9.8m/s^2$. Also assume that the mass of each spring is zero and that these springs satisfy Hooke's law exactly.



18. Recall that the initial position vector $\mathbf{x}_0$ and the final position vector $\mathbf{x}(T)$ store the positions, measured in meters, of each mass at equilibrium when $t = 0$ and when $t = T$ respectively. Suppose we measure

$$\mathbf{x}_0 = \begin{bmatrix} x_1(0) \\ x_2(0) \\ x_3(0) \\ x_4(0) \end{bmatrix} = \begin{bmatrix} 0.200 \\ 0.400 \\ 0.600 \\ 0.800 \end{bmatrix} \qquad \mathbf{x}(T) = \begin{bmatrix} x_1(T) \\ x_2(T) \\ x_3(T) \\ x_4(T) \end{bmatrix} = \begin{bmatrix} 0.249 \\ 0.498 \\ 0.698 \\ 0.849 \end{bmatrix}$$

where each entry is given in meters. Using this information, which of the following vectors gives the force vector $\mathbf{f}_s$ that encodes the forces stored in each spring in this system?

A. $\begin{bmatrix} 1.960 \\ 0.490 \\ 0.000 \\ -0.490 \\ -1.960 \end{bmatrix}$ B. $\begin{bmatrix} 1.960 \\ 0.980 \\ 0.980 \\ 1.960 \end{bmatrix}$ C. $\begin{bmatrix} 1.470 \\ 0.490 \\ 0.000 \\ 0.490 \\ 1.470 \end{bmatrix}$ D. $\begin{bmatrix} 1.470 \\ 0.490 \\ 0.490 \\ 1.470 \end{bmatrix}$ E. $\begin{bmatrix} 0.049 \\ 0.049 \\ 0.000 \\ -0.049 \\ -0.049 \end{bmatrix}$

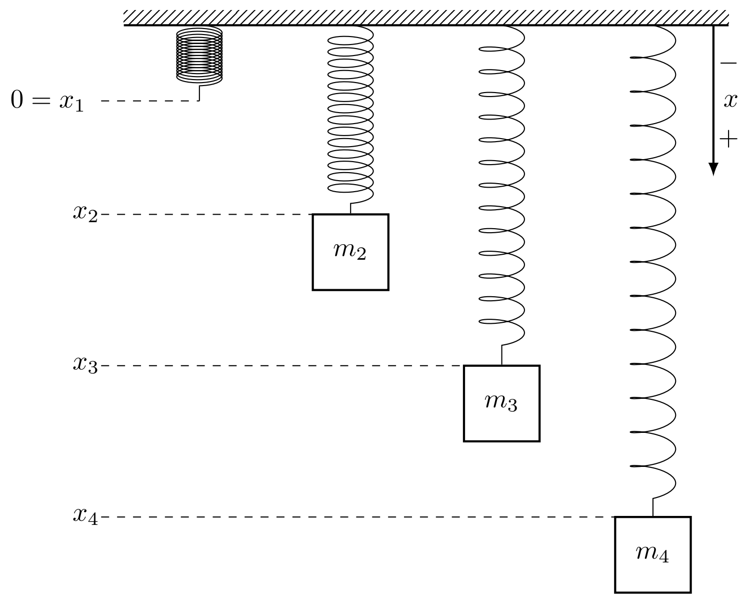
19. Under the same assumptions as the problem above, which of the following gives the mass vector

$$\mathbf{m} = \begin{bmatrix} m_1 & m_2 & m_3 & m_4 \end{bmatrix}^T$$

measured in kg, used to produce this position data?

A. $\begin{bmatrix} 0.200 \\ 0.100 \\ 0.100 \\ 0.200 \end{bmatrix}$ B. $\begin{bmatrix} 1.470 \\ 0.490 \\ 0.490 \\ 1.470 \end{bmatrix}$ C. $\begin{bmatrix} 0.250 \\ 0.200 \\ 0.200 \\ 0.250 \end{bmatrix}$ D. $\begin{bmatrix} 2.450 \\ 1.960 \\ 1.960 \\ 2.450 \end{bmatrix}$ E. $\begin{bmatrix} 0.150 \\ 0.050 \\ 0.050 \\ 0.150 \end{bmatrix}$

20. Consider the experiment below. Suppose we hang three masses on the same spring and record the position data for that spring. Assume the spring constant is known to be $k = 5$ N/m. Assume also that the acceleration due to earth's gravity is $g = 9.8N/kg$. Finally, suppose that the mass of the spring is zero and that this spring satisfy Hooke's law exactly.



In order to model the relationship between the displacement of the movable end of the spring and the internal force stored in the spring, we introduce two 4×1 vectors given by

$$\mathbf{m} = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \end{bmatrix} = \begin{bmatrix} 0.0 \\ 0.1 \\ 0.2 \\ 0.3 \end{bmatrix}, \qquad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

Each entry m_i is measured in kg. The entries of the position vector x_i, measured in meters. We know $x_1 = 0$m and the other entries $x_2, x_3, x_4 \in \mathbb{R}$ can be calculated from our knowledge of vector $\mathbf{m}$ and Hooke's Law. Which of the following gives the vector $\mathbf{x}$ in this situation?

A. $\begin{bmatrix} 0.0 \\ 0.1 \\ 0.2 \\ 0.3 \end{bmatrix}$ B. $\begin{bmatrix} 0.000 \\ 0.196 \\ 0.392 \\ 0.588 \end{bmatrix}$ C. $\begin{bmatrix} 0.00 \\ 0.02 \\ 0.04 \\ 0.06 \end{bmatrix}$ D. $\begin{bmatrix} 0.0 \\ 0.5 \\ 1.0 \\ 1.5 \end{bmatrix}$ E. $\begin{bmatrix} 0.0 \\ 4.9 \\ 9.8 \\ 14.7 \end{bmatrix}$

Free Response

$\boxed{10}$ 21. Suppose you are enrolled in a math course in which your final percent score is calculated as a weighted average. Below is a table that describes the important details of this class's grading scheme:

Grade Category on Syllabus	Total Points Available	Percentage Weight
Homework	200	10%
Projects	500	15%
Exam 1	100	20%
Exam 2	100	20%
Final Exam	100	35%

Suppose the teacher of this class does NOT have a grade replacement policy for your exam scores. With this in mind, respond to the following three questions.

A. Set up a vector model $\mathbf{g} \in \mathbb{R}^5$ that encodes all aspects of your course grade. Define each entry of $\mathbf{g}$ and describe your choices.

B. Demonstrate how to use the inner-product operation to calculate your final grade in this class.

C. Suppose on the night before the final, you know you've earned the following scores:

Grade Category on Syllabus	Points You Earned
Homework	186
Projects	420
Exam 1	82
Exam 2	90

Assuming you want to get above a 85% in this class, determine the minimum percent score you will need to earn on the final exam to achieve your goal. Show your work.

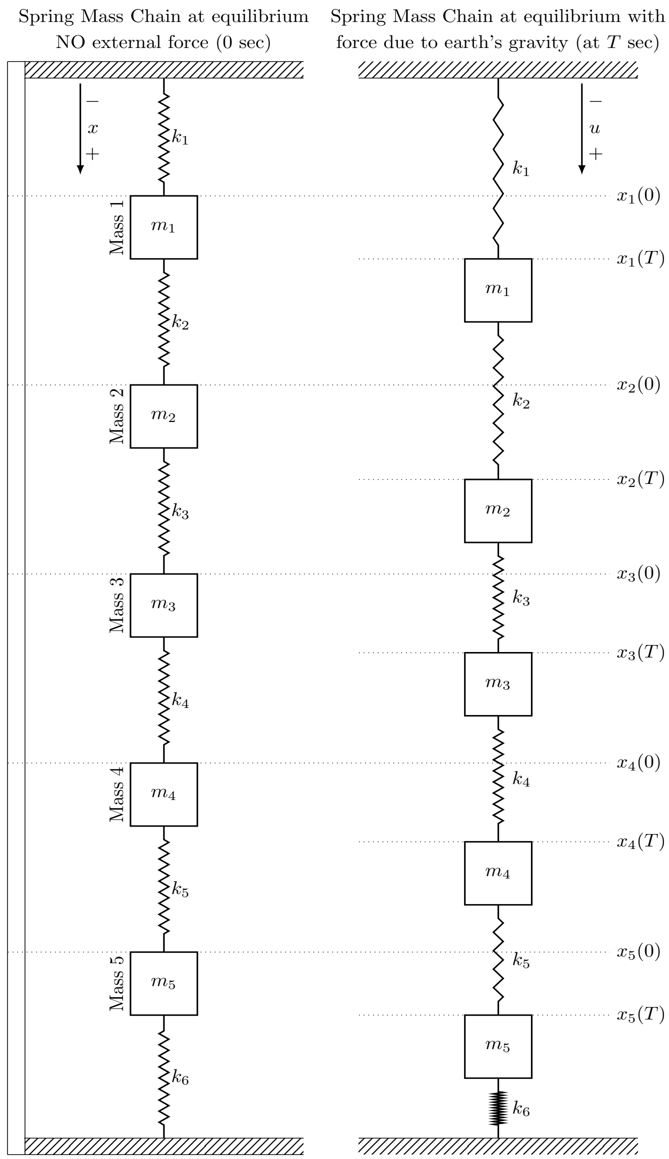
$\boxed{10}$ 22. Describe, in detail, each of the following problems. For each problem, your should:

i. Identify the problem statement
ii. Identify the given and unknown quantities (explicitly identify relevant dimensions)
iii. Identify the function description of this problem (explicitly discuss domain, codomain and range)
iv. Describe how each problem is similar to and different from the other problem.

A. The Matrix-Vector Multiplication Problem

B. The Square Linear-Systems Problem

$\boxed{10}$ 23. Mass-spring problem



A. Generate vector models (using appropriate matrices and vectors) to define

$$\mathbf{x}_0, \mathbf{x}(T), \text{ and } \mathbf{u}$$

where these vectors represent the initial position vector, the final position vector, and the displacement vector, respectively (as discussed in class and in our lesson notes).

B. Show how to calculate the elongation vector $\mathbf{e}$ as a matrix-vector product

$$\mathbf{e} = A\mathbf{u}$$

Write the entry-by-entry definition of matrix A and explain how you derived the equation for each coefficient e_i in this vector. Your answer should include specific references to the diagram of the 5-mass, 6-spring chain above.

C. Show how to calculate the spring force vector $\mathbf{f}_s$ as a matrix-vector product

$$\mathbf{f}_s = C\mathbf{e}$$

Write the entry-by-entry definition of matrix C and discuss how Hooke's law is used to create the vector of forces for each spring.

D. Create "free-body" diagrams that show all forces acting on each mass m_i. Use these diagrams to derive the vector

$$\mathbf{y} = -A^T \mathbf{f}_s$$

of internal forces. Also, show how to combine your equation for $\mathbf{y}$ with equations from parts B and C to form the stiffness matrix K. Note, you do not have to find the entry-by-entry definition of K.

E. Use Newton's second law to derive the matrix equation

$$M\ddot{\mathbf{u}} + K\mathbf{u} = \mathbf{f}_e$$

where $\mathbf{f}_e$ represents the vector of external forces on each mass. Show the entry-by-entry definition of the mass matrix M.

Challenge Problem

24. (Optional, Extra Credit, Challenge Problem)

    Let $\mathbf{x} \in \mathbb{R}^n$ be a column vector. Recall that we defined the 2-norm of $\mathbf{x}$ to be

    $$\|\mathbf{x}\|_2 = \left(\sum_{i=1}^{n} x_i^2 \right)^{1/2}$$

    This is one example of a much larger class of vector norms, known as $p-$norms. To create a $p-$norm, we choose a real number $p \geq 1$ and set

    $$\|\mathbf{x}\|_p = \left(\sum_{i=1}^{n} |x_i|^p \right)^{1/p}$$

    Using this definition, we can set $p = \infty$ and define the $\infty-$norm (read "infinity norm"), using the following definition

    $$\|\mathbf{x}\|_\infty = \max_{1 \leq i \leq n} |x_i|$$

    Prove $\lim\limits_{p \to \infty} \|\mathbf{x}\|_p = \|\mathbf{x}\|_\infty$

Use for Scratch Work