Name : ____________________ **Class Number:** _____

Math 1D: Lesson 1 Suggested Problems

Theoretic Problems: Discussed in-class

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1. Let $f : D \subseteq \mathbb{R}^2 \longrightarrow \mathbb{R}$ be a continuous function on a rectangular region

$$D = \{(x, y) \,:\, a \leq x \leq b \text{ and } c \leq y \leq d\}$$

Derive the limit definition for the double integral of a function:

$$\iint\limits_D f(x, y)\ dA = \lim_{\Delta \to 0} \sum_{k=1}^{n} f(x_k^*, y_k^*)\ \Delta A_k$$

   A. Explain how to set up the general partition of the rectangular region D and to enumerate subregions from $k = 1, 2, ..., n$.

   B. Explain how to choose a sample point (x_k^*, y_k^*) from the kth subregion of the partition.

   C. Explain how to translate the Riemann sum

$$\sum_{k=1}^{n} f(x_k^*, y_k^*)\ \Delta A_k$$

   into the integral by taking a limit with respect to Δ where Δ is the maximum diagonal length of the subregions.

2. Let $f : D \subseteq \mathbb{R}^2 \longrightarrow \mathbb{R}$ be a continuous function on a rectangular region

$$D = \{(x, y) \,:\, a \leq x \leq b \text{ and } c \leq y \leq d\}$$

Recall the definition of the iterated integral:

$$\iint\limits_D f(x, y)\ dA = \int_a^b A(x)\ dx \qquad \text{where } A(x) = \int_c^d f(x, y)\ dy$$

$$= \int_a^b \int_c^d f(x, y)\ dy\ \ dx$$

   A. Interpret the function $A(x)$ as the area under a curve created by intersecting the surface $f(x, y)$ with a plane having a normal vector $\mathbf{n} = \langle 1, 0, 0 \rangle$ going through the point $(x, 0, 0)$.

   B. Explain why $A(x)$ is a function of x

   C. Explain why the integral that defines $A(x)$ is taken with respect to variable y.

Problems Solved in Jeff's Handwritten Notes

3. Example 13.1.1 and 13.1.2 p. 965 - 967
4. Example 13.1.3 p. 968 - 969
5. Example 13.1.4 p. 969

Suggested Problems: Answers in Back of Book

3. Exercise 13.1.9 p. 971
4. Exercise 13.1.17 p. 971
5. Exercise 13.1.27 p. 971
6. Exercise 13.1.51 p. 972