

1. Let f(x) be a function. The graph below is of the derivative f'(x). Use it to solve this problem.

- a. Identify the intervals on which f(x) is concave up and concave down. Justify your answers.
- b. Identify the intervals on which f(x) is increasing and decreasing. Justify your answers.
- c. Identify the extreme values of the function f(x) using the second derivative test. Justify your answer.

2. A small bike company selling utility bicycles for daily commuting has been in business for four years. This company has recorded annual sales (in tens of thousands of dollars) as follows:



This data is plotted in the figure next to the table above. Although the data do not exactly lie on a straight line, we can create a linear model to fit this data.

- a. For the set of 4 data points given above in the form  $\{(t_i, y_i)\}_{i=1}^4$ , assume this data can be approximately modeled by a line y(t) = mt + b. For each data point, describe how to form the error  $e_i(m, b)$  between the collected output  $y_i$  and the modeled output  $y(t_i)$ .
- b. Using the model for the errors, set up the least-squares problem for this input data. In particular, create a two variable function  $E_2(m, b)$  that you can use to solve create the "best-fit" model for this data. Explain in detail the choices that you made to construct your error function  $E_2$ .
- c. Describe how we will use multivariable calculus to create the line of best fit.

3. A ride-sharing app called Super was recently released in 2009. During the first six years of it's business operations, the Super app has seen spectacular growth in its number of users:



a. For the set of 7 data points given above in the form  $\{(t_i, y_i)\}_{i=1}^7$ , assume this data can be approximately modeled by a quadratic function

$$y(t) = a_0 + a_1 t + a_2 t^2$$

for unknown constants  $a_0, a_1, a_2$ . For each data point, describe how to form the error  $e_i(m, b)$  between the collected output  $y_i$  and the modeled output  $y(t_i)$ .

b. Using the model for the errors, set up the least-squares problem for this input data. In particular, create a two variable function  $E_2(a_0, a_1, a_2)$  that you can use to solve create the "best-fit" model for this data. Explain in detail the choices that you made to construct your error function  $E_2$ .