Name : ____________________ **Class Number**: _____

Exam 2, Version 1A
Math 1D: Calculus IV

What are the rules of this exam?

- PLEASE DO NOT TURN THIS PAGE UNTIL TOLD TO DO SO!
- It is a violation of the Foothill Academic Integrity Code to, in any way, assist another person in the completion of this exam. Please keep your own work covered up as much as possible during the exam so that others will not be tempted or distracted. Thank you for your cooperation.
- No notes (other than your note card), books, or classmates can be used as resources for this exam.
- Please turn off your cell phones during this exam. No cell phones will be allowed on your desk.

How long is this exam?

- This exam is scheduled for a 110 minute class period.
- Make sure you have 5 sheets of paper (10 pages front and back) including this cover page.
- There are a total of 6 separate questions (50 points) on this exam including:
  - 5 Free-Response Questions (50 points)
  - 1 Optional, Extra Credit Challenge Problem (5 points)

What can I use on this exam?

- You may use up as many note sheets that are no larger then 11 inches by 8.5 inches as you would like. You may write on both sides of these note sheets. PLEASE SUBMIT ALL OF YOUR NOTE SHEETS WITH YOUR EXAM.
- You are allowed to use calculators for this exam. Examples of acceptable calculators include TI 83, TI 84, and TI 86 calculators. You are not allowed to use any calculator with a Computer Algebra System including TI 89 and TI NSpire. If you have a question, please ask your instructor about this.

How will I be graded on the Free-Response Questions?

- Read the directions carefully. Show all your work for full credit. In most cases, a correct answer with no supporting work will NOT receive full credit. What you write down and how you write it are the most important means of getting a good score on this exam. Neatness and organization are IMPORTANT!
- You will be graded on proper use of integral notation.

Use for Scratch Work

1. Consider the single-variable, vector-valued function:

$$\mathbf{r}(t) = \langle x(t), y(t) \rangle$$

Suppose that the functions $x(t)$ and $y(t)$ have continuous derivatives for all $t \in [a, b]$. Let's construct the limit definition of the derivative $\mathbf{r}'(t)$.

A. (6 points) Find two points on the curve C, call them P_0 and P. Construct the vector-valued equation for the secant line through these two points.

B. (6 points) Using the proper limit, construct the derivative vector $\mathbf{r}'(t)$ as the vector that defines the direction of the tangent line. Also, explain how we know that

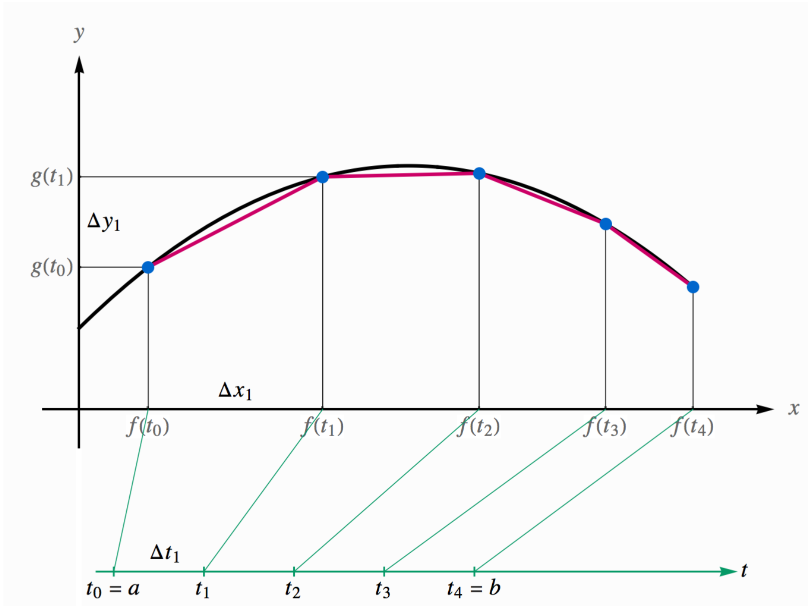
$$\mathbf{r}'(t) = \langle\ x'(t)\,,\ y'(t)\ \rangle$$

2. Suppose that we have a parameterized curve $C = \{\, \mathbf{r}(t) \,:\, a \le t \le b \,\}$ where $\mathbf{r}(t) = \langle x(t), y(t) \rangle$. Suppose that the functions $x(t)$ and $y(t)$ have continuous derivatives for all $t \in [a, b]$.

   A. (6 points) Let L represent the total arc length of the curve C. Using the diagram below, explain why we can approximate L, the arc length of the curve C, as

$$L \approx \sum_{k=1}^{n} \sqrt[2]{\,(\Delta x_k)^2 + (\Delta y_k)^2}$$

   Notice that in the diagram, we assume that $n = 4$.



B. (6 points) Explain how to use the mean value theorem and a limit to derive the precise arc length formula

$$L = \int_a^b \|\mathbf{r}'(t)\|_2 \ dt$$

Make sure to explain how the limit affects each term in the approximation and results in the stated definition for calculating the arc length L of the curve C.

3. (8 points) Evaluate the integral:

$$\iiint\limits_{D} e^{\left(x^2+y^2+z^2\right)^{3/2}} \; dV$$

where D is the unit ball in $\mathbb{R}^3$ centered at the origin. (Hint: try a change of variables into spherical coordinates).

4. (8 points) Compute the line integral:

$$\int_C f \ ds$$

where $f : D \subseteq \mathbb{R}^2 \longrightarrow \mathbb{R}$ is given by $f(x, y) = e^{x+y}$ and C is the line segment from the point $O(0, 0)$ to the point $P(2, 1)$

5. (10 points) Compute the circulation of the vector field

$$F(x, y) = \langle 2y, -2x \rangle$$

along the unit circle C, oriented counterclockwise.

Optional Challenge Problem

6. (5 points) State and prove the fundamental theorem of calculus.