Name : ____________________ **Class Number:** ______

Math 1C: Lesson 9 Suggested Problems

Theoretic Problems: Discussed in-class

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1. Let $f : D \subseteq \mathbb{R} \longrightarrow \mathbb{R}$ be a single-variable, real-valued function that is differentiable on the domain D. Derive the limit definition of the ordinary derivative of f at input a, given by:

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

Make sure to discuss all five steps we used in this process. Your solution should demonstrate a strong concept image associated with this idea that includes verbal, spatial, and symbolic descriptions linked to the concept definition known as the ordinary derivative. Remember, the stronger your concept image is associated with this foundational idea, the easier other types of derivatives will be as we work to generalize this concept to different contexts related to multivariable functions.

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2. Let $f : D \subseteq \mathbb{R}^2 \longrightarrow \mathbb{R}$ be a two-variable, real-valued function that is differentiable on the domain D. Discuss the five step process for constructing the partial derivative of f with respect to x. In particular:

   A. Discuss each of the five steps used to derive the limit definition of the partial derivative in slope notation given by

   $$f_x(a, b) = \lim_{x \to a} \frac{f(x, b) - f(a, b)}{x - a}$$

   Make sure to explicitly describe this concept geometrically and refer back to lines that exist in the intersection between the surface $z = f(x, y)$ and the plane $y = b$.

   B. Explain, in detail, how this limit definition of $f_x(a, b)$ relates to Path 1 for taking multivariable limits from Lesson 8.

   C. Using the proper algebraic substitution, create the limit definitions of the partial derivatives in derivative notation given by

   $$f_x(a, b) = \lim_{h \to 0} \frac{f(a + h, b) - f(a, b)}{h}$$

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3. Let $f : D \subseteq \mathbb{R}^2 \longrightarrow \mathbb{R}$ be a two-variable, real-valued function that is differentiable on the domain D. Discuss the five step process for constructing the partial derivative of f with respect to y. In particular:

   A. Discuss each of the five steps used to derive the limit definition of the partial derivative in slope notation given by

   $$f_y(a, b) = \lim_{y \to b} \frac{f(a, y) - f(a, b)}{y - b}$$

   Make sure to explicitly describe this concept geometrically and refer back to lines that exist in the intersection between the surface $z = f(x, y)$ and the plane $x = a$.

   B. Explain, in detail, how this limit definition of $f_y(a, b)$ relates to Path 2 for taking multivariable limits from Lesson 8.

C. Using the proper algebraic substitution, create the limit definitions of the partial derivatives in derivative notation given by

$$f_y(a,b) = \lim_{h\to 0} \frac{f(a,b+h) - f(a,b)}{h}$$

Problems Solved in Jeff's Handwritten Notes

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4. Let $f : D \subseteq \mathbb{R}^2 \longrightarrow \mathbb{R}$ be a two-variable, real-valued function given by

$$f(x,y) = x^3 + x^2y^3 - 2y^2$$

A. Find $f_x(x,y)$ and use this result to calculate $f_x(2,1)$.

B. Find $f_y(x,y)$ and use this result to calculate $f_y(2,1)$.

C. Find the higher-order derivatives f_{xx}, f_{xy}, f_{yx}, and f_{yy}

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5. Let $f : D \subseteq \mathbb{R}^2 \longrightarrow \mathbb{R}$ be a two-variable, real-valued function given by

$$f(x,y) = \sin\left(\frac{x}{1+y}\right)$$

Find $f_x(x,y)$ and $f_y(x,y)$.

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6. Let $f : D \subseteq \mathbb{R}^3 \longrightarrow \mathbb{R}$ be a three-variable, real-valued function given by

$$f(x,y,z) = e^{x\,y} \cdot \ln(z)$$

Find $f_x(x,y,z)$, $f_y(x,y,z)$, and $f_z(x,y,z)$.

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7. Let $f : D \subseteq \mathbb{R}^3 \longrightarrow \mathbb{R}$ be a three-variable, real-valued function given by

$$f(x,y,z) = 3\,x\,y\,\arcsin(y\,z)$$

Find $f_x(x,y,z)$, $f_y(x,y,z)$, and $f_z(x,y,z)$.

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8. Consider the following relation

$$x^3 + y^3 + z^3 + 6xyz = 1$$

Assume that $z = z(x,y)$ and use implicit differentiation to find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$.