lesson8 Activities.

1. Find the domain and range of $f(x,y)=\sqrt{80-5x^2-5y^2}$

Domain: vaild input

since $\sqrt{a}$, $a\geq 0$, we know: $80-5x^2-5y^2\geq 0$

$\iff x^2+y^2\leq 16$.

$Dom(f)=\{(x,y)\in\mathbb{R}^2 : x^2+y^2\leq 16\}$

domain of $f(x,y)$ is a disk center at $(0,0)$ and radius 4.

Range: since $f(x,y)=\sqrt{80-5x^2-5y^2}\geq 0$.

$\Rightarrow\quad 80-5x^2-5y^2\geq 0$ (square)

$\Rightarrow\quad 0\leq 5x^2+5y^2\leq 80$ (≥ 0 b/c $x^2+y^2\geq 0$).

$\Rightarrow\quad 0\leq x^2+y^2\leq 16$

$\Rightarrow\quad 0\geq 5(-x^2-y^2)\geq -80$ ($\times -1$ change sign).

$\Rightarrow\quad 80\geq 80-5x^2-5y^2\geq 0$

$\Rightarrow\quad 0\leq\sqrt{80-5x^2-5y^2}\leq 4\sqrt{5}$.

$Rng(f)=[0,4\sqrt{5}]$.

Finish #2 on Page 3

2. Evaluate limit $\lim\limits_{(x,y)\to(1,3)}\dfrac{\sqrt{x+y}-2}{x+y-4}$

limit law does not apply (quotient rule not applicable)

test path 1: $y=3$ $\quad\lim\limits_{(x,3)\to(1,3)}\dfrac{\sqrt{x+y}-2}{x+y-4}=\dfrac{(\sqrt{x+3}-2)(}{(x-1)(\sqrt{x}}$

test path 2: $x=1$ $\quad\lim\limits_{(1,y)\to(1,3)}\dfrac{\sqrt{x+y}-2}{x+y-4}=\dfrac{\sqrt{1+y}-2}{y-3}$

since $\dfrac{\sqrt{x+3}-2}{x-1}\neq\dfrac{\sqrt{1+y}-2}{y-3}$

By two path test, the limit Does Not Exist.

4:00 PM – 5 PM @#2!

1/27 3:40 PM – 4:00 PM.

3. use 2-path test to prove limit DNE: $\lim\limits_{(x,y)\to(0,0)} \frac{y^4-2x^2}{y^4+x^2}$

solution: path1: $y=0$ $\lim\limits_{(x,0)\to(0,0)} \frac{0-2x^2}{0+x^2} = \boxed{-2}$

path2: $x=0$ $\lim\limits_{(0,y)\to(0,0)} \frac{y^4-0}{y^4+0} = \boxed{1}$

since two path leads to different limit, the limit DNE.

4. For what values of m does $\lim\limits_{(x,y)\to(0,0)} \frac{3xy^2}{y^4+x^2} = 0.6$ along path $x=my^2$?

solution:

$$\lim_{(my^2,y)\to(0,0)} \frac{3xy^2}{y^4+x^2} = \frac{3my^4}{y^4+m^2y^4} = \frac{y^4(3m)}{y^4(1+m^2)}$$

let $\frac{3m}{1+m^2} = 0.6$, $\boxed{m = \frac{5\pm\sqrt{21}}{2}}$

5. Evaluate the limit $\lim\limits_{(x,y)\to(0,0)} \frac{y^2}{x^4+y^2} = 0.6$

solution:

path1: $y=0$ $\lim\limits_{(x,0)\to(0,0)} \frac{0}{x^4+0} = 0$

path2: $x=0$ $\lim\limits_{(0,y)\to(0,0)} \frac{y^2}{0+y^2} = 1$

since two path leads to different limit, the limit DNE.

lesson 8 Page 2/3

1/28 1:36PM – 1:44PM. 15min

1:48PM – 1:55 PM

6. Find the limit $\lim\limits_{(x,y)\to(0,0)} \dfrac{\sin(x^2+y^2)}{x^2+y^2}$

Solution:

let $x^2+y^2=r^2$, replace x^2+y^2 with r^2

$\Rightarrow \lim\limits_{(r^2\to 0)} \dfrac{\sin(r^2)}{r^2}$ let $u=r^2$

$\Rightarrow \lim\limits_{(\theta\to 0)} \dfrac{\sin\theta}{\theta} = \boxed{1}$

Note:

Prove $\lim\limits_{x\to 0} \dfrac{\sin x}{x} \overset{H}{=} \lim\limits_{x\to 0} \dfrac{\cos x}{1} = 1$

(use L' Hopital's Rule).

2. Evaluate the limit: $\lim\limits_{(x,y)\to(1,3)} \dfrac{\sqrt{x+y}-2}{x+y-4}$

Solution:

since $x=1, y=3$ leads to $\frac{0}{0}$ inconclusive,

we want to cancel the term $(x+y-4)$ on bottom.

Let's use conjugate:

$$\lim_{(x,y)\to(1,3)} \frac{\sqrt{x+y}-2}{x+y-4} = \lim_{(x,y)\to(1,3)} \frac{(\sqrt{x+y}-2)(\sqrt{x+y}+2)}{(x+y-4)(\sqrt{x+y}+2)}$$

$$= \lim_{(x,y)\to(1,3)} \frac{\cancel{x+y-4}}{\cancel{(x+y-4)}(\sqrt{x+y}+2)}$$

Note: $(x+y-4)$ is cancelled!

(Then we plug in $x=1$, $y=3$,)

$= \boxed{\frac{1}{4}}$.