Name : ______________________ **Class Number**: _____

Exam 1, Version 1A
Math 1D: Calculus IV

What are the rules of this exam?

- PLEASE DO NOT TURN THIS PAGE UNTIL TOLD TO DO SO!
- It is a violation of the Foothill Academic Integrity Code to, in any way, assist another person in the completion of this exam. Please keep your own work covered up as much as possible during the exam so that others will not be tempted or distracted. Thank you for your cooperation.
- No notes (other than your note card), books, or classmates can be used as resources for this exam.
- Please turn off your cell phones during this exam. No cell phones will be allowed on your desk.

How long is this exam?

- This exam is scheduled for a 110 minute class period.
- Make sure you have 5 sheets of paper (10 pages front and back) including this cover page.
- There are a total of 6 separate questions (50 points) on this exam including:
  - 5 Free-Response Questions (50 points)
  - 1 Optional, Extra Credit Challenge Problem (5 points)

What can I use on this exam?

- You may use up to three note sheets that are no larger then 11 inches by 8.5 inches. You may write on both sides of these note sheets. PLEASE SUBMIT ALL OF YOUR NOTE SHEETS WITH YOUR EXAM.
- You are allowed to use calculators for this exam. Examples of acceptable calculators include TI 83, TI 84, and TI 86 calculators. You are not allowed to use any calculator with a Computer Algebra System including TI 89 and TI NSpire. If you have a question, please ask your instructor about this.

How will I be graded on the Free-Response Questions?

- Read the directions carefully. Show all your work for full credit. In most cases, a correct answer with no supporting work will NOT receive full credit. What you write down and how you write it are the most important means of getting a good score on this exam. Neatness and organization are IMPORTANT!
- You will be graded on proper use of integral notation.

1. Let $f : D \subseteq \mathbb{R} \longrightarrow \mathbb{R}$ be a continuous function on an interval $D = \{x \, : \, a \leq x \leq b\} \subseteq \mathbb{R}$. In this problem, we will derive the limit definition for the single integral of a function:

$$\int_a^b f(x) \; dx = \lim_{\Delta \to 0} \sum_{k=1}^{n} f(x_k^*) \; \Delta x_k$$

   A. (6 points) Explain how to set up a general partition of D, how to choose a sample input value x_k^* from the kth subregion of the partition of the region D and to enumerate subregions from $k = 1, 2, ..., n$.

B. (6 points) Explain how to translate the Riemann sum $\sum_{k=1}^{n} f(x_k^*)\,\Delta x_k$ into the integral by taking a limit with respect to Δ where Δ is the maximum size of the subregions. With this in mind, please explain each symbol in integral notation.

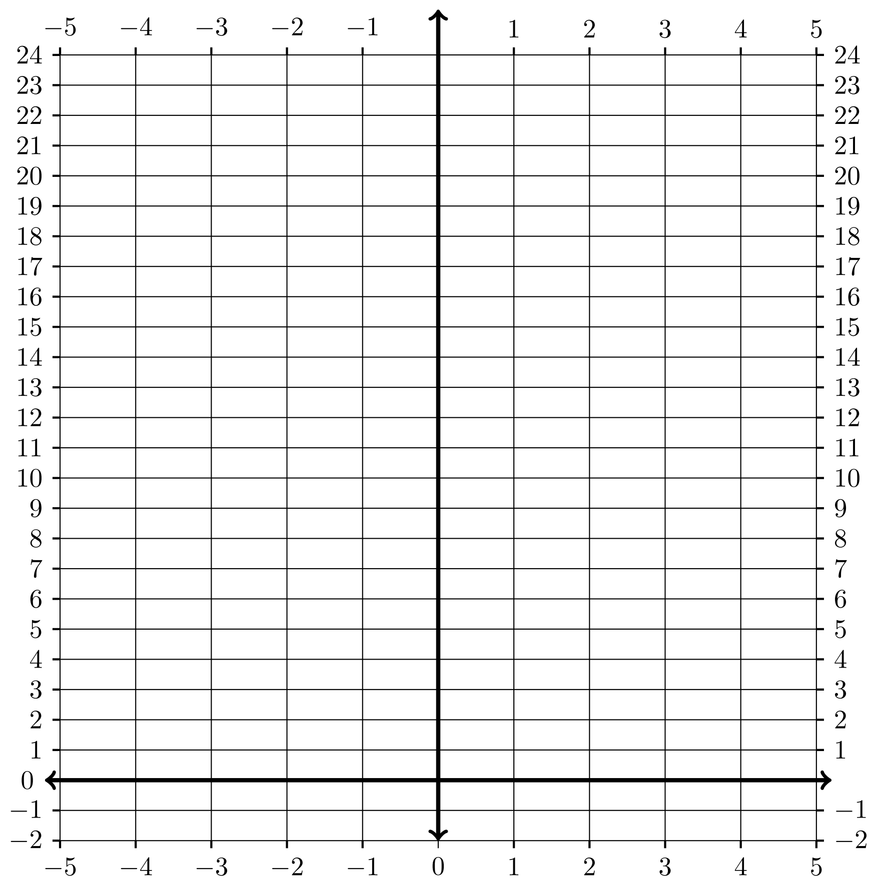
2. Consider the following integral

$$\iint_D f(x, y)\ dA$$

where the integrand $f(x, y) = x + y$ and $D \subseteq \mathbb{R}^2$ is the region bounded below by $y = |x|$ and above $y = 20 - x^2$

A. (6 points) Fill out the table below and sketch the region of integration

x	$y = \|x\|$	$y = 20 - x^2$
-5		
-4		
-3		
-2		
-1		
0		
1		
2		
3		
4		
5		



B. (6 points) Evaluate the integral: $\iint\limits_D f(x,y)\, dA$ described in problem 2 above.

3. (6 points) Evaluate the following integral:

$$\int_{-1}^{1} \int_{-1}^{2} \int_{0}^{1} 6\,x\,y\,z \;\; dy\; dx\; dz$$

4. Let $f : D \subseteq \mathbb{R}^2 \longrightarrow \mathbb{R}$ be a continuous function on a polar rectangle

$$D = \{(r, \theta) \, : \, a \leq r \leq b \text{ and } \alpha \leq \theta \leq \beta\}$$

where $z = f(r, \theta)$ is given in polar coordinates.

A. (6 points) Explain how to set up the uniform discretization (a regular partition) of the polar rectangle D and derive the formula for the area of the kth sector of our partition as $\Delta A_k = r_k^* \cdot \Delta r \cdot \Delta\theta$

B. (6 points) Now, explain why we define of the double integral of our function f on the polar rectangle as:

$$\iint\limits_D f(r,\theta)\ dA = \int\limits_{\alpha}^{\beta} \int\limits_{a}^{b} f(r,\theta)\ \cdot r\ dr\ \ d\theta$$

and explain why the integral has a factor of r in the differential form dA.

5. (8 points) Use a double integral to find the volume of the solid bounded between the paraboloids:

$$z = x^2 + y^2 \qquad \text{and} \qquad z = 64 - 4x^2 - 4y^2.$$

Be sure to explain your reasoning and show your work.

Optional Challenge Problem

6. (5 points) Show that

$$\int_{-\infty}^{\infty} e^{-x^2}\,dx = \sqrt{\pi}.$$