Lesson 4: Handout
Reference: Brigg's "Calculus: Early Transcendentals, Second Edition"
Topics: Section 13.3: Double Integrals in Polar Coordinates, p. 984 - 994

Theorem 13.3. *p. 986* ***Double Integrals over Polar Rectangular Regions***

Let $f : D \subseteq \mathbb{R}^2 \to \mathbb{R}$ be a continuous function (where the domain is written in polar coordinates) on the region D in the $xy-$plane given by

$$D = \{(r, \theta) \,:\, 0 \leq a \leq r \leq b, \alpha \leq \theta \leq \beta\}$$

where $\beta - \alpha \leq 2\pi$. Then

$$\iint_D f(r,\theta) dA = \int_\alpha^\beta \int_a^b f(r,\theta) \cdot r \; dr \, d\theta$$

Warning: Do NOT forget the factor of r in the differential form $dA = r \; dr \, d\theta$ that results from measuring "sizes" of polar regions

Theorem 13.4. *p. 988* ***Double Integrals over More General Polar Regions***

Let $f : D \subseteq \mathbb{R}^2 \to \mathbb{R}$ be a continuous function (where the domain is written in polar coordinates) on the region D in the $xy-$plane given by

$$D = \{(r, \theta) \,:\, 0 \leq g(\theta) \leq r \leq h(\theta), \alpha \leq \theta \leq \beta\}$$

where $\beta - \alpha \leq 2\pi$. Then

$$\iint_D f(r,\theta) \, dA = \int_\alpha^\beta \int_{g(\theta)}^{h(\theta)} f(r,\theta) \cdot r \; dr \, d\theta$$

Theorem. *p. 989* ***Areas of Polar Region***

The area of the region D in the $xy-$plane, written in polar coordinates, defined by

$$D = \{(r, \theta) \,:\, 0 \leq g(\theta) \leq r \leq h(\theta), \alpha \leq \theta \leq \beta\}$$

is given by the double integral

$$\text{area of } D = \iint_D 1 \, dA = \int_\alpha^\beta \int_{g(\theta)}^{h(\theta)} r \; dr \, d\theta$$