Name : _____ **Class Number:** _____

Math 1C: MD Lesson 12 Suggested Problems

Theoretic Problems: Discussed in-class

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1. **Derive the equation for a tangent plane** Suppose we are given a surface in $\mathbb{R}^3$ that is defined implicitly as a level surface using equation

$$F(x, y, z) = 0$$

where the function $F : D \to \mathbb{R}$ is differentiable for all domain values in $D \subseteq \mathbb{R}^3$.

   A. Derive the equation for the tangent plane to the level surface at point (a, b, c) on this surface. Make sure to mention the geometric interpretation of the gradient vector with respect to the tangent plane.

   B. Suppose we have a surface in $\mathbb{R}^3$ defined by function $z = f(x, y)$ for differentiable function f. Derive the equation for a tangent plane to this surface at the point $(a, b, f(a, b))$ by relating this situation back to your work in part A.

Problems Solved in Jeff's Handwritten Notes

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2. Consider the implicit relation for the ellipsoid defined by equation

$$\frac{x^2}{9} + \frac{y^2}{25} + z^2 - 1 = 0.$$

   Using this equation, please do the following:

   A. Find the equation for the tangent plane to the ellipsoid at the point $\left(0, 4, \frac{3}{5}\right)$

   B. Find any point(s) on the ellipsoid with a horizontal tangent plane.

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3. Find the equation of the tangent plane to the elliptic paraboloid defined by the explicit equation

$$z = f(x, y) = 2x^2 + y^2$$

   at the point $(1, 1, 3)$.

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4. Define a two-variable, real-valued function given by

$$f(x, y) = \frac{5}{x^2 + y^2}$$

   Find the linear approximation to f at the point $(-1, 2, 1)$.

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5. Find the linear approximation to $f(x, y) = x\, e^{x\, y}$ at the point $(1, 0, 1)$.

Suggested Problems

6. Find the equations for the tangent planes to the surface given by

$$z^2 - \frac{x^2}{16} - \frac{y^2}{9} - 1 = 0$$

at point $P_1(4, 3, -\sqrt{3})$ and point $P_2(-8, 9, \sqrt{14})$.

7. Find the point(s) at which the surface given by equation

$$x^2 + 2y^2 + z^2 - 2x - 2z = 0$$

has a horizontal tangent plane.

Optional Challenge Problems

8. Exercise 12.7.60 p. 938
9. Exercise 12.7.62 p. 938
10. Exercise 12.7.66 p. 938