Name : ____________________ **Class Number:** _____

Math 1D: Lesson 4 Suggested Problems

Theoretic Problems: Discussed in-class

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1. Let $f : D \subseteq \mathbb{R}^2 \longrightarrow \mathbb{R}$ be a continuous function on a polar rectangle

$$D = \{(r, \theta) \,:\, a \le r \le b \text{ and } \alpha \le \theta \le \beta\}$$

where $z = f(r, \theta)$ is given in polar coordinates.

   A. Explain how to set up the uniform discretization (a regular partition) of the polar rectangle D and how to enumerate subregions from $k = 1, 2, ..., n$.

   B. Explain how to choose a sample point (x_k^*, y_k^*) from the kth subregion of the partition using the midpoint rule.

   C. Explain how to translate the Riemann sum

$$\sum_{k=1}^{n} f(x_k^*, y_k^*)\ \Delta A_k$$

   into the integral by taking a limit with respect to Δ where Δ is the maximum value of Δr and $\Delta\theta$.

   D. Derive the formula for the area of the kth sector of our partition as $\Delta A_k = r_k^* \cdot \Delta r \cdot \Delta\theta$

   E. Using parts A - D above, explain why the differential form dA used to measure "sizes" of points encoded in polar coordinates has a factor of r in it where $dA = r \cdot dr \cdot d\theta$

   F. Now, explain why we define of the double integral of our function f on the polar rectangle as:

$$\iint_D f(r, \theta)\ dA = \int_\alpha^\beta A(\theta)\ d\theta \qquad \text{where } A(\theta) = \int_a^b f(r, \theta)\ \cdot r\ dr$$

$$= \int_\alpha^\beta \int_a^b f(r, \theta)\ \cdot r\ dr\ \ d\theta$$

   G. Interpret the function $A(\theta)$ as the area under a curve created by intersecting the surface $f(r, \theta)$ with the appropriate plane.

   H. Explain why $A(\theta)$ is a function of θ

   I. Explain why the integral that defines $A(\theta)$ is taken with respect to variable r.

Problems Solved in Jeff's Handwritten Notes

3. Example 13.3.1 p. 986
4. Example 13.3.3 p. 987
5. Example 13.3.5 p. 990

Suggested Problems: Answers in Back of Book

3. Example 13.3.2 p. 986 - 987
4. Example 13.3.4 parts a and b on p. 988 - 989
5. Exercise 13.3.31 p. 992
6. Exercise 13.3.55 p. 993

Optional Challenge Problems

3. Exercise 13.3.71 p. 994