Name : ____________ Math 1C, Lesson 11: In-Class Problems **Class Number**: ______

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1. **Calculating and interpreting the Directional Derivative**

   A. Derive the limit definition for directional derivative. Make explicit connections between this limit definition and the limit definition of the ordinary derivative. Explain how these definitions utilize a limiting process to transform the slope of a secant line into the slope of a tangent line at a particular point.

   B. Relate the limit definition of the directional derivative to the limit definition of partial derivatives. In particular, find the direction vector $\mathbf{u}$ such that

   $$D_{\mathbf{u}}f(a,b) = f_x(a,b) \qquad \text{or} \qquad D_{\mathbf{u}}f(a,b) = f_y(a,b)$$

   c. Derive, using the multivariable chain rule, the dot product formula for the directional derivative.

   d. Use the cosine formula for the dot product and the dot product formula for the directional derivative to explain why the direction of the gradient is the direction steepest ascent. Interpret this result as it relates to the given surfaces.

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2. **The Gradient and Level Curves** The level curves to the surface $z = x^2 + y^2$ are circles in $\mathbb{R}^2$ centered at the origin. Consider each of the following input points:

   A. $P(3, -2)$

   B. $P(-2, -1)$

   C. $P(-5, 12)$

   Use this explicit equation for the elliptic paraboloid and the given points to do the following:

   i. Graph the contour curve and level curve associated with each input point.

   ii. Determine gradient vector at each input point.

   iii. Graph the gradient vector on the level curves from part i. assuming that the tail of each vector is the given point P.

   iv. Find a parametric equation for the tangent line to the level curve at each point P

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3. **The Gradient and Steepest Descent** Consider the paraboloid

   $$f(x,y) = 16 - \frac{x^2}{4} - \frac{y^2}{16},$$

   Let P be the point on a given level curve of $f(x,y)$, as described in each part below. Compute the slope of the line tangent to the level curve at P and verify that the tangent line is orthogonal to the gradient at that point.

   A. Level curve $f(x,y) = 0$ including point $P(0, 16)$

   B. Level curve $f(x,y) = 12$ including point $P(4, 0)$

   C. Level curve $f(x,y) = 12$ including point $P(2\sqrt{3}, 4)$