Name : ________________________ **Class Number:** ______

Math 1D: Lesson 2 Suggested Problems

Theoretic Problems: Discussed in-class

---

1. Let $f : D \subseteq \mathbb{R}^2 \longrightarrow \mathbb{R}$ be a continuous function on a nonrectangular region encoded in cartesian coordinates. Derive the limit definition for the double integral of a function:

$$\iint\limits_D f(x,y)\ dA = \lim_{\Delta \to 0} \sum_{k=1}^{n} f(x_k^*, y_k^*)\ \Delta A_k$$

   A. Explain how to set up the general partition of the rectangular region D and to enumerate subregions from $k = 1, 2, ..., n$.

   B. Explain how to choose a sample point (x_k^*, y_k^*) from the kth subregion of the partition.

   C. Explain how to translate the Riemann sum

$$\sum_{k=1}^{n} f(x_k^*, y_k^*)\ \Delta A_k$$

   into the integral by taking a limit with respect to Δ where Δ is the maximum diagonal length of the subregions.

---

2. Let $f : D \subseteq \mathbb{R}^2 \longrightarrow \mathbb{R}$ be a continuous function on a nonrectangular, $y-$simple region encoded in cartesian coordinates

$$D = \{(x,y)\ :\ a \le x \le b \text{ and } g(x) \le y \le h(x)\}$$

Recall the definition of the iterated integral:

$$\iint\limits_D f(x,y)\ dA = \int_a^b A(x)\ dx \qquad \text{where } A(x) = \int_{g(x)}^{h(x)} f(x,y)\ dy$$

$$= \int_a^b \int_{g(x)}^{h(x)} f(x,y)\ dy\ \ dx$$

   A. Interpret the function $A(x)$ as the area under a curve created by intersecting the surface $f(x,y)$ with a plane having a normal vector $\mathbf{n} = \langle 1, 0, 0\rangle$ going through the point $(x, 0, 0)$.

   B. Explain why $A(x)$ is a function of x

   C. Explain why the integral that defines $A(x)$ is taken with respect to variable y.

3. Let $f : D \subseteq \mathbb{R}^2 \longrightarrow \mathbb{R}$ be a continuous function on a nonrectangular, $x-$simple region encoded in cartesian coordinates

$$D = \{(x, y) \, : \, g(y) \leq x \leq h(y) \text{ and } c \leq y \leq d\}$$

Recall the definition of the iterated integral:

$$\iint\limits_D f(x,y)\ dA = \int\limits_c^d A(y)\ dy \qquad \text{where } A(y) = \int\limits_{g(y)}^{h(y)} f(x,y)\ dx$$

$$= \int\limits_c^d \int\limits_{g(y)}^{h(y)} f(x,y)\ dx\ \ dy$$

A. Interpret the function $A(y)$ as the area under a curve created by intersecting the surface $f(x, y)$ with a plane having a normal vector $\mathbf{n} = \langle 0, 1, 0 \rangle$ going through the point $(0, y, 0)$.

B. Explain why $A(y)$ is a function of y.

C. Explain why the integral that defines $A(y)$ is taken with respect to variable x.

Problems Solved in Jeff's Handwritten Notes

---

3. Example 13.2.1 p. 974 - 975
4. Example 13.2.2 p. 976
5. Example 13.2.6 p. 980

Suggested Problems: Answers in Back of Book

---

3. Exercise 13.2.23 p. 981
4. Exercise 13.2.33 p. 981
5. Exercise 13.2.47 p. 981
6. Exercise 13.2.63 p. 982
7. Exercise 13.2.79 p. 983

Optional Challenge Problems

---

3. Exercise 13.2.88 p. 983