Name : ______________________ © Jeffrey A. Anderson **Class Number**: _____

Sample Exam 1
Math 1C: Calculus III

What are the rules of this exam?

- PLEASE DO NOT TURN THIS PAGE UNTIL TOLD TO DO SO!
- It is a violation of the Foothill Academic Integrity Code to, in any way, assist another person in the completion of this exam. Please keep your own work covered up as much as possible during the exam so that others will not be tempted or distracted. Thank you for your cooperation.
- No notes (other than your note card), books, or classmates can be used as resources for this exam.
- Please turn off your cell phones during this exam. No cell phones will be allowed on your desk.

How long is this exam?

- This exam is scheduled for a 110 minute class period.
- Make sure you have 6 sheets of paper (12 pages front and back) including this cover page.
- There are a total of 20 separate questions (100 points) on this exam including:
  - 5 True/False Questions (10 points)
  - 15 Multiple Choice Questions (60 points)
  - 3 Free-Response Questions (30 points)
  - 1 Optional, Extra Credit Challenge Problem (10 points)

What can I use on this exam?

- You may use one HANDWRITTEN note sheer that is no larger then 11 inches by 8.5 inches. You may write on both sides of this note card. PLEASE SUBMIT YOUR NOTECARD WITH YOUR EXAM.
- You are allowed to use calculators for this exam. Examples of acceptable calculators include TI 83, TI 84, and TI 86 calculators. You are not allowed to use any calculator with a Computer Algebra System including TI 89 and TI NSpire. If you have a question, please ask your instructor about this.

How will I be graded on the Free-Response Questions?

- Read the directions carefully. Show all your work for full credit. In most cases, a correct answer with no supporting work will NOT receive full credit. What you write down and how you write it are the most important means of getting a good score on this exam. Neatness and organization are IMPORTANT!
- You will be graded on proper use of the notation we discussed in our lessons.

True/False

(10 points: 2 points each) For the problems below, circle T if the answer is true and circle F is the answer is false. After you've chosen your answer, mark the appropriate space on your Scantron form. Notice that letter A corresponds to true while letter B corresponds to false.

1. T F $f_y(a,b) = \lim_{y \to b} \frac{f(a,y) - f(a,b)}{y - b}$.

2. T F For any vectors $\mathbf{u}, \mathbf{v} \in \mathbb{R}^3$, $(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{u} = 0$.

3. T F The set of points $\{(x,y,z) \,:\, x^2 + y^2 = 1\}$ is a circle.

4. T F If $f(x,y) \to L$ as $(x,y) \to (a,b)$ along every straight line through (a,b),, then

$$\lim_{(x,y)\to(a,b)} f(x,y) = L.$$

5. T F If $\mathbf{x}, \mathbf{y} \in \mathbb{R}^3$, then $|\mathbf{x} \cdot \mathbf{y}| \leq \|\mathbf{x}\|_2 \, \|\mathbf{y}\|_2$

Multiple Choice

(60 points: 4 points each) For the problems below, circle the correct response for each question. After you've chosen your answer, mark your answer on your Scantron form.

6. Which of the following is a unit vector point in the direction of vector $\mathbf{x} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$?

A. $\begin{bmatrix} 3 \\ -2 \end{bmatrix}$ B. $\frac{1}{\sqrt{13}} \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ C. $\frac{1}{\sqrt{13}} \begin{bmatrix} 3 \\ -2 \end{bmatrix}$ D. $\frac{1}{\sqrt{5}} \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ E. $\begin{bmatrix} 2/3 \\ 1 \end{bmatrix}$

7. Consider the vectors

$$\mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}, \qquad \mathbf{y} = \begin{bmatrix} -2 \\ 0 \\ 4 \end{bmatrix}.$$

Which of the following vectors gives $\mathbf{x} \cdot \mathbf{y}$?

A. 14 B. 10 C. $-12\mathbf{k}$ D. -10 E. -14

8. Find values of $b \in \mathbb{R}$ such that the vectors $\begin{bmatrix} -11 \\ b \\ 2 \end{bmatrix}$ and $\begin{bmatrix} b \\ b^2 \\ b \end{bmatrix}$ are orthogonal.

A. $0, 11, -3$ B. $0, -11, 2$ C. $0, 2, -2$ D. $0, 3, -3$ E. $0, 11, 2$

9. Given $\mathbf{x} = (4, 0)$ and $\mathbf{y} = (5, 2)$, which of the following is the projection of vector $\mathbf{x}$ onto the vector $\mathbf{y}$?

A. $(5, 0)$ B. $\left(\frac{40}{27}, \frac{16}{27}\right)$ C. $\left(\frac{100}{29}, \frac{40}{29}\right)$ D. $(4, 2)$ E. $\left(\frac{100}{\sqrt{29}}, \frac{40}{\sqrt{29}}\right)$

10. Consider the vectors

$$\mathbf{x} = \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix} = 2\mathbf{i} + 3\mathbf{k}, \qquad \mathbf{y} = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} = \mathbf{j} - \mathbf{k}.$$

Which of the following vectors gives $\mathbf{x} \times \mathbf{y}$?

A. $-3\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$ B. $-3\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$ C. -3 D. $-3\mathbf{k}$ E. $3\mathbf{i} - 2\mathbf{j} - 2\mathbf{k}$

11. Find an equation of the line through the point $(1, 2, 3)$ and parallel to the plane $x - y + z = 100$:

A. $x - y + z - 2 = 0$ B. $x - y + z + 2 = 0$ C. $x + 2y + 3z = 100$

D. $x - 1 = 2 - y = z - 3$ E. $x - 1 = \dfrac{y+1}{2} = \dfrac{z-1}{3}$

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12. Given $f(x, y) = \sqrt{x^2 + y^2}$, find f_{xx} :

A. $\dfrac{xy}{\left(x^2+y^2\right)^{1/2}}$ B. $\dfrac{x^2}{\left(x^2+y^2\right)^{3/2}}$ C. $\dfrac{y^2}{\left(x^2+y^2\right)^{3/2}}$ D. $\dfrac{y}{\left(x^2+y^2\right)^{1/2}}$ E. $\dfrac{x}{\left(x^2+y^2\right)^{1/2}}$

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13. Find an equation for the line through the point $(3, -1, 2)$ and perpendicular to the plane $2x - y + z + 10 = 0$.

A. $\dfrac{x-3}{2} = \dfrac{y+1}{-1} = z - 2$ B. $\dfrac{x+2}{2} = \dfrac{y-1}{-1} = z - 2$ C. $\dfrac{x-2}{3} = \dfrac{y+1}{-1} = \dfrac{x-2}{2}$

D. $3x - y + 2z + 10 = 0$ E. $3x - 2y + z + 10 = 0$

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14. Find the area of the triangle with vertices at the points $(0, 0, 0)$, $(1, 0, -1)$ and $(1, -1, 2)$.

A. $\sqrt{11}$ B. $\sqrt{6}$ C. $\dfrac{\sqrt{6}}{2}$ D. $\dfrac{\sqrt{11}}{2}$ E. 1

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15. The equation of the sphere with center $(4, -1, 3)$ and radius $\sqrt{5}$ is

A. $(x-4)^2 + (y+1)^2 + (z-3)^2 = 5$

B. $(x-4)^2 + (y+1)^2 + (z-3)^2 = \sqrt{5}$

C. $(x-4)^2 + (y+1)^2 + (z-3)^2 = 25$

D. $(x+4)^2 + (y-1)^2 + (z+3)^2 = 5$

E. $(x-4)^2 + (y-1)^2 + (z-3)^2 = 5$

16. Find the distance between the point $(-1, -1, -1)$ and the plane $x + 2y + 2z - 1 = 0$

A. 0 B. 6 C. -2 D. -6 E. 2

17. Let $f(x, y) = e^{\sin(x)} + x^5 y + \ln\left(1 + y^2\right)$. Find f_{yx} :

A. $5x^4$ B. $\dfrac{2y}{1+y^2}$ C. $20x^3 y$ D. $e^{\sin(x)}\cos(x)$ E. $e^{\sin(x)}\cos(x) + x^5 + \dfrac{2y}{1+y^2}$

18. Find the limit $\displaystyle\lim_{(x,y)\to(0,0)} \frac{2x^4 y^2}{x^4 + 3y^4}$

A. 2 B. $\dfrac{2}{3}$ C. 0 D. $\dfrac{1}{2}$ E. Does NOT exist

19. Find the parametric equations of the intersection of the planes $x - z = 0$ and $x - y + 2z + 3 = 0$

A. The line given by $x(t) = -2 - t, y(t) = 1 - 3t$ and $z(t) = -t$.
B. The line given by $x(t) = 1 + t, y(t) = 6 - t$ and $z(t) = 1 + 2t$.
C. The plane $3x + 3y - 3z + 3 = 0$
D. The line given by $x(t) = -t, y(t) = 3 - 3t$ and $z(t) = -t$.
E. The line given by $x(t) = 1 + t, y(t) = 6$ and $z(t) = 1 - t$.

20. Given $\mathbf{x} = (2, 0, 1)$ and $\mathbf{v} = (4, 1, 2)$, what is the area of the parallelogram formed by the vectors $\mathbf{x}$ and $\mathbf{y}$?

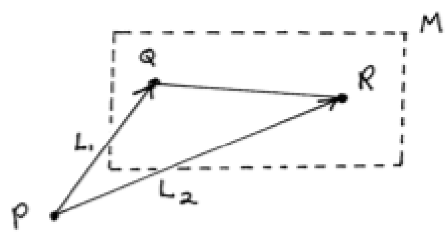
A. $2\sqrt{5}$ B. $\sqrt{5}$ C. $2\sqrt{3}$ D. $3\sqrt{2}$ E. $4\sqrt{2}$

Free Response

21. Consider the lines

$$\mathbf{L}_1(t) = \begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix} = \begin{bmatrix} t \\ 2t \\ 1-t \end{bmatrix} \qquad \text{and} \qquad \mathbf{L}_2(s) = \begin{bmatrix} x(s) \\ y(s) \\ z(s) \end{bmatrix} = \begin{bmatrix} 1-s \\ 2+s \\ -2s \end{bmatrix}$$

and the plane M given by equation $10x - 2y + 3z = 0$. The line $\mathbf{L}_1(t)$ intersects plane M at point Q. The line $\mathbf{L}_2(s)$ intersects plane M at point R. Lines $\mathbf{L}_1(t)$ and $\mathbf{L}_2(s)$ intersect at point P. Compute the area of the triangle PQR. The diagram below may help you visualize this problem.



22. Let $z = z(x, y)$ be defined implicitly by equation

$$x^2 + y^2 - z^2 = 3xyz$$

Compute $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ at point $(3, 1, 1)$.

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23. A. Use a scalar projection to show that the distance from point $P(x_1, y_1)$ to line $ax + by + c = 0$ is

$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}.$$

Draw a diagram and explain your reasoning in detail using full sentences.

B. Use this formula to find the distance from the point $(-2, 3)$ to the line $3x - 4y + 5 = 0$.

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Challenge Problem

24. (Optional, Extra Credit, Challenge Problem) Let A be the area of the region in the first quadrant of the cartesian plane bounded by the line $y = \frac{1}{2}x$, the x-axis, and the ellipse $\dfrac{1}{9}x^2 + y^2 = 1$. Find the positive number m such that A is equal to the area of the region in the first quadrant bounded by the line $y = mx$, the y-axis and the ellipse $\dfrac{1}{9}x^2 + y^2 = 1$.

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Use for Scratch Work