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1. Let $f(x)$ be a function. The graph below is of the derivative $f^{\prime}(x)$. Use it to solve this problem.

a. Identify the intervals on which $f(x)$ is concave up and concave down. Justify your answers.
b. Identify the intervals on which $f(x)$ is increasing and decreasing. Justify your answers.
c. Identify the extreme values of the function $f(x)$ using the second derivative test. Justify your answer.
2. A small bike company selling utility bicycles for daily commuting has been in business for four years. This company has recorded annual sales (in tens of thousands of dollars) as follows:


This data is plotted in the figure next to the table above. Although the data do not exactly lie on a straight line, we can create a linear model to fit this data.
a. For the set of 4 data points given above in the form $\left\{\left(t_{i}, y_{i}\right)\right\}_{i=1}^{4}$, assume this data can be approximately modeled by a line $y(t)=m t+b$. For each data point, describe how to form the error $e_{i}(m, b)$ between the collected output $y_{i}$ and the modeled output $y\left(t_{i}\right)$.
b. Using the model for the errors, set up the least-squares problem for this input data. In particular, create a two variable function $E_{2}(m, b)$ that you can use to solve create the "best-fit" model for this data. Explain in detail the choices that you made to construct your error function $E_{2}$.
c. Describe how we will use multivariable calculus to create the line of best fit.
3. A ride-sharing app called Super was recently released in 2009. During the first six years of it's business operations, the Super app has seen spectacular growth in its number of users:

| Year | Number of Users <br> (in Millions) |
| :---: | :---: |
| 0 | 0.0 |
| 1 | 0.1 |
| 2 | 0.8 |
| 3 | 1.5 |
| 4 | 2.3 |
| 5 | 3.2 |
| 6 | 5.3 |


a. For the set of 7 data points given above in the form $\left\{\left(t_{i}, y_{i}\right)\right\}_{i=1}^{7}$, assume this data can be approximately modeled by a quadratic function

$$
y(t)=a_{0}+a_{1} t+a_{2} t^{2}
$$

for unknown constants $a_{0}, a_{1}, a_{2}$. For each data point, describe how to form the error $e_{i}(m, b)$ between the collected output $y_{i}$ and the modeled output $y\left(t_{i}\right)$.
b. Using the model for the errors, set up the least-squares problem for this input data. In particular, create a two variable function $E_{2}\left(a_{0}, a_{1}, a_{2}\right)$ that you can use to solve create the "best-fit" model for this data. Explain in detail the choices that you made to construct your error function $E_{2}$.

