**Name** : ______________ © Jeffrey A. Anderson **Class Number**: ____

# **Sample Exam** 2
## Math 1C: Calculus III

**What are the rules of this exam?**

- PLEASE DO NOT TURN THIS PAGE UNTIL TOLD TO DO SO!
- It is a violation of the Foothill Academic Integrity Code to, in any way, assist another person in the completion of this exam. Please keep your own work covered up as much as possible during the exam so that others will not be tempted or distracted. Thank you for your cooperation.
- No notes (other than your note card), books, or classmates can be used as resources for this exam.
- Please turn off your cell phones during this exam. No cell phones will be allowed on your desk.

**How long is this exam?**

- This exam is scheduled for a 110 minute class period.
- Make sure you have 5 sheets of paper (10 pages front and back) including this cover page.
- There are a total of 24 separate questions (100 points) on this exam including:
  - 5 True/False Questions (10 points)
  - 15 Multiple Choice Questions (60 points)
  - 3 Free-Response Questions (30 points)
  - 1 Optional, Extra Credit Challenge Problem (10 points)

**What can I use on this exam?**

- You may use one note card that is no larger then 11 inches by 8.5 inches. You may write on both sides of this note card. PLEASE SUBMIT YOUR NOTECARD WITH YOUR EXAM.
- You are allowed to use calculators for this exam. Examples of acceptable calculators include TI 83, TI 84, and TI 86 calculators. You are not allowed to use any calculator with a Computer Algebra System including TI 89 and TI NSpire. If you have a question, please ask your instructor about this.

**How will I be graded on the Free-Response Questions?**

- Read the directions carefully. Show all your work for full credit. In most cases, a correct answer with no supporting work will NOT receive full credit. What you write down and how you write it are the most important means of getting a good score on this exam. Neatness and organization are IMPORTANT!
- You will be graded on proper use of notation as discussed in our lesson notes.

## True/False

(15 points: 3 points each) For the problems below, circle T if the answer is true and circle F is the answer is false. After you've chosen your answer, mark the appropriate space on your Scantron form. Notice that letter A corresponds to true while letter B corresponds to false.

1. T F If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ converges.

2. T F The series $\sum_{n=1}^{\infty} 3ne^{-n^2}$ diverges

3. T F If $f$ has a local minimum at point $(a, b) \in \mathbb{R}^2$, then

$$D_{\mathbf{u}f(a,b)} = 0$$

for any unit vector $\mathbf{u} \in \mathbb{R}^2$.

4. T F If $a_n > 0$ and $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} (-1)^n a_n$ converges.

5. T F Suppose $f : \mathbb{R}^3 \to \mathbb{R}$. If $\nabla f = 0$ at a point $\mathbf{x} \in \mathbb{R}^3$, then $f$ has a local extreme value at point $\mathbf{x}$.

## Multiple Choice

(45 points: 3 points each) For the problems below, circle the correct response for each question. After you've chosen your answer, mark your answer on your Scantron form.

6. Let $z = \sin(x \cdot y)$ and let $x = x(t)$ and $y = y(t)$ be functions of $t$. Suppose

$$x(1) = 0, \qquad y(1) = 1, \qquad x'(1) = 2, \qquad y'(1) = 3.$$

Find $\dfrac{dz}{dt}$ when $t = 1$.

A. 1 B. 2 C. 3 D. 4 E. 5

7. The series $\sum_{n=0}^{\infty} r^n$ converges if and only if:

A. $-1 < r < 1$ B. $-1 \le r \le 1$ C. $-1 \le r < 1$ D. $-1 < r \le 1$ E. $r < 1$

8. Find the direction of maximum increase of the function $f(x, y, z) = x\,e^{-y} + 3\,z$ at the point $(1, 0, 4)$.

A. $\begin{bmatrix} -1 \\ -1 \\ 3 \end{bmatrix}$ B. $\begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}$ C. $\begin{bmatrix} -1 \\ 3 \\ 3 \end{bmatrix}$ D. $\begin{bmatrix} -1 \\ -3 \\ 3 \end{bmatrix}$ E. $\begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$

9. Which of the following series converge?

$$1)\sum_{n=1}^{\infty} \frac{1}{n} \qquad 2)\sum_{n=1}^{\infty} \frac{(-1)^n \cdot n}{\ln(n)} \qquad 3)\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$$

A. 1 B. 2 C. 3 D. 2, 3 E. None

10. Find the limit of the sequence $a_n = 2 + \left(-\frac{4}{5}\right)^n$:

A. 2 B. $\frac{6}{5}$ C. $-\frac{4}{5}$ D. $-2$ E. $\frac{4}{5}$

11. Find the shortest distance from the origin to the surface $z^2 = 2xy + 2$

A. $\dfrac{1}{\sqrt{2}}$ B. $\sqrt{2}$ C. $\dfrac{1}{2}$ D. 2 E. 1

12. The series $\sum_{n=1}^{\infty} \dfrac{1}{n^\alpha}$ converges if and only if

A. $1 < \alpha$ B. $\alpha < 1$ C. $-1 < \alpha < 1$ D. $\alpha \geq 1$ E. $-1 < \alpha$

13. Find the minimum value of the function $f(x, y) = x\,y$ subject to the constraint that $x^2 + y^2 = 2$:

A. 1 B. 2 C. $-1$ D. $\dfrac{3}{2}$ E. $-\dfrac{3}{2}$

14. Find the values of $x$ for which the series $\sum_{n=1}^{\infty} (x-1)^n$ :

A. $-2 < x < 0$ B. $0 < x \leq 2$ C. $0 < x < 2$ D. $0 \leq x \leq 2$ E. $-2 \leq x < 0$

15. Find the directional derivative of the function

$$f(x, y) = y^2 \cdot \ln(x)$$

at the point $(1, 2)$ in the direction of the vector $(3, 4) = 3\mathbf{i} + 4\mathbf{j}$:

A. $\dfrac{5}{16}$ B. 12 C. $\dfrac{5}{12}$ D. $\dfrac{16}{5}$ E. $\dfrac{12}{5}$

16. Find an equation of the tangent plane to the surface $\sqrt{x}+\sqrt{y}+\sqrt{z}=4$ at the point $(4,1,1)$.

A. $2x+y-z=1$
B. $x+2y+2z=8$
C. $x-2y+4z=0$
D. $x+y+z=6$
E. $2x+y+z=10$

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17. Which of the following series converge?

$$1)\sum_{n=1}^{\infty}\frac{3^{2n}}{2^{3n}} \qquad 2)\sum_{n=1}^{\infty}\frac{1}{(n+1)^3} \qquad 3)\sum_{n=1}^{\infty}\frac{n+1}{\sqrt{n^3+2}}$$

A. None   B. 1   C. 2   D. 3   E. $2,3$

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18. Determine how many critical points the function $f(x,y)=x\,y-x^2\,y-x\,y^2$ has:

A. 1   B. 2   C. 3   D. 4   E. 5

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19. How many terms of the alternating series

$$\sum_{n=1}^{\infty}(-1)^{n+1}n^{-2}$$

must we add in order to be sure that the partial sum $s_n$ is within 0.0001 of the sum $s$.

A. 10   B. 300   C. 30   D. 100   E. 1000

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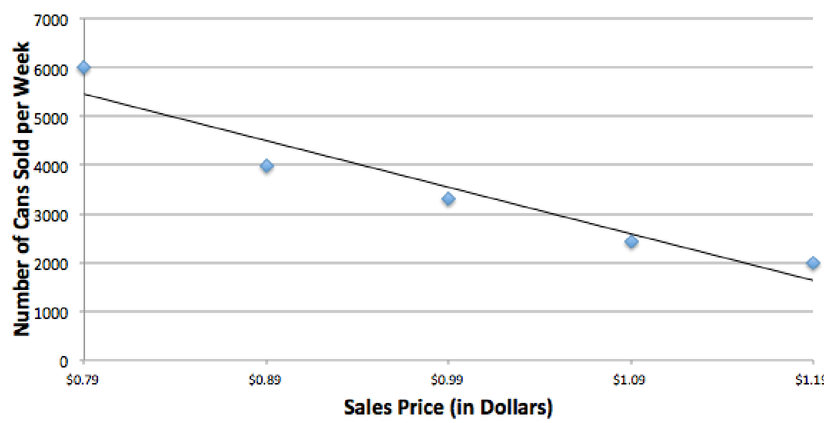
20. Let $f(x,y)=\dfrac{x}{y}+\dfrac{y}{x}$. Find the gradient vector $\nabla f$:

A. $\begin{bmatrix}2y\\2x\end{bmatrix}$   B. $\begin{bmatrix}x\\y\end{bmatrix}$   C. $\begin{bmatrix}\dfrac{1}{y}-\dfrac{y}{x^2}\\[2ex]\dfrac{1}{x}-\dfrac{x}{y^2}\end{bmatrix}$   D. $\begin{bmatrix}-y/x^2\\-x/y^2\end{bmatrix}$   E. $\begin{bmatrix}y\\x\end{bmatrix}$

## Free Response

21. (10 points) A company test-markets a new canned energy drink made of all natural ingredients in 5 cities of equal size on the West Coast of the US. The selling price (in dollars) and the number of drinks sold per week is each of the cities is listed as follows

| City | Price | Sales/Week |
|---|---|---|
| 1 | 0.79 | 6000 |
| 2 | 0.89 | 3980 |
| 3 | 0.99 | 3300 |
| 4 | 1.09 | 2440 |
| 5 | 1.19 | 1990 |




This data is plotted in the figure next to the table above. Although the data do not exactly lie on a straight line, we can create a linear model to fit this data.

A. Set up the least squares problem to fit this data to a linear model

$$S(p) = c_1 + c_2 p$$

where $S$ is the sales per week and $p$ is the price.

B. Explain how you would use multivariable calculus to find the line of best fit.

22. (10 points)

A. Find the value of $\sum_{n=2}^{\infty} \frac{3^n + 5^n}{15^n}$. Show your work.

B. Show that series $\sum_{n=2}^{\infty} \frac{1}{n \cdot (\ln(n))^p}$ converges if $p > 1$ and diverges if $p \leq 1$.

23. (10 points) Let $f : \mathbb{R}^2 \to R$ be a differentiable, multivariable function. Let $\mathbf{u}$ be a unit vector.

A. Derive the dot product formula for the limit definition of the directional derivative $D_{\mathbf{u}}f$.

B. Prove that the gradient vector is in the direction of maximum increase.

## Challenge Problem

24. (Optional, Extra Credit, Challenge Problem)

Suppose that a sequence $\{a_n\}_{n=1}^{\infty}$ satisfies

$$0 < a_n \leq a_{2n} + a_{2n+1}$$

for all $n \geq 1$. Prove that the series $\sum_{n=1}^{\infty} a_n$ diverges.

**Use for Scratch Work**