Name : ____________________ **Class Number:** _____

Math 1D: Lesson 5 Suggested Problems

Theoretic Problems: Discussed in-class

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1. Let $f : D \subseteq \mathbb{R}^n \longrightarrow \mathbb{R}$ be a continuous function on a region D in $\mathbb{R}^n$ where f is encoded using input variables written in cartesian coordinates.

   A. For $n = 1$, explain how the constructed limit definition of the integral

   $$\int\limits_D f \; d\omega = \int\limits_D f(x) \; dx = \lim_{\Delta \to 0} \sum_{k=1}^{n} f(x_k^*) \; \Delta x_k$$

   relies on a geometric interpretation of the codomain. Specifically, explain why we use the concepts of "length" and "height" to interpret the single-variable integral as the "area" under a curve.

   B. For $n = 1$, reinterpret the integral $\int\limits_D f \; d\omega$ without using the concept of "height" by describing a modeling scenario in which we can use our single-variable integral to calculate a physical quantity without require that we interpret the output of f as the "height" under a curve. Within this application, please explain each idea behind the limiting process in the Riemann sum under this revised interpretation.

   C. For $n = 2$, explain how the constructed limit definition of the integral

   $$\int\limits_D f \; d\omega = \iint\limits_D f(x, y) \; dA = \lim_{\Delta \to 0} \sum_{k=1}^{n} f(x_k^*, y_k^*) \; \Delta A_k$$

   relies on a geometric interpretation of the codomain. Specifically, explain why we use the concepts of "length," "width," and "height" to interpret the double-variable integral as the "volume" under a surface.

   D. For $n = 2$, reinterpret the integral $\int\limits_D f \; d\omega$ without using the concept of "height" by describing a modeling scenario in which we can use our double-variable integral to calculate a physical quantity without require that we interpret the output of f as the "height" under a curve. Within this application, please explain each idea behind the limiting process in the Riemann sum under this revised interpretation.

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2. Let $f : D \subseteq \mathbb{R}^3 \longrightarrow \mathbb{R}$ be a continuous function on a region D in $\mathbb{R}^3$ where $f(x, y, z)$ is encoded using input variables written in cartesian coordinates.

   A. Explain why it is not possible to interpret the output $f(x, y, z) \in \mathbb{R}$ as the "height" of some curve in this case.

   B. Create some modeling application in which we might want to use integration of a function $f : D \subseteq \mathbb{R}^3 \longrightarrow \mathbb{R}$ to better understand the physical world around you.

   C. Within the context of this modeling application, derive the limit definition of the triple integral

$$\int_D f\ d\omega = \iint_D f(x, y, z)\ dV = \lim_{\Delta \to 0} \sum_{k=1}^{n} f(x_k^*, y_k^*, z_k^*)\ \Delta V_k$$

   D. Within this application, please explain each idea behind the limiting process in the Riemann sum.

   E. Explain why Jeff is asking you to interpret the triple integral within the context of a modeling application without using a geometric interpretation of the codomain of the function f?

Problems Solved in Jeff's Handwritten Notes

3. Example 13.4.1 p. 997
4. Example 13.4.2 p. 997 - 998
5. Example 13.4.3 p. 998 - 999

Suggested Problems: Answers in Back of Book

3. Example 13.4.4 p. 1000 - 1001
4. Exercise 13.4.9 p. 1002
5. Exercise 13.4.15 p. 1003
6. Exercise 13.4.41 p. 1005

Optional Challenge Problems

3. Exercise 13.4.61 p. 1006
4. Exercise 13.4.62 p. 1006
5. Exercise 13.4.63 p. 1006
6. Exercise 13.4.64 p. 1006
7. Exercise 13.4.65 p. 1006