Lesson 0: Welcome to Math 1C

- ☐ Intake survey & student names
- ☐ Review class webpage & online resources
- ☐ Handout Study Skills HW 1 (due at start of class 2)
- ☐ Lesson 0 Lecture content

Major Themes of Math 1C

Part 1: Multivariable Differentiation

Let $f : D \longrightarrow \mathbb{R}$ be a multivariable function where $D \subseteq \mathbb{R}^2$ or $D \subseteq \mathbb{R}^3$.

Notation: This symbol is the "subset" symbol, written as

$$\subseteq$$

in English, we read "D is a subset of $\mathbb{R}^3$"

In part 1 of math 1C, we study partial derivatives and the gradient operator.

We will construct the necessary tools to find

2-Dimensional Input : $\underbrace{\vec{\nabla} f}_{\text{the gradient of function } f(x,y)}(x,y) = \langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \rangle$

the partial derivative of f with respect to y

the partial derivative of f with respect to x

3-Dimensional Input : $\vec{\nabla} f(x,y,z) = \langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \rangle$

the partial derivative of f with respect to z

Partial derivatives and the gradient operator generalize the techniques for finding ordinary derivatives of single variable functions.

The gradient of a function f with multiple inputs is the multidimensional version of a "derivative". We will use this tool to find maximum & minimum values of multivariable functions (think least squares).

Story-line: So far in your calculus classes, you've studied two powerful operations

Single Variable:

$$\frac{d}{dx}\Big[\underbrace{F(x)}_{\text{given}}\Big] = \underbrace{f(x)}_{\text{unknown derivative function}} = F'(x)$$

$\longleftarrow$ Math 1A studies the "forward" problem of ordinary differentiation

$$\frac{d}{dx}\Big[\underbrace{F(x)}_{\text{unknown antiderivative function}}\Big] = \underbrace{f(x)}_{\text{given derivative function}} = F'(x)$$

$\longleftarrow$ Math 1B studies the "backward"/inverse problem

Recall: In Math 1B we used special notation to study our inverse problem

$$\frac{d}{dx}\big[F(x)\big] = f(x) \iff F(x) = \int_{x_0}^{x} f(\tau)\, d\tau$$

These are now part of the "standard" operations you can use in your professional life

Add/subtract : $+/-$, Exponentiate $x^n / x^{1/n}$, $n \in \mathbb{R}$

Multi/Divide : $\cdot / \div$, Exp / Logarithms $a^x / \log_a(x)$ ...

Multivariable: $\vec{\nabla}[F(x,y)] = \vec{f}(x,y)$ ← Part 1 of Math 1C studies the "forward" problem of partial differentiation

given multivariable function

unknown vector-valued "derivative" function

$\vec{\nabla}[F(x,y)] = \vec{f}(x,y)$ ← Math 1D studies the backward problem

unknown

given vector-valued "derivative" function (known as a vector field)

Thus, in part 1 of this class we will explore how to find partial derivatives and the gradient:

$$\vec{\nabla}[F(x,y)] = \left\langle \frac{\partial F}{\partial x}, \frac{\partial F}{\partial y} \right\rangle.$$

Example 1: Modeling Real-World Data with Least Squares

A very famous application of multivariable differentiation theory is known as the least-squares problem. Let's consider a subset of 14 data points from the famous Keeling Curve data of CO_2 particles in the atmosphere near the Mauna Loa observatory in Hawaii:

$$(x_1, y_1) = (1980, 338.7)$$

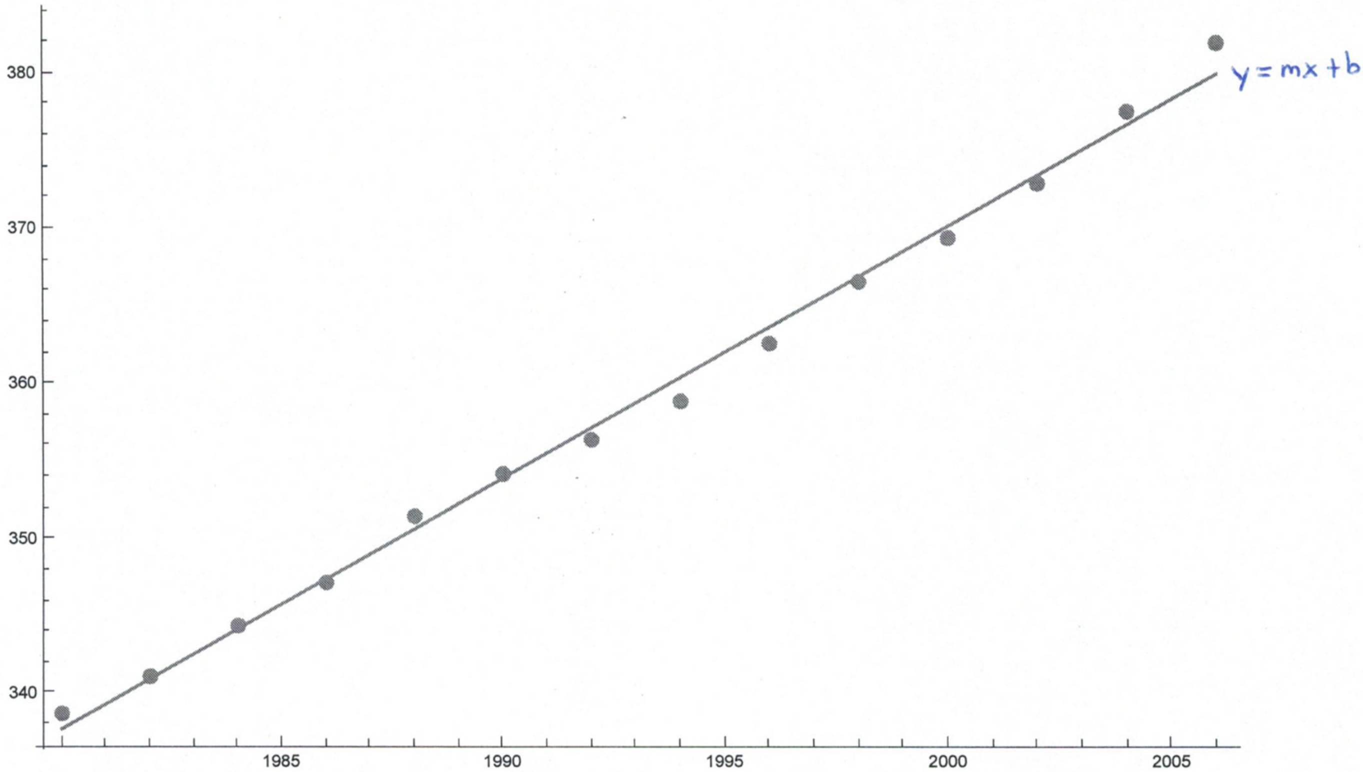
$$(x_2, y_2) = (1982, 341.1)$$

$$(x_3, y_3) = (1984, 344.4)$$

$$\vdots$$

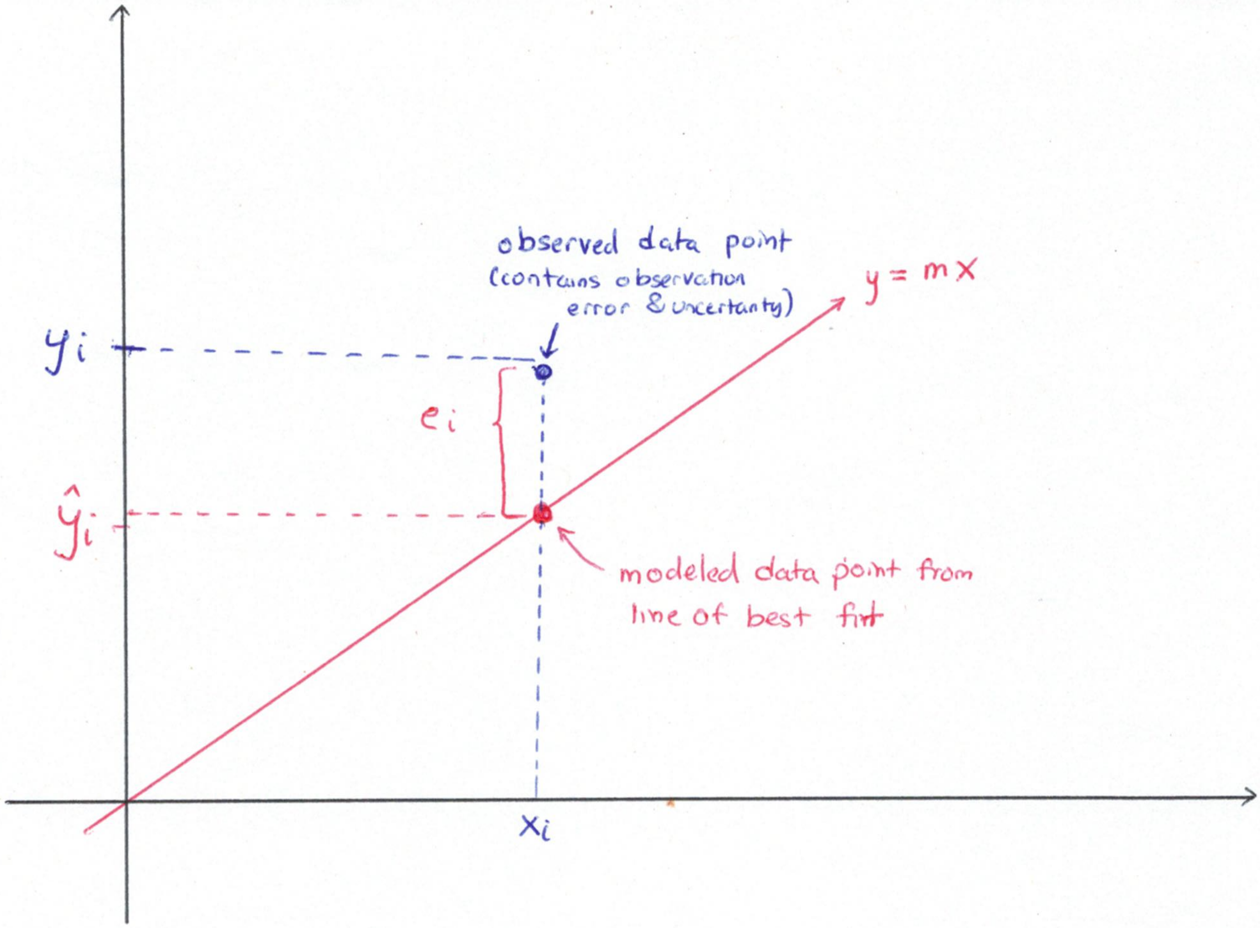
$$(x_{14}, y_{14}) = (2006, 381.9)$$

For more about this data, see the Mathematica notebook associated with Lesson 0.



Above, we graph the 14 points of this data set on a single axis and look for a pattern. We notice the data seems to "approximately" follow a linear model given by $y = m \cdot x + b$.

However, the unknown slope m and unknown y-intercept b need to be chosen to "best" fit the collected data. This is where we do something clever.



We defined the error between our collected data output y_i and our modeled output $\hat{y}_i = m x_i + b$ to be

$$e_i = y_i - \hat{y}_i$$

$$= y_i - (m x_i + b)$$

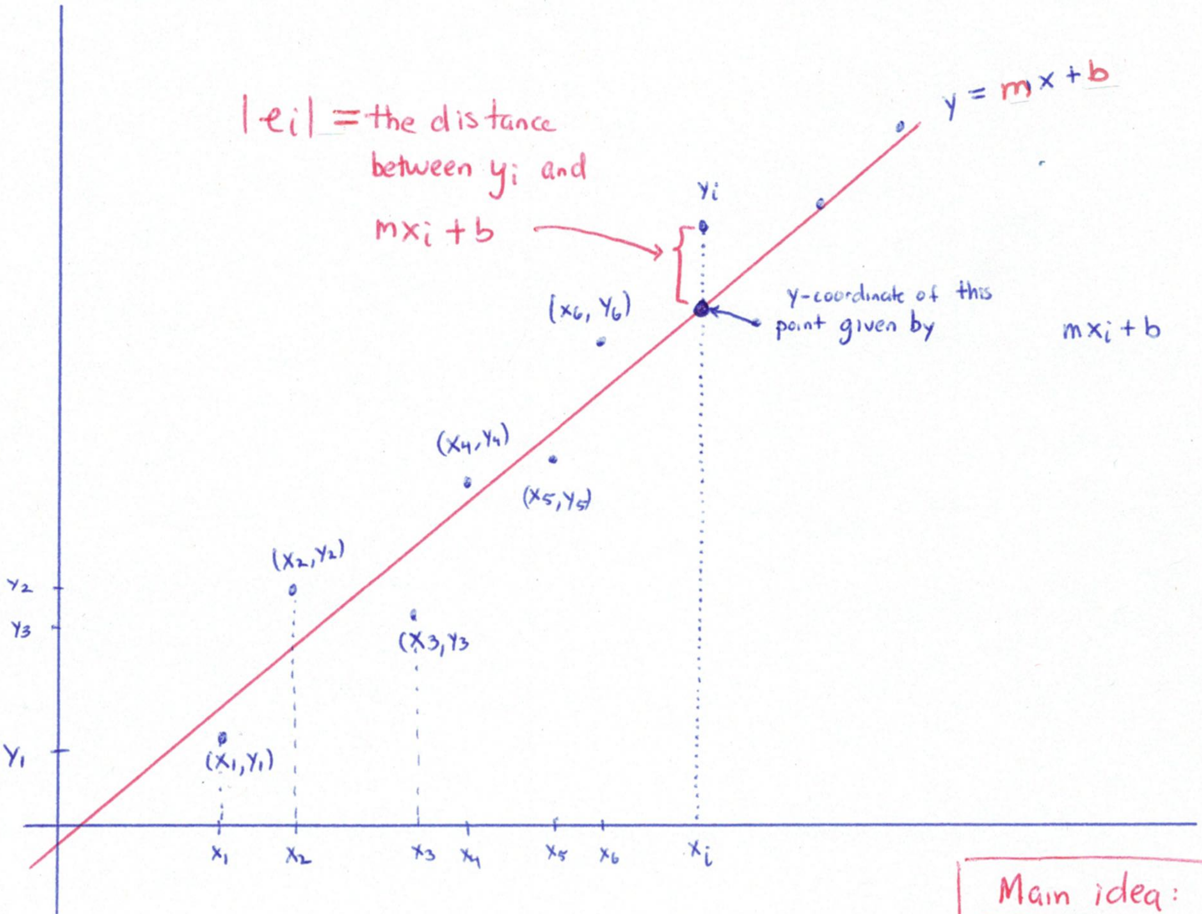
$$= y_i - m x_i - b$$

Here we stick with most general formulation for simplicity. In the case we want to force $b=0$, we will enhance this general formulation with special techniques (for more take statistics) :)

Example 2, continued...)

We notice that this data is probally fit by model

$$y = \underset{\text{unknown slope}}{m}\, x + \underset{\text{unknown y-intercept}}{b}$$



Main idea:

find m, b to fit the given data is a multivariable problem

Define $e_i = \underset{\text{given}}{y_i} - (\overset{\text{unknown}}{m}\, \underset{\text{given}}{x_i} + \overset{\text{unknown}}{b})$

Find $m, b \in \mathbb{R}$ such that $\sum e_i^2$ is minimum

Two independent, real valued inputs

$$f(m, b) = \left[\sum_{i=1}^{14} \left(y_i - (m x_i + b) \right)^2 \right]$$ ← single real number output

Again, we see in the general case, we will ~~f~~ define

a function

□ A fantastic question here is why do we use

$\sum (e_i)^2$ and not $\sum |e_i|$

error squared

absolute value of error

$$f(m,b) = \sum_{i=1}^{14} [e_i]^2$$

$$= \sum_{i=1}^{14} [y_i - \hat{y}_i]^2$$

collected output from experiment

unknown output from desired best fit model

unknown slope of best fit line

unknown y-intercept of best fit line

$$= \sum_{i=1}^{14} [y_i - (m\, x_i + b)]^2$$

collected data input from experiment

Now the goal is to minimize $f(m,b)$:

□ Find $m, b \in \mathbb{R}$ such that $f(m,b)$ achieves a global minimum at those points.

Natural Question: How do we do this? Part 1 of this class will help us answer this question.